STUDENT NUMBER/NAME

St Aloysius' College

Year 11 Preliminary Examination

2015

MATHEMATICS (2 Unit)

General Instructions

Reading time – 5 minutes

Working time – 2 hours

- Write using black or blue pen only with diagrams in pencil
- Board approved calculators may be used
- Examination papers must NOT be removed from the examination room
- Attempt all questions

Total marks: 70

Section I:

- 10 objective response questions worth 1 mark each.
- Give your answers on the Section I answer sheet.
- **Only the letter will be considered for marking.**

Section II:

- 4 questions worth 15 marks each consisting of shorter part-questions.
- Attempt all questions.
- Marks for each part are shown in the margin.
- Hand in a booklet for each question, even if not attempted.
- If a second booklet is used place it inside the first.

Section I

10 marks

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

1. Given that $m = (\sqrt{5} - \sqrt{3})^2 + \sqrt{60}$, find m in simplest form.

(A) 8

(B) 24

(C) $8 - 4\sqrt{15}$

(D) $24 - 4\sqrt{15}$

2. A function is defined by the rule:

$$f(x) = \begin{cases} 0 & x \leq -3 \\ -1 & -3 < x < 0 \\ x & x \geq 0 \end{cases}$$

Find $f(-3) + f(-2) + f(2)$

(A) –3

(B) –1

(C) 1

(D) 3

3. If the straight lines $2x + 3y = 4$ and $x + ay = 7$ are perpendicular, find the value of a.

(A) $-\frac{3}{2}$

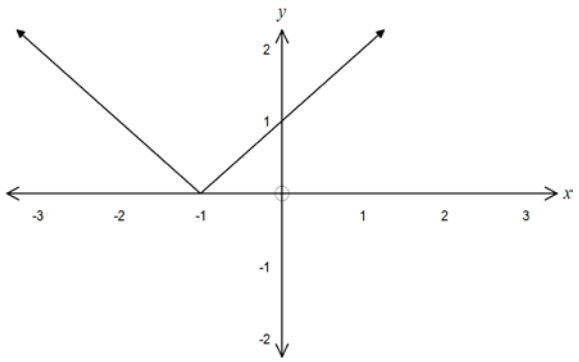
(B) $-\frac{2}{3}$

(C) $\frac{2}{3}$

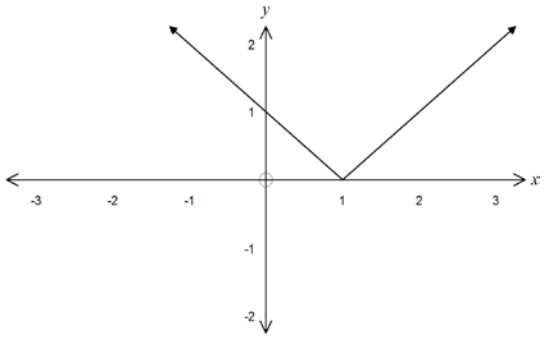
(D) $\frac{3}{2}$

4. Which of the following graphs best represents the function $y = |x + 1|$.

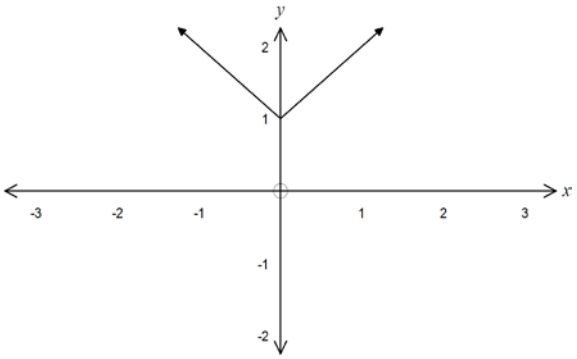
(A)



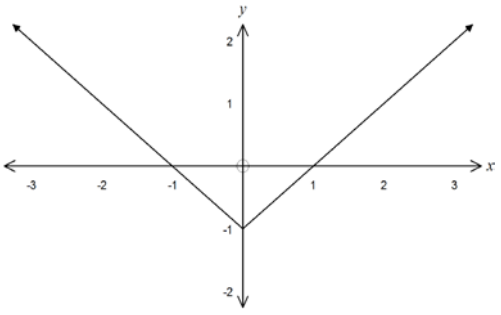
(B)



(C)



(D)



5. If $\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$, find the value of a (to 3 decimal places) when $b = 2.08$ and $c = 6.45$.

