Shore School

2015

Year 11

Preliminary Task 5

Yearly Examination

Mathematics

Examination Number:

Set:

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board approved calculators may be used
- Answer Questions 1 – 10 on the Multiple Choice answer sheet provided
- Start each of Questions 11 – 14 in a new writing booklet
- In Questions 11−14, show relevant mathematical reasoning and/or calculations
- Write your examination number on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your examination number and "N/A"

Note: Any time you have remaining should be spent revising your answers.

Total marks – 70

Section I Pages 2−5

10 marks

- Attempt Questions 1−10
- Allow about 15 minutes for this section

Section II Pages 6−11

60 marks

- Attempt Questions 11−14
- Allow about 1 hour and 45 minutes for this section

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Section I

10 marks
Attempt Questions 1−10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1−10.

1 What is the simplified expression of $(x-3)^2 - 4x(x+5)$?

(A) $-3x^2 + 20x - 9$

(B) $-3x^2 + 14x + 9$

(C) $-3x^2 - 26x + 9$

(D) $-3x^2 - 20x - 9$

2 A watch sells for \$148.00, including 10% GST. What is the price before GST is added?

(A) \$13.45

(B) \$14.80

(C) \$133.30

(D) \$134.55

3 What is the factorised form of $x^3 - 8$?

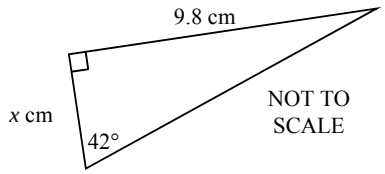
(A) $(x-2)(x^2+2x+4)$

(B) $(x-2)(x^2-2x+4)$

(C) $(x-2)(x^2+4x+4)$

(D) $(x-2)(x^2-4x+4)$

4 What is the correct expression for the value of x?



(A) $x = \dfrac{9.8}{\tan 42°}$

(B) $x = 9.8 \tan 42°$

(C) $x = \dfrac{9.8}{\sin 42°}$

(D) $x = 9.8 \sin 42°$

5 What is the gradient of the tangent to the curve $y = 4x^2$ at the point where $x = 3$?

(A) -24

(B) $-\dfrac{1}{24}$

(C) 24

(D) $\dfrac{1}{24}$

6 What is the perpendicular distance between the point $(-3, 7)$ and the line $2x - y + 5 = 0$?

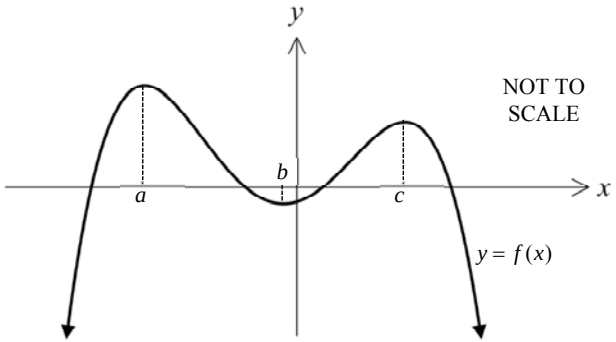
(A) $4\sqrt{3}$

(B) $\dfrac{8\sqrt{5}}{5}$

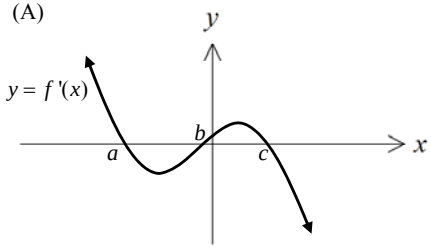
(C) $\dfrac{8\sqrt{3}}{3}$

(D) $\dfrac{4\sqrt{58}}{29}$

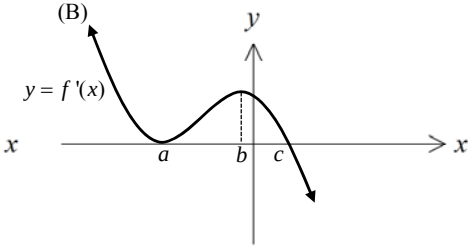
7 What is the graph of the gradient function for the curve $y = f(x)$?



