

## St Aloysius' College <br> Year 11 Preliminary Examinations <br> 2016

## MATHEMATICS

## General Instructions

Reading time - 5 minutes
Working time - 2 hours

- Write using black or blue pen only
- Board approved calculators may be used
- Examination papers must NOT be removed from the examination room
- Attempt all questions

Total marks: 70

## Section I:

- 10 objective response questions worth 1 mark each.
- Give your answers on the Section I answer sheet.
- Only the letter will be considered for marking.


## Section II:

- 4 questions worth 15 marks each consisting of shorter part-questions.
- Attempt all questions.
- Marks for each part are shown in the margin.
- Hand in a booklet for each question, even if not attempted.
- If a second booklet is used place it inside the first.


## 10 marks

Attempt Questions 1 - 10 .
Allow about 15 minutes for this section.
Use the multiple-choice answer sheet for Questions 1-10.

1. What is the derivative of the expression $6 x^{4}+9 x^{2}-12 x+9$ ?
(A) $24 x^{3}+18 x-12$
(B) $4 x^{3}+2 x-12$
(C) $24 x^{3}+18 x^{2}-12$
(D) $24 x^{3}+18 x-3$
2. What is the domain of the function $f(x)=\sqrt{3 x-2}$ ?
(A) $-\frac{2}{3}>x>\frac{2}{3}$
(B) $-\frac{2}{3} \geq x \geq \frac{2}{3}$
(C) $x>\frac{2}{3}$
(D) $x \geq \frac{2}{3}$
3. What is the gradient of the tangent to the curve $y=4 x^{2}-3 x+2$ at the point where $x=2$ ?
(A) 10
(B) 12
(C) 13
(D) 15
4. What is the equation of the circle shown in the diagram below?

(A) $(x-2)^{2}+y^{2}=9$
(B) $(x-2)^{2}+y^{2}=3$
(C) $(x+2)^{2}+y^{2}=9$
(D) $(x+2)^{2}+y^{2}=3$
5. In baseball, the batter runs around a square whose sides have a length of 27.4 metres, as shown in the diagram. A player is standing at the point $P, 9.1$ metres from $B$. $A B P$ is a straight line.


## NOT TO SCALE

What is the distance of the player at $P$ from the point $D$, correct to 1 decimal place?
(A) 28.8 m
(B) 29.6 m
(C) 38.7 m
(D) 45.6 m
6. What are the $x$ coordinates of the points of intersection of the parabola $y=x^{2}+6 x-3$ and the straight line $y=2 x+9$ ?
(A) $x=-4,8$
(B) $x=-6,2$
(C) $x=3,2$
(D) $x=-2,4$
7. If $\cos \alpha=\frac{a}{b}$, where $\alpha$ is an acute angle, what is the value of $\cos \left(180^{\circ}+\alpha\right)$ ?
(A) $\frac{\sqrt{b^{2}-a^{2}}}{b}$
(B) $\frac{a}{b}$
(C) $-\frac{a}{b}$
(D) $-\frac{b}{a}$
8. The diagram shows $\triangle A B C$ and $\triangle E D C . \quad A B=E D$ and $\angle B A C=\angle D E C$. Which statement below is not sufficient to prove that $\triangle A B C$ and $\triangle E D C$ are congruent?

(A) $B C=C D$
(B) $A C=E C$
(C) $\angle A B C=\angle E D C$
(D) $\angle A C B=\angle E C D$
9. What are the solutions of $\cot \theta=0$, where $0 \leq \theta \leq 360^{\circ}$ ?
(A) no solutions
(B) $90^{\circ}$ and $270^{\circ}$
(C) $0^{\circ}, 180^{\circ}$ and $360^{\circ}$
(D) $45^{\circ}$ and $225^{\circ}$
10. An infinite geometric series has a first term of 3 and a limiting sum of $\frac{9}{5}$. What is the common ratio?
(A) $\quad-0 . \dot{3}$
(B) $\quad-0 . \dot{6}$
(C) -1.5
(D) $\quad-3.75$