(A) 0.635

(B) 0.636

(C) 1.572

(D) 1.573

6. Differentiate $y = 15\sqrt[5]{x}$.

(A) $15x^{-\frac{4}{5}}$

(B) $3\sqrt[5]{x^4}$

(C) $75\sqrt[5]{x}$

(D) $\frac{3}{\sqrt[5]{x^4}}$

7. Simplify $\cos A(\sec A + \tan A)$

(A) $1 + \cos A$

(B) $1 + \sin A$

(C) $1 + \tan A$

(D) $\sin A$

8. The quadrilateral *PQRS* is a rhombus. Consider these two statements.

(I) Sides *PQ* and *QR* are equal.
(II) The diagonals *PR* and *QS* bisect each other.

Which of the following best describes statement (I) and (II)?

(A) Both statements are correct.

(B) Only statement I is correct.

(C) Only statement II is correct.

(D) Both statements are incorrect.

9. Solve $|3x - 1| = 4x + 2$.

(A) No solution

(B) $x = -3$

(C) $x = -\frac{1}{7}$

(D) $x = -3$ and $x = -\frac{1}{7}$

10. The line $y = 5x - 1$ is tangent to the curve $y = x^2 + 3x$ at the point A.

Find the coordinates of A.

(A) $(-2, -2)$

(B) $(-1.5, -2.25)$

(C) $(1, 4)$

(D) $(0, 0)$

End of Section I

Section II

60 marks

Attempt Questions 11 – 14.

Allow about 1 hour and 45 minutes for this section.

Answer each question in a separate writing booklet.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

(a) The fuel tank on my new car was 40% full. I added 28 litres and then found it was 75% full. How much fuel does the tank hold? **1**

(b) Evaluate $\sqrt{\dfrac{6.749 \times (3.22)^2}{17.413}}$ correct to 3 significant figures. **1**

(c) Solve $\dfrac{3x-2}{5} = \dfrac{x}{4} + 3$. **3**

(d) Find the exact value of $t^3 - 2t^2 + 2t + 1$ when $t = 2\sqrt{3}$. **2**

(e) Express $\dfrac{x+1}{x^2-x} - \dfrac{x-1}{x^2+x}$ as a single fraction in its lowest form. **3**

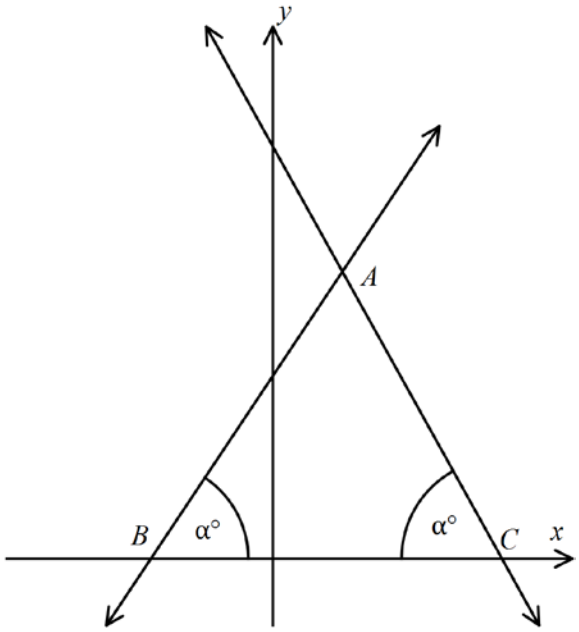
(f) Solve $2\sin^2\theta - 1 = 0$ for $0° \leq \theta \leq 360°$. **2**

(g) Prove that $\dfrac{1}{\sin^2\theta} + \dfrac{1}{\cos^2\theta} = \sec^2\theta \operatorname{cosec}^2\theta$. **3**

End of Question 11

Question 12 (15 marks) Start a new booklet

(a) The vertices of a triangle are $A(1,4)$ and $B(-1,0)$ and C, where C lies on the x-axis and $\angle ABC = \angle ACB = \alpha$.



