Shore School

2016

Year 11

Preliminary Task 4

Yearly Examination

Examination Number:
Set:

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Board approved calculators may be used
- Answer Questions 1 – 10 on the Multiple Choice answer sheet provided
- Start each of Questions 11 – 14 in a new writing booklet
- In Questions 11−14, show relevant mathematical reasoning and/or calculations
- Write your examination number on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your examination number and "N/A"
- A BOSTES reference sheet is provided.

Note: Any time you have remaining should be spent revising your answers.

Total marks – 70

Section I Pages 2−5

10 marks

- Attempt Questions 1−10
- Allow about 15 minutes for this section

Section II Pages 6−10

60 marks

- Attempt Questions 11−14
- Allow about 1 hour and 45 minutes for this section

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Section I

10 marks
Attempt Questions 1−10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1−10.

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1 What is 0.00412248 written in scientific notation, correct to 4 significant figures?

(A) 4.1225×10^{-2}

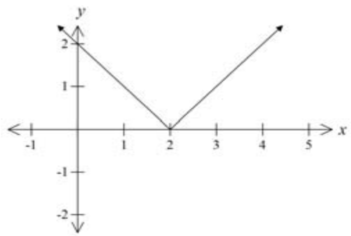
(B) 4.122×10^{-2}

(C) 4.1225×10^{-3}

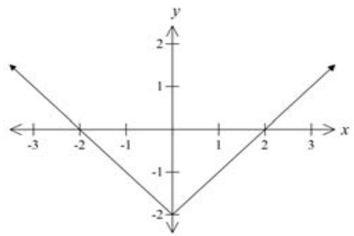
(D) 4.122×10^{-3}

2 Which graph best represents $y=|x-2|$?

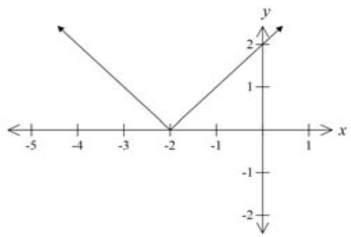
(A)



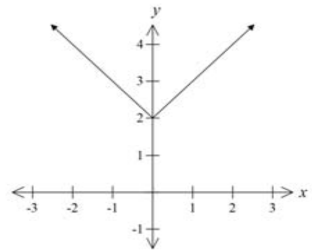
(B)



(C)



(D)



3 Which of the following is equal to $\frac{1}{2\sqrt{5}-\sqrt{3}}$?

(A) $\frac{2\sqrt{5}-\sqrt{3}}{7}$

(B) $\frac{2\sqrt{5}+\sqrt{3}}{7}$

(C) $\frac{2\sqrt{5}-\sqrt{3}}{17}$

(D) $\frac{2\sqrt{5}+\sqrt{3}}{17}$

4 Which inequality defines the domain of the function $f(x)=\frac{1}{\sqrt{x+4}}$?

(A) $x>-4$

(B) $x\geq -4$

(C) $x<-4$

(D) $x\leq -4$

5 What is the gradient of the normal to the curve $y=2x^3$ at the point where $x=2$?

(A) –24

(B) $-\frac{1}{24}$

(C) 24

(D) $\frac{1}{24}$

6 Which equation represents the line perpendicular to $2x-3y-8=0$, passing through the point (2,0) ?

(A) $3x+2y-4=0$

(B) $3x+2y-6=0$

(C) $3x-2y+4=0$

(D) $3x-2y-6=0$

7 What are the solutions of $2\cos x=-\sqrt{3}$ for $0°\leq x\leq 360°$?

(A) $x=30°$ and $330°$

(B) $x=60°$ and $300°$

(C) $x=150°$ and $210°$

(D) $x=120°$ and $240°$

8 What is the best description of the nature of the roots of $3x^2-7x+2=0$?

(A) two real, irrational roots

(B) one real, irrational root

(C) two real, rational roots

(D) one real, rational root

9 What is the value of $\lim_{x\to -2}\frac{x^2-4}{x+2}$?

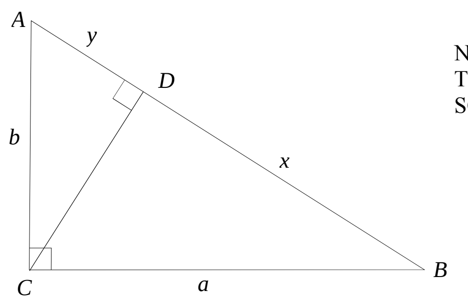
(A) Undefined

(B) –4

(C) 0

(D) 4

10 Let $BC = a$, $AC = b$, $BD = x$ and $AD = y$. Triangle ADC is similar to triangle ACB.



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Which of the following statements is correct?