## Section II

## 60 marks

Attempt Questions 11 - 14.
Allow about 1 hour and 45 minutes for this section.
Answer each question in a separate writing booklet.
In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)
(a) Evaluate $\sqrt{\frac{3^{2}-1}{4^{3}+1}}$ correct to 1 decimal place.
(b) Find the gradient of a line that is perpendicular to the line $3 x-y+1=0$.
(c) Evaluate $\left(1.3 \times 10^{-10}\right) \div\left(5.5 \times 10^{-4}\right)$, expressing your answer in scientific notation correct to 3 significant figures.
(d) Find the exact roots of the equation $2 x^{2}-4 x+1=0$.
(e) Solve $|3 a-5|<4$.
(f) Factorise $3 x^{3}-8 x^{2}+4 x$.
(g) Rationalise the denominator of $\frac{2+\sqrt{2}}{1+\sqrt{2}}$.
(h) Solve $2 \log _{a} x-\log _{a} 4=2 \log _{a} 8$

Question 12 (15 marks) Start a new booklet
(a) The points $A$ and $B$ have coordinates $(3,8)$ and $(-2,-4)$ respectively.
(i) Find the equation of the line that is parallel to $A B$ and that cuts the $y$-axis at $y=-3$. Give your answer in general form.
(ii) Find the midpoint of $A B$.
(iii) Find the length of $A B$.
(iv) Using your answers to (ii) and (iii), find the equation of the circle that has $A B$ as a diameter.
(v) Show that the point $(4,8)$ lies outside this circle.
(b) For the arithmetic sequence $2,7,12,17, \ldots \ldots \ldots$.
(i) Find a formula for the $n$th term
(ii) Find the $27^{\text {th }}$ term 1
(iii) Find the sum of the first 27 terms
(c) On a number plane, shade the region given by $x \geq 0, y \geq 0$ and $y \leq 4-x$.

Question 13 (15 marks) Start a new booklet
(a) What is the value of $\sum_{r=1}^{5}(4 r+1)$ ?
(b) Find the exact value of $\sin 240^{\circ}$
(c) Show that $\frac{\tan \theta \sec \theta}{1+\tan ^{2} \theta}=\sin \theta$
(d) Find the equation of the normal to the curve $y=x^{4}+2 x^{2}-14$ at the point where $x=2$.
(e) Find the coordinates of the point on the curve $y=(2 x-3)^{2}$ where the tangent is parallel to the line $y=4 x-1$.
(f) The third term of a geometric series is $\frac{3}{2}$ and the sixth term is 12 .
(i) Find the first term and the common ratio. 2
(ii) Find the tenth term 1
(iii) Find the sum of the first 10 terms $\mathbf{1}$

Question 14 (15 marks) Start a new booklet
(a) In the diagram, $\triangle A B C$ is an isosceles triangle with $A B=A C . A D$ bisects $\angle P A C$ and meets $B C$ produced at $D$. AP is parallel to $B D . \angle P A D=37^{\circ}$


## NOT TO SCALE

Copy or trace this diagram into your writing booklet.
(i) Find $\angle A C D$
(ii) Hence find, giving reasons, $\angle B A C$
(b) Show that $\frac{d}{d x}\left[3 x^{2}(4 x-1)^{3}\right]=6 x(10 x-1)(4 x-1)^{2}$.
(c) Differentiate $\frac{\sqrt{x}-1}{\sqrt{x}+1}$, giving your answer as a single fraction in surd form.

## Question 14 Continued

(d) A cruise ship leaves port $P$ on a bearing of $120^{\circ}$. It sails at a speed of $30 \mathrm{~km} / \mathrm{h}$.

After travelling for 300 km , the cruise ship stops at the point Q for one hour.

It then travels on a bearing of $15^{\circ}$ and sails at a speed of $42 \mathrm{~km} / \mathrm{h}$.

A container ship leaves the port $P$ on a bearing of $075^{\circ}$. It sails at a speed of $25 \mathrm{~km} / \mathrm{h}$. The paths of the container ship and the cruise ship meet at the point $R$.


NOT TO
SCALE

Copy the diagram clearly into your answer booklet
(i) Show that $\angle P R Q=60^{\circ}$.
(ii) Use the sine rule to show that the length of $P R$, to the nearest kilometre is 335 km .
(iii) Use the cosine rule to show that the length of $Q R$, to the nearest kilometre is 245 km .
(iv) The cruise ship leaves port $P$ at 8.30 a.m. At what time should the container ship leave $P$ if the two ships are to be at point $R$ at the same time?

## End of paper