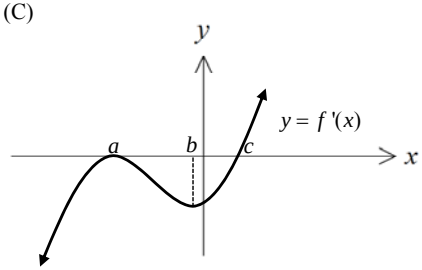
(A)



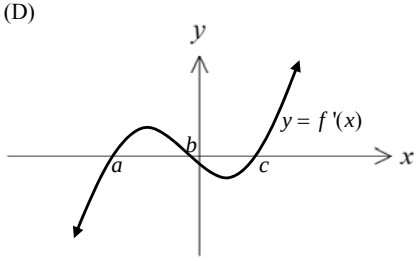
(B)



(C)



(D)



8 For what values of k will $x^2 + (k+2)x + 4 = 0$ have no real roots?

(A) $-6 \le k \le 2$

(B) $-6 < k < 2$

(C) $k \le -6,\ k \ge 2$

(D) $k < -6,\ k > 2$

9 How many solutions of the equation $\sin(2x) = 0.5$ lie between 0° and 360° ?

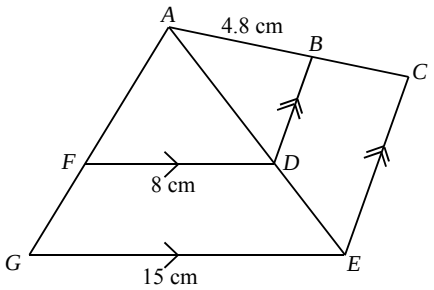
(A) 1

(B) 2

(C) 3

(D) 4

10 In the diagram $FD = 8$ cm, $GE = 15$ cm and $AB = 4.8$ cm.
What is the length of BC?



NOT TO SCALE

(A) 2.6 cm

(B) 4.2 cm

(C) 7.2 cm

(D) 9.0 cm

Section II

60 marks
Attempt Questions 11−14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11−14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Evaluate $\sqrt[3]{\dfrac{7\pi^2}{6}}$ correct to 3 significant figures. **2**

(b) Simplify $\dfrac{x^2-2x-8}{4y-12} \times \dfrac{y^2-9}{6x-24}$. **2**

(c) Solve $|x-8| = 3x-4$. **3**

(d) Express $\dfrac{5}{\sqrt{3}-1}$ in simplified form with a rational denominator. **2**

(e) State the domain and range for $y = \sqrt{49-x^2}$. **2**

(f) The function $g(x)$ is defined as: **2**

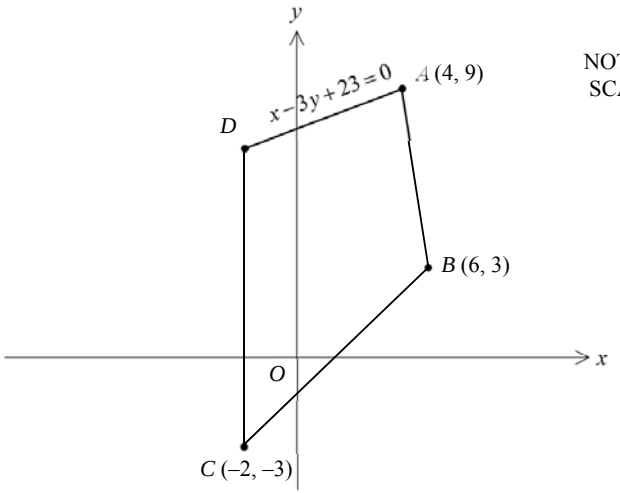
$$g(x) = \begin{cases} x^2 & \text{for} \quad -5 \le x \le 0 \\ x-3 & \text{for} \quad 0 < x < 2 \\ 5 & \text{for} \quad x \ge 2 \end{cases}$$

Evaluate $g(-3) - g(2) + g(1)$.