(A) $\dfrac{b}{y} = \dfrac{a}{x}$

(B) $\dfrac{b}{y} = \dfrac{x+y}{a}$

(C) $\dfrac{b}{y} = \dfrac{x+y}{b}$

(D) $\dfrac{b}{y} = \dfrac{a}{b}$

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Solve $7 - 3x \geq 22$. **2**

(b) Write, using an algebraic technique, $0.3\dot{2}\dot{4}$ as a fraction in its simplest terms. **2**

(c) Solve $|x - 3| = 2x - 4$. **3**

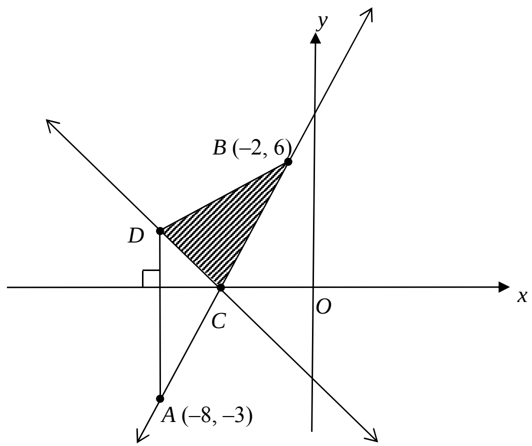
(d) Simplify $\dfrac{x^2 - 4x - 5}{4x^2 + 4x + 4} \times \dfrac{x^3 - 1}{x^2 - 1}$. **3**

(e) On the number plane shade the region defined by $x^2 + y^2 < 9$ and $y \geq x + 3$. **3**

(f) A regular polygon has an interior angle of $171°$. **2**
How many sides does the polygon have?

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) $A(-8, -3)$ and $B(-2, 6)$ are two points on the number plane. The line AB intersects the x-axis at C. The line CD makes an angle of $135°$ with the positive x-axis. The line AD is parallel to the y-axis.



NOT TO SCALE

(i) Given the equation of the line AB is $3x-2y+18=0$, find the co-ordinates of C. **1**

(ii) Show that the equation of the line CD is $x+y+6=0$. **2**

(iii) Find the co-ordinates of D. **1**

(iv) Find the exact length of CD, in its simplest form. **1**

(v) Find the perpendicular distance from B to the line CD. **2**

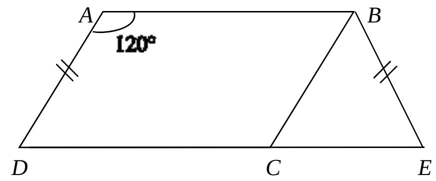
(vi) Find the area of ΔBCD. **1**

(b) Simplify $\cot\theta-\cot\theta\cos^2\theta$. **2**

(c) Show that $f(x)=x^3-x$ is an odd function. **2**

Question 12 continues on page 8

(d) $ABCD$ is a parallelogram with $\angle DAB=120°$. The side DC is produced to E so that $AD=BE$. **3**



NOT TO SCALE

(i) Copy or trace the diagram into your writing booklet.

(ii) Prove that ΔBCE is an equilateral triangle.