(i) Find the coordinates of the midpoint of AB. **1**

(ii) Show that $\tan\alpha = 2$. **2**

(iii) Show that AB has equation $y = 2x + 2$. **1**

(iv) Explain why AC has a gradient of -2, and hence find its equation in general form. **2**

(v) Find the coordinates of C and hence the area of ΔABC. **2**

(vi) Find the length of AC and the perpendicular distance from B to AC. **2**

(b) State the range of $y = 1 - 2x^2$. **1**

Question 12 continues on the next page

Question 12 Continued

(c) Differentiate the following with respect to x

(i) $3x^2 - 5x + 4$ **1**

(ii) $(5x + 2)^4$ **1**

(iii) $3x^2(3x^4 - x)$ **2**

End of Question 12

Question 13 (15 marks) Start a new booklet

(a) Two ships sail from the town of Posthawk (*P*). The ship Longview (*L*) sails on a bearing of 118° and the ship Quest (*Q*) on a bearing of 276°. Both ships sailed for 3 hours, the Longview at a speed of 8 knots and the Quest at a speed of 12 knots.

(1 knot = 1 nautical mile per hour)

(i) Draw a diagram to show the distance and direction of the two ships from *P* after 3 hours. **2**

(ii) Find the distance, in nautical miles, between the 2 ships after 3 hours. **3**

(Answer correct to 1 decimal place)

(iii) Find the bearing of the Quest from the Longview after 3 hours. **2**

(Answer to the nearest degree)

(b) Shade the region represented by $9 < x^2 + y^2 \leq 25$ **3**

(c) Given that $\cos\theta = \frac{\sqrt{3}}{2}$ and $180° \leq \theta \leq 360°$, find the exact value of $\sin\theta$. **2**

(d) Solve for *x*: **3**

$$\log(x) + \log(x-3) = \log 28$$

End of Question 13

Question 14 (15 marks) Start a new booklet

(a) Find the exact value of $\cot(-135°)$. **1**

(b) For the circle with equation $x^2 + y^2 + 6x - 8y = 0$:

(i) Show that the centre of the circle is (–3, 4) and hence find the radius. **2**

(ii) Show that the circle passes through the origin. **1**

(iii) The origin is at one end of a diameter of the circle. **1**
Find the coordinates of the other end of this diameter.

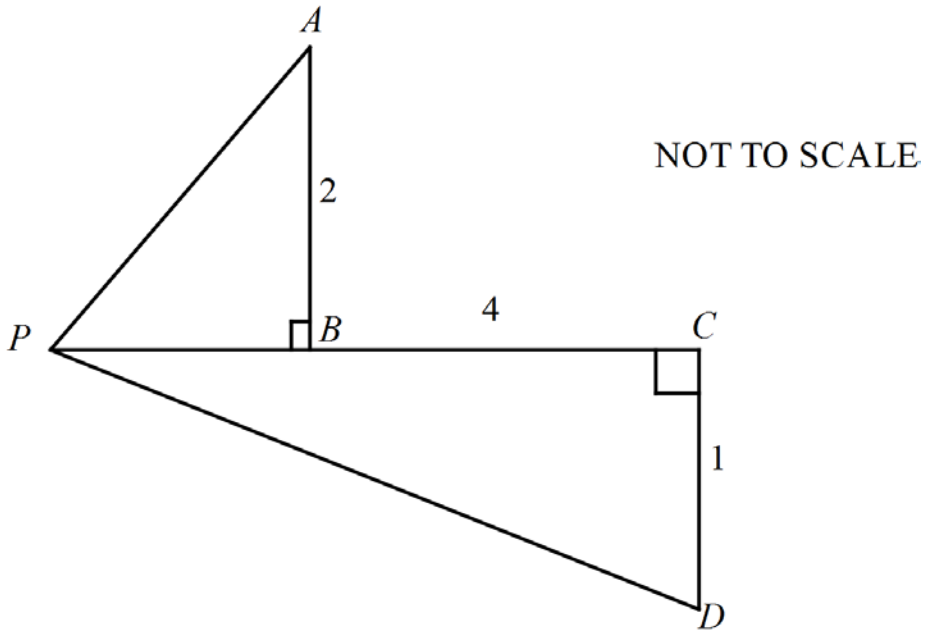
(c) If $y = \dfrac{\sqrt{x+1}}{x}$ show that $y' = \dfrac{-x-2}{2x^2\sqrt{x+1}}$. **3**

(d) Differentiate $y = x^2 + bx + c$ and hence find the values of b and c if the line $3x + y - 5 = 0$ is a normal to the curve at the point X (3,-1). **3**

Question 14 continues on the next page

Question 14 Continued

(e) In the diagram $AB = 2$, $BC = 4$ and $CD = 1$.



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If $PD^2 = 2PA^2$:

(i) Show that $PD^2 = PB^2 + 8PB + 17$

(ii) Hence find PB

End of Examination