(g) Differentiate $\dfrac{2x-1}{3x+2}$. **2**

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) *ABCD* is a kite with vertices *A* (4, 9), *B* (6, 3), *C* (–2, –3) and *D*.
The line *AD* has equation $x-3y+23=0$.



NOT TO SCALE

(i) Find the distance *BC*. **1**

(ii) Show the equation of diagonal *AC* is $2x-y+1=0$ **2**

(iii) Given the line *CD* is parallel to the *y*-axis, find the coordinates of point *D*. **1**

(iv) Show that the diagonals of the kite are perpendicular. **1**

(b) Solve $\sqrt{3}\tan\theta=1$ for $0°\le\theta\le360°$. **2**

(c) The roots of the quadratic equation $3x^2+6x+k=0$ are α and β.

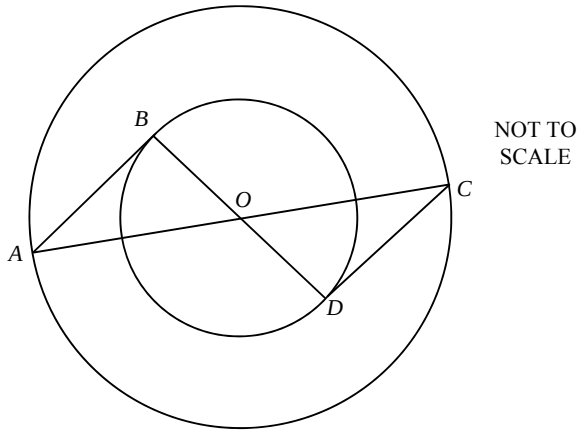
(i) Find the value of $\alpha+\beta$. **1**

(ii) Find the value of *k* if $\alpha^2+\beta^2=-6$. **3**

Question 12 continues on page 8

Question 12 (continued)

(d) Two circles have the same centre *O*. Lines *AC* and *BD* intersect at *O*.



NOT TO SCALE

(i) Prove $\Delta AOB\equiv\Delta COD$. **2**

(ii) Hence, or otherwise, show $AB\parallel CD$. **2**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet

(a) A function is defined by $f(x) = x^2 + 5x$.

(i) Show that $f(x+h) = x^2 + 2xh + h^2 + 5x + 5h$. **1**

(ii) Hence, use the formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to differentiate $f(x) = x^2 + 5x$ from first principles. **2**

(b) Consider the two equations $y = -x^2$ and $x + y + 6 = 0$.

(i) Solve the two equations simultaneously to find their points of intersection. **2**

(ii) Sketch $y = -x^2$ and $x + y + 6 = 0$ on the same number plane, labelling their points of intersection. **2**

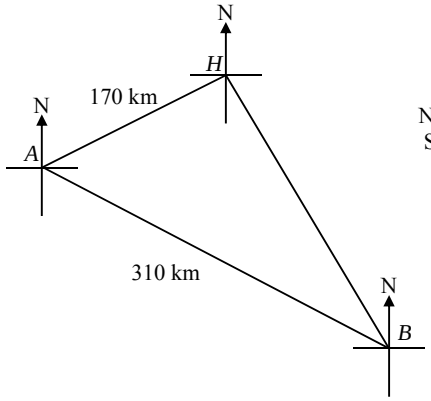
(c) Shade the region represented by $x^2 + y^2 \leq 25$ and $x < 3$. **2**
Do not find their points of intersection.

(d) Differentiate $f(x) = 5x^2(2x-1)^3$ leaving your answer in fully factorised form. **3**

(e) Find the equation of the normal to the curve $y = x^3 + 5x^2 - 4$ at the point where $x = 1$. **3**

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) Ship *A* and Ship *B* leave the same harbour, *H*. Ship *A* sails on a bearing of 254° for 170 km. Ship *B* sails on a bearing of 117° until it is 310 km from ship *A*.