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet

(a) If α and β are roots of the equation $2x^2 + 3x - 4 = 0$, find the value of

(i) $\alpha + \beta$ **1**

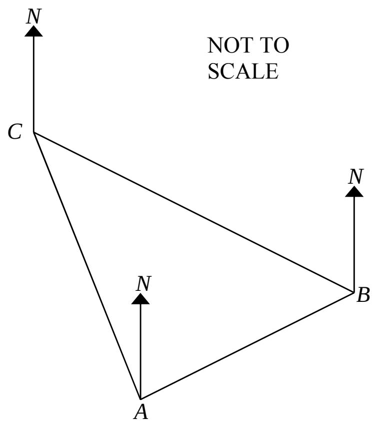
(ii) $\alpha\beta$ **1**

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ **2**

(iv) $(\alpha - \beta)^2$ **2**

(b) Find the values of *A*, *B* and *C* for which $2x^2 + 3x + 1 \equiv A(x+2)^2 + B(x+2) + C$. **3**

(c) A ship sails 50 km from Port A to Port B on a bearing of $063°$ and then sails 130 km from Port B to Port C on a bearing of $296°$.



(i) Copy or trace the diagram into your writing booklet showing all the above information. **1**

(ii) Show that $\angle ABC = 53°$. **1**

(iii) Find the distance from Port *A* to Port *C*. Give you answer correct to the nearest kilometre. **2**

(iv) Find the bearing of Port *A* from Port *C*. Give your answer correct to the nearest degree. **2**

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) Differentiate the following with respect to *x*

(i) $y = (3x+4)^5$ **2**

(ii) $y = \dfrac{x^2+5}{x-2}$ **2**

(b) Differentiate $y = x^2(3-2x)^4$ with respect to *x*. Answer in fully factored form. **3**

(c) Find the equation of the tangent to the curve $y = x^3 + 2x^2 - 5x$ at the point $(-3, 6)$. **2**

(d) A function is defined by $f(x) = 2x^2 + 7x - 3$.

(i) Show that $f(x+h) = 2x^2 + 4xh + 2h^2 + 7x + 7h - 3$. **1**

(ii) Hence, differentiate $f(x) = 2x^2 + 7x - 3$ from first principles. **2**

(e) The function $f(x) = \sqrt{4x-1}$ has a tangent with gradient 2 at point *N*.

(i) Find the co-ordinates *N*. **2**

(ii) Find the equation of the normal to the curve at *N*. **1**

END OF PAPER

Year 11 Mathematics Preliminary Exam 2016

Section 1

1. $0.00412248 = 4.122 \times 10^{-3}$ (D)

2. (A)

3. $\frac{1}{2\sqrt{5}-\sqrt{3}} = \frac{1}{2\sqrt{5}-\sqrt{3}} \times \frac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}+\sqrt{3}}$

$= \frac{2\sqrt{5}+\sqrt{3}}{4\times 5-3}$

$= \frac{2\sqrt{5}+\sqrt{3}}{17}$ (D)

4. $f(x) = \frac{1}{\sqrt{x+4}}$

$x+4 > 0$

$x > -4$ (A)

5. $y = 2x^3$

$\frac{dy}{dx} = 6x^2$ at $x = 2$

$= 6(2)^2$

$= 24$

Gradient of the tangent $= 24$

Gradient of the normal $= \frac{-1}{24}$

as $m_1 m_2 = -1$ for perpendicular lines. (B)

6. $2x - 3y - 8 = 0$

$-3y = -2x + 8$

$y = \frac{2}{3}x - \frac{8}{3}$

$m_1 = \frac{2}{3}$

As $m_1 m_2 = -1$, for perpendicular lines $m_2 = \frac{-3}{2}$

$y - y_1 = m(x - x_1)$ $\quad (2,0), m = \frac{-3}{2}$

$y - 0 = \frac{-3}{2}(x-2)$

$2y = -3(x-2)$

$2y = -3x+6$

$\therefore 3x + 2y - 6 = 0$ (B)

7. $2\cos x = -\sqrt{3}$

$\cos x = -\frac{\sqrt{3}}{2}$

Related angle $\cos 30° = \frac{\sqrt{3}}{2}$ $\quad$ S | A ✓, T ✓ | C

$\therefore x = 180°-30°, 180°+30°$

$= 150°, 210°$ (C)

8. $3x^2 - 7x + 2 = 0$ $\quad a=3, b=-7, c=2$

$\Delta = b^2 - 4ac$

$= (-7)^2 - 4\times3\times2$

$= 25$

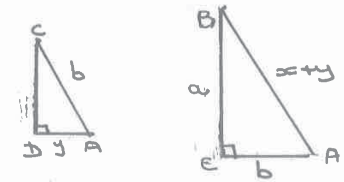
$\therefore$ There are 2 real, rational roots (C)

9. $\lim_{x\to -2} \frac{x^2-4}{x+2} = \lim_{x\to -2} \frac{(x-2)(x+2)}{(x+2)}$

$= \lim_{x\to -2} x - 2$

$= -4$ (B)

10.