NOT TO SCALE

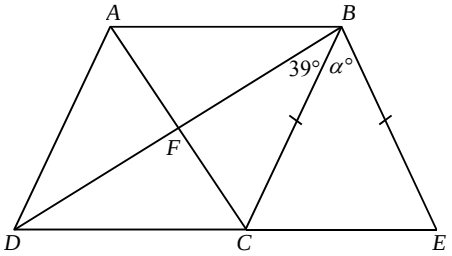
Copy or trace this diagram into your writing booklet.

(i) Show that $\angle AHB = 137°$. **1**

(ii) Hence, or otherwise, find the bearing of Ship *A* from Ship *B*. **3**
Give your answer correct to the nearest degree.

(b) Prove $\frac{(1-\cos\theta)(1+\cos\theta)}{\cos^2\theta} = \tan^2\theta$. **2**

(c) *ABCD* is a rhombus, with *DC* produced to *E*. $\angle FBC = 39°$ and $BC = BE$. **3**



NOT TO SCALE

Find the value of α giving reasons.

Question 14 continues on page 11

Question 14 (continued)

(d) Solve $9^x - 6(3^x) - 27 = 0$. **3**

(e) A curve has equation $y = kx^3 + 2$, where k is a constant. At the point where $x = 2$ the tangent to the curve is inclined at an angle of 60° with the positive direction of the x-axis. Find the exact value of k. **3**

END OF PAPER

Multi Choice

① C
② D
③ A
④ A
⑤ C
⑥ B
⑦ A
⑧ B
⑨ D
⑩ B

① $(x-3)^2 - 4x(x+5) = x^2 - 6x + 9 - 4x^2 - 20x$
$= -3x^2 - 26x + 9$ (C)

② $x \times 1.1 = 148.00$
$x = 134.55$ (D)

③ $x^3 - 8 = (x-2)(x^2+2x+4)$ (A)

④ $\tan 42 = \frac{9.8}{x}$
$x = \frac{9.8}{\tan 42}$ (A)

⑤ $y = 4x^2$
$\frac{dy}{dx} = 8x$
at $x=3$ $M = 8 \times 3$
$= 24$ (C)

⑥ $(-3, 7)$ $2x - y + 5 = 0$
$d = \frac{|2x-3 + -1x7 + 5|}{\sqrt{2^2 + (-1)^2}}$
$= \frac{|-8.1|}{\sqrt{5}}$
$= \frac{8}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$
$= \frac{8\sqrt{5}}{5}$ (B)

⑦ (A)

⑧ $x^2 + (k+2)x + 4 = 0$
$\Delta < 0$
$(k+2)^2 - 4x1x4 < 0$
$k^2 + 4k + 4 - 16 < 0$
$k^2 + 4k - 12 < 0$
critical points inequality.
$(k+6)(k-2) = 0$
$k = -6$, $k = 2$

$-6 < k < 2$ (B)

⑨ $\sin(2x) = 0.5$ over $0 \le 2x \le 720$
$2x = \sin^{-1}(0.5)$
$2x = 30, 150, 390, 510$
$x = 15, 75, 195, 255$
∴ 4 solutions (D)

⑩ $\frac{4.8 + 8c}{4.8} = \frac{15}{8}$
$4.8 + 8c = 9$
$8c = 4.2$ (B)

Question 11

a) $\sqrt{\frac{7\pi^2}{6}} = 2.258129\ldots$
$= 2.26$

b) $\frac{x^2-2x-8}{4y-12} \times \frac{y^2-9}{6x-24} = \frac{(x-4)(x+2)}{4(y-3)} \times \frac{(y-3)(y+3)}{6(x-4)}$
$= \frac{(x+2)(y+3)}{24}$

c) $|x-8| = 3x-4$

$x - 8 = 3x - 4$
$-4 = 2x$
$x = -2$
test LHS $= |-2-8|$
$= 10$
RHS $= 3x - 2 - 4$
$= -10$
∴ not a solution

$x - 8 = -(3x-4)$
$x - 8 = -3x + 4$
$4x = 12$
$x = 3$
LHS $= |3-8|$
$= 5$
RHS $= 3x3 - 4$
$= 5$
∴ is a solution

d) $\frac{5}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{5(\sqrt{3}+1)}{(\sqrt{3})^2 - 1^2}$
$= \frac{5(\sqrt{3}+1)}{2}$

e) $y = \sqrt{49 - x^2}$

domain $x: -7 \le x \le 7$
range $y: 0 \le y \le 7$

f) $g(x) = \begin{cases} x^2 & -5 \le x \le 0 \\ x-3 & 0 < x < 2 \\ 5 & x \ge 2 \end{cases}$

$g(-3) - g(2) + g(1) = (-3)^2 - 5 + (1-3)$

$= 9 - 5 - 2$

$= 2$

g) $\frac{d}{dx}\frac{2x-1}{3x+2} = \frac{vu' - uv'}{v^2}$ $\quad u = 2x-1$ $\quad v = 3x+2$

$u' = 2$ $\quad v' = 3$

$= \frac{2(3x+2) - 3(2x-1)}{(3x+2)^2}$

$= \frac{6x+4-6x+3}{(3x+2)^2}$

$= \frac{7}{(3x+2)^2}$

QUESTION 12

a) i) $d = \sqrt{(3+3)^2 + (6+2)^2}$

$= \sqrt{100}$

$= 10$

ii) $m_{AC} = \frac{9+3}{4+2}$

$= \frac{12}{6}$

$= 2$ $\quad (4, 9)$

$y - 9 = 2(x-4)$

$y - 9 = 2x - 8$

$0 = 2x - y + 1$

iii) x-value of $D = -2$

$\therefore x - 3y + 23 = 0$

$-2 - 3y + 23 = 0$

$21 = 3y$

$y = 7$

$\therefore D(-2, 7)$

iv) $m_{AC} = 2$.

$m_{BD} = \frac{3-7}{\quad}$

$= \frac{-4}{8}$

$= -\frac{1}{2}$

$m_{AC} \times m_{BC} = 2 \times -\frac{1}{2}$

$= -1$

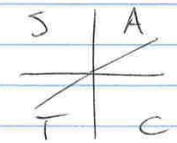
$\therefore$ perpendicular

b) $\sqrt{3}\tan\theta = 1$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\theta = 30°, 210°$



c) $3x^2 + 6x + k = 0$

i) $\alpha + \beta = -\frac{b}{a}$

$= -\frac{6}{3}$

$= -2$

ii) $\alpha^2 + \beta^2 = -6$

$(\alpha+\beta)^2 - 2\alpha\beta = -6$

$\alpha\beta = \frac{c}{a}$

$= \frac{k}{3}$

$\therefore (-2)^2 - 2\times\frac{k}{3} = -6$

$4 - \frac{2k}{3} = -6$

$-\frac{2k}{3} = -10$

$-2k = -30$

$k = 15$

d) i) In ΔAOB and ΔCOD

- $OB = OD$ (radii of small circle)
- $OA = OC$ (radii of large circle)
- $\angle BOA = \angle DOC$ (vertically opposite $\angle$s are equal)

$\therefore \Delta AOB \equiv \Delta COD$ (SAS)

ii) $\angle BAO = \angle DCO$ (matching $\angle$s in congruent Δs)

$\therefore AB \parallel CD$ (alternate $\angle$s are equal)

QUESTION 13

a) $f(x) = x^2 + 5x$

i) $f(x+h) = (x+h)^2 + 5(x+h)$

$= x^2 + 2xh + h^2 + 5x + 5h$

ii) $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 5x + 5h - (x^2 + 5x)}{h}$