$\therefore \frac{b}{y} = \frac{x+y}{b}$ (C)

Question 11

a) $7 - 3x \geq 22$

$-3x \geq 15$

$x \leq -5$

b) Let $x = 0.\dot{3}\dot{2}\dot{4}$

$= 0.32424\ldots\ldots$

$1000x = 324.2424\ldots\ldots$

$10x = 3.2424\ldots\ldots$

$990x = 321$

$x = \frac{321}{990}$

$x = \frac{107}{330}$

$\therefore 0.3\dot{2}\dot{4} = \frac{107}{330}$

c) $|x-3| = 2x-4$

$x - 3 = 2x - 4$ $\qquad$ $x-3 = -(2x-4)$

$-x = -1$ $\qquad$ $x-3 = -2x+4$

$x = 1$ $\qquad$ $3x = 7$

$\qquad$ $x = 2\frac{1}{3}$

CHECK:

L.H.S $= |x-3|$

$= |1-3|$

$= 2$

R.H.S $= 2x-4$

$= 2(1)-4$

$= -2$

L.H.S $\neq$ R.H.S

$\therefore x = 1$ is not a solution

CHECK:

L.H.S $= |x-3|$

$= |2\frac{1}{3}-3|$

$= \frac{2}{3}$

R.H.S $= 2x-4$

$= 2(2\frac{1}{3})-4$

$= \frac{2}{3}$

L.H.S = R.H.S

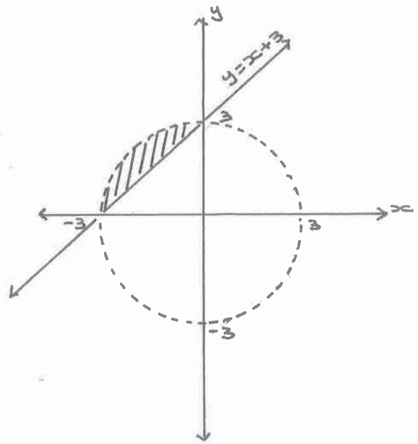
$\therefore x = 2\frac{1}{3}$ is a solution

$\therefore x = 2\frac{1}{3}$

d) $\frac{x^2-4x-5}{4x^2+4x+4} \times \frac{x^3-1}{x^2-1}$

$= \frac{(x-5)(x+1)}{4(x^2+x+1)} \times \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)}$

$= \frac{x-5}{4}$



Test (0,0) in $x^2+y^2<9$
$0+0<9$ is true

$\therefore$ Shade inside the circle

Test (0,0) $y \geq x+3$
$0 \geq 0+3$ is false

$\therefore$ Shade on the other side.

f) exterior angle $= 180° -$ interior angle

$= 180° - 171°$

$= 9°$

number of sides $= \frac{360}{9}$

$= 40$

OR. interior angle $= \frac{(n-2)\times 180°}{n}$

$171 = \frac{(n-2)\times 180}{n}$

$171n = 180n - 360$

$-9n = -360$

$n = 40$

Question 12

a) i) $3x - 2y + 18 = 0$ at C, $y = 0$

$3x = -18$

$x = -6$

∴ The co-ordinates of C are $(-6, 0)$

ii) $m_{CD} = \tan 135^\circ$

$= -1$

$y - y_1 = m(x - x_1)$ C(−6,0)