$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 5x + 5h - x^2 - 5x}{h}$

$= \lim_{h\to 0} \frac{h(2x + h + 5)}{h}$

$= 2x + 5$

b) i) $y = -x^2$ ①

$x + y + 6 = 0$ ②

sub ① into ②

$x - x^2 + 6 = 0$

$0 = x^2 - x - 6$

$0 = (x-3)(x+2)$

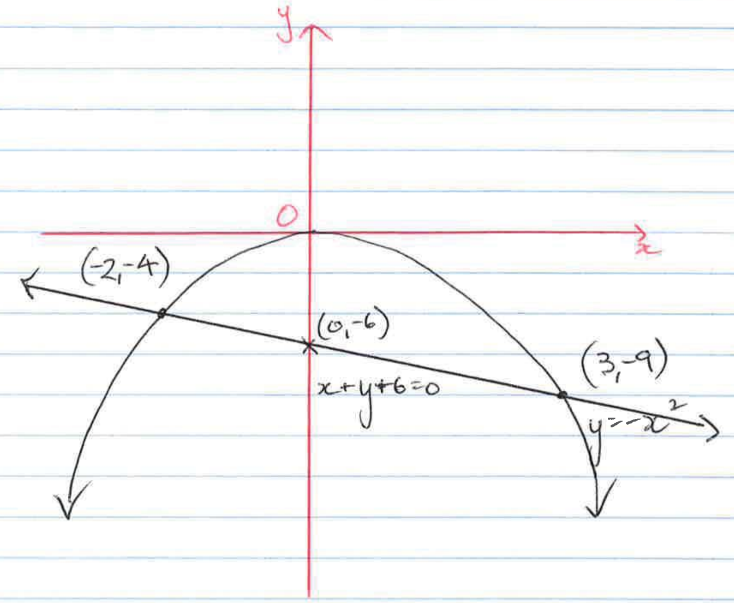
$\therefore x = 3, \quad x = -2$

$\therefore y = -3^2 \qquad y = -(-2)^2$

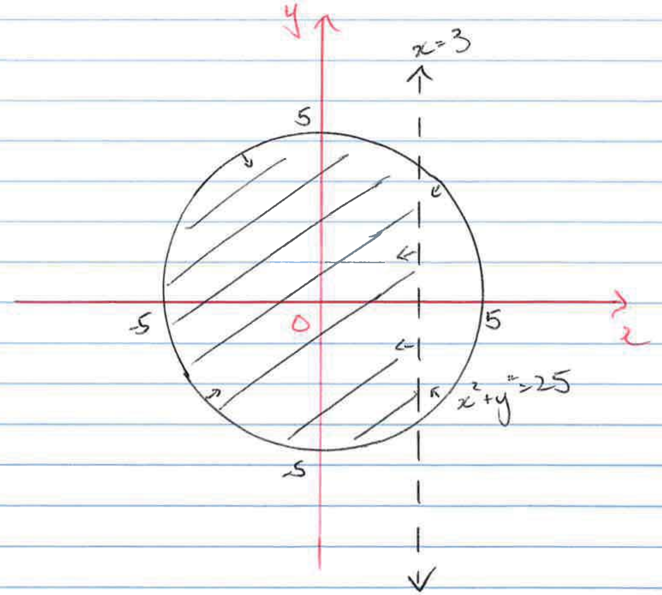
$= -9 \qquad = -4$

$(3, -9) \qquad (-2, -4)$

ii)



c) $x^2+y^2=25$ (circle at (0,0) with radius 5)
$x=3$ (vertical line at $x=3$)



d) $f(x)=5x^2(2x-1)^3$

$u=5x^2$ $\quad v=(2x-1)^3$
$u'=10x$ $\quad v'=3(2x-1)^2\times 2$
$\quad =6(2x-1)^2$

$f'(x)=vu'+uv'$
$=10x(2x-1)^3+5x^2\times 6(2x-1)^2$
$=10x(2x-1)^3+30x^2(2x-1)^2$
$=10x(2x-1)^2[2x-1+3x]$
$=10x(2x-1)^2(5x-1)$

e) $y=x^3+5x^2-4$
$y'=3x^2+10x$

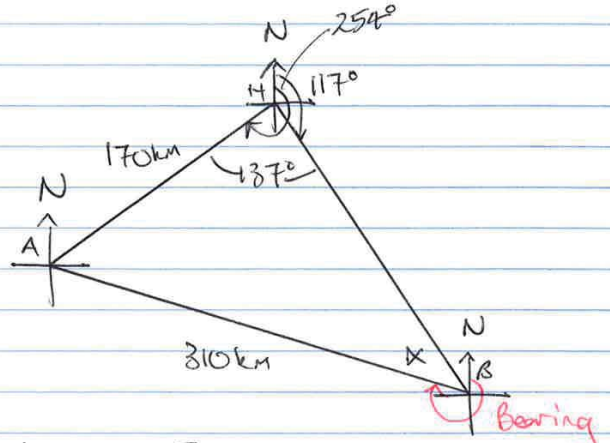
at $x=1$
$m=3\times1^2+10\times1$
$=13$
$m_{\perp}=-\frac{1}{13}$

at $x=1$ $\quad y=1^3+5\times1^2-4$
$=2$
$(1,2)$

$\therefore y-y_1=m(x-x_1)$
$y-2=-\frac{1}{13}(x-1)$
$-13y+26=x-1$
$0=x+13y-27$

QUESTION 14

a)



i) $\angle AHB = 254 - 117$
$= 137°$

ii) $\frac{\sin x}{170} = \frac{\sin 137}{310}$

$\sin x = \frac{170 \sin 137}{310}$

$x = \sin^{-1}\left(\frac{170 \sin 137}{310}\right)$

$x = 21.962..$
$x = 22°$

cointerior Ls.

$\therefore$ Bearing $= 360 - 22 - (180 - 117)°$
$= 360 - 22 - 63$
$= 275°$

b) $\frac{(1-\cos\theta)(1+\cos\theta)}{\cos^2\theta} = \tan^2\theta$

$LHS = \frac{(1-\cos\theta)(1+\cos\theta)}{\cos^2\theta}$

$= \frac{1-\cos^2\theta}{\cos^2\theta}$

$= \frac{\sin^2\theta}{\cos^2\theta}$

$= \tan^2\theta$

$= RHS$

c) $\angle ABD = 39°$ (diagonals of a rhombus bisect Ls)
$\angle ADC = 39 + 39$ (opposite Ls in a rhombus are equal)
$= 78°$
$\angle BCE = \angle ADC$ (corresponding Ls, AD $\parallel$ BC)
$= 78°$
$\angle BEC = \angle BCE$ (base Ls of isosceles $\triangle$ are equal)
$= 78°$
$\therefore x = 180 - 78 - 78$ ($\angle$ sum $\triangle$ is 180°)
$= 24°$

d) $9^x - 6(3^x) - 27 = 0$
let $u = 3^x$

$\therefore u^2 - 6u - 27 = 0$
$(u-9)(u+3) = 0$

$\therefore u = 9$ or $u = -3$
$3^x = 9$ $\quad$ $3^x = -3$
$x = 2$ $\quad$ no solution
$\therefore x = 2$

e) $y = kx^3 + 2$

$m = \tan 60$

$= \sqrt{3}$

at $x = 2$, $m = \sqrt{3}$

$y = kx^3 + 2$

$\frac{dy}{dx} = 3kx^2$

$\frac{dy}{dx} = m$

$\therefore \sqrt{3} = 3kx^2$ at $x = 2$.

$\sqrt{3} = 3k \times 2^2$

$\sqrt{3} = 12k$

$k = \frac{\sqrt{3}}{12}$