$y - 0 = -1(x + 6)$

$y = -x - 6$

∴ $x + y + 6 = 0$

iii) at D, $x = -8$

$x + y + 6 = 0$

$-8 + y + 6 = 0$

$y = 2$

∴ D is $(-8, 2)$

iv) $CD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-6+8)^2 + (0-2)^2}$

$= \sqrt{8}$

∴ $CD = 2\sqrt{2}$

iv) $d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$

$= \left|\frac{-2 + 6 + 6}{\sqrt{1^2 + 1^2}}\right|$ B(−2,6)

$= \frac{10}{\sqrt{2}}$

$= \frac{10\sqrt{2}}{2}$

∴ $d = 5\sqrt{2}$

v) $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 2\sqrt{2} \times 5\sqrt{2}$

$= 10$

∴ Area is 10 units^2

b) $\cot\theta - \cot\theta\cos^2\theta = \cot\theta(1 - \cos^2\theta)$

$= \cot\theta \times \sin^2\theta$

$= \frac{\cos\theta}{\sin\theta} \times \sin^2\theta$

$= \cos\theta\sin\theta$

c) $f(x) = x^3 - x$

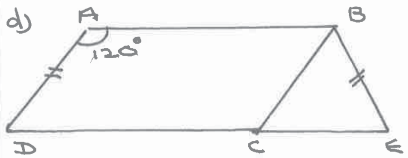
$f(-x) = (-x)^3 - (-x)$

$= -x^3 + x$

$-f(x) = -(x^3 - x)$

$= -x^3 + x$

As $f(-x) = -f(x)$ the function is odd.

d) 

$B\hat{C}D = D\hat{A}B$ · opposite angles of a parallelogram are equal

$= 120^\circ$

$B\hat{C}E = 180^\circ - 120^\circ$, a straight angle is 180°

$= 60^\circ$

$AD = BC$ · opposite angles of a parallelogram are equal

$B\hat{C}E = B\hat{E}C$ · In an isosceles triangle equal angles are opposite equal sides

$= 60^\circ$

$C\hat{B}E = 180^\circ - 60 - 60^\circ$ · angle sum of a triangle is 180°

$= 60^\circ$

∴ △BCE is an equilateral triangle as all angles are 60°

Question 13

a) $2x^2 + 3x - 4 = 0$

$a = 2$, $b = 3$, $c = -4$

i) $\alpha + \beta = \frac{-b}{a}$

$= \frac{-3}{2}$

ii) $\alpha\beta = \frac{c}{a}$

$= \frac{-4}{2}$

$= -2$

iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$

$= \frac{-\frac{3}{2}}{-2}$

$= \frac{3}{4}$

iv) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= \left(\frac{3}{2}\right)^2 - 4 \times -2$

$= 10\frac{1}{4}$

b) $2x^2 + 3x + 1 \equiv A(x+2)^2 + B(x+2) + C$

$= Ax^2 + 4Ax + 4A + Bx + 2B + C$

$= Ax^2 + (4A + B)x + (4A + 2B + C)$

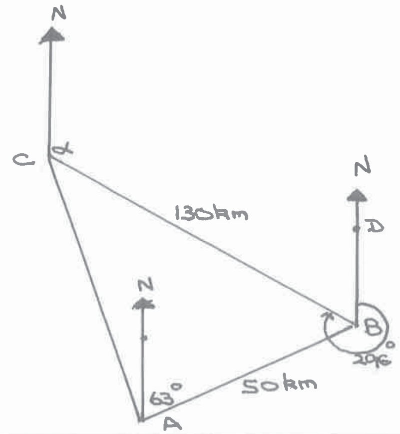
∴ $A = 2$ $4A + B = 3$ $4A + 2B + C = 1$

$4(2) + B = 3$ $4(2) + 2(-5) + C = 1$

$B = -5$ $-2 + C = 1$

$C = 3$

∴ $A = 2$, $B = -5$, $C = 3$

c) 

$D\hat{B}C = 360^\circ - 296^\circ$ · angles about a point add up to 360°

$= 64^\circ$

$D\hat{B}A = 180^\circ - 63^\circ$ · cointerior angles add up to 180° if the lines are parallel

$= 117^\circ$

$A\hat{B}C = 117^\circ - 64^\circ$

$= 53^\circ$

ii) $b^2 = a^2 + c^2 - 2ac\cos B$

$= 130^2 + 50^2 - 2 \times 130 \times 50 \cos 53^\circ$

$\doteq 11576.4047$

$b \doteq 107.5937019$

∴ AC is 108 km to nearest km

iii) $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$= \frac{130^2 + 107.6^2 - 50^2}{2 \times 130 \times 107.6}$

$= 0.92857\ldots$

$C = 21.7856\ldots$

$= 22^\circ$ to the nearest deg.

$\alpha = 180^\circ - 64^\circ$ · cointerior angles are equal if the lines are parallel

$= 116^\circ$

∴ Bearing of A from C $= 116^\circ + 22^\circ$

$= 138^\circ$

Question 14

a) i) $y = (3x+4)^5$

$\frac{dy}{dx} = 5(3x+4)^4 \times 3$

$= 15(3x+4)^4$

ii) $y = \frac{x^2+5}{x-2}$

$u = x^2+5 \qquad v = x-2$

$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = 1$

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{(x-2)2x - (x^2+5)\times 1}{(x-2)^2}$

$= \frac{2x^2-4x-x^2-5}{(x-2)^2}$

$= \frac{x^2-4x-5}{(x-2)^2}$

$= \frac{(x-5)(x+1)}{(x-2)^2}$

b) $y = x^2(3-2x)^4$

$u = x^2 \qquad v = (3-2x)^4$

$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = 4(3-2x)^3 \times -2$

$= -8(3-2x)^3$

$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

$= (3-2x)^4 \times 2x + x^2 \times -8(3-2x)^3$

$= 2x(3-2x)^4 - 8x^2(3-2x)^3$

$= 2x(3-2x)^3[3-2x-4x]$

$= 2x(3-2x)^3(3-6x)$

$= 6x(3-2x)^3(1-2x)$

c) $y = x^3 + 2x^2 - 5x$

$\frac{dy}{dx} = 3x^2 + 4x - 5 \quad$ at $x = -3$

$= 3(-3)^2 + 4(-3) - 5$

$= 27 - 12 - 5$

$= 10$

$y - y_1 = m(x - x_1)$

$y - 6 = 10(x - -3)$

$y - 6 = 10x + 30$

$y = 10x + 36$

d) $f(x) = 2x^2 + 7x - 3$

i) $f(x+h) = 2(x+h)^2 + 7(x+h) - 3$

$= 2(x^2+2xh+h^2) + 7x + 7h - 3$

$= 2x^2 + 4xh + 2h^2 + 7x + 7h - 3$

ii) $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{2x^2+4xh+2h^2+7x+7h-3-2x^2-7x+3}{h}$

$= \lim_{h\to 0} \frac{4xh+2h^2+7h}{h}$

$= \lim_{h\to 0} \frac{h(4x+2h+7)}{h}$

$= \lim_{h\to 0} 4x+2h+7$

$\therefore f'(x) = 4x+7$

c) $f(x) = \sqrt{4x-1}$

$= (4x-1)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(4x-1)^{-\frac{1}{2}} \times 4$

$= 2(4x-1)^{-\frac{1}{2}}$

$= \frac{2}{\sqrt{4x-1}}$

But $f'(x) = 2$

$\frac{2}{\sqrt{4x-1}} = 2$

$\therefore \sqrt{4x-1} = 1$

$4x - 1 = 1$

$4x = 2$

$x = \frac{1}{2}$

$f(x) = \sqrt{4x-1}$

$f(\frac{1}{2}) = \sqrt{4(\frac{1}{2})-1}$

$= 1$

$\therefore$ The co-ordinates of N are $(\frac{1}{2}, 1)$

ii) gradient of the tangent $= 2$

gradient of the normal $= -\frac{1}{2}$

as $m_1 m_2 = -1$ for perpendicular lines.

$y - y_1 = m(x - x_1) \qquad (\frac{1}{2}, 1)$

$y - 1 = -\frac{1}{2}(x - \frac{1}{2}) \qquad m = -\frac{1}{2}$

$2y - 2 = -x + \frac{1}{2}$

$2x + 4y - 5 = 0$ is the equation of the normal