St Aloysius' College
Year 11 Preliminary Examination
2017

MATHEMATICS

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Board-approved calculators may be used
- In Questions 11–14, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I

10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II

60 marks

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 What is the domain of the function $y = \sqrt{x+4}$?

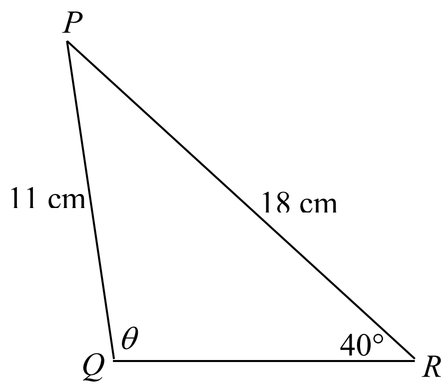
(A) $y \geq -0$

(B) $y > 0$

(C) $x \geq -4$

(D) $x > -4$

2 The diagram shows the triangle PQR where $PQ = 11$ cm, $PR = 18$ cm and $\angle PRQ = 40°$. Which expression correctly gives the value of $\sin\theta$?



(A) $\dfrac{18}{11\sin 40°}$

(B) $\dfrac{11}{18\sin 40°}$

(C) $\dfrac{11\sin 40°}{18}$

(D) $\dfrac{18\sin 40°}{11}$

3 $A(3, y)$ and $B(7, 2)$ are points on the number plane. The gradient of AB is $m = -3$.

What is the value of y?

(A) 14

(B) $\frac{1}{14}$

(C) -10

(D) $-\frac{1}{10}$

4 What are the solutions of the equation $|3x - 5| = 10$?

(A) $x = -5,\ 15$

(B) $x = -5,\ 5$

(C) $x = -1\frac{2}{3},\ 5$

(D) $x = -5,\ 1\frac{2}{3}$

5 How many solutions are there to the equation $x^3 - x^2 - 5x = 0$?

(A) 1

(B) 3

(C) 0

(D) 2

6 Solve $\sin\theta = \frac{1}{\sqrt{2}}, 0 \le \theta \le 360^\circ$.

(A) $\theta = 45^\circ, 135^\circ$

(B) $\theta = 135^\circ, 225^\circ$

(C) $\theta = 225^\circ, 315^\circ$

(D) $\theta = 315^\circ, 45^\circ$

7 Simplify $\lim_{h \to 0} \frac{4(x+h)^2 - 4x^2}{h}$

(A) $8h$

(B) $8xh + 8h$

(C) $8x + 8h$

(D) $8x$

8 The line $y = mx - 2$ is a tangent to the curve $y = 3x^2 - 2x + 1$ at the point $(1, 2)$.

What is the value of m?

(A) 4

(B) $\frac{1}{2}$

(C) –4

(D) 2

9 The function $y = f(x)$ is defined as:

$$f(x) = \begin{cases} x^2 + 1 & x < 0 \\ 1 - x^2 & x \geq 0 \end{cases}$$

What is the range of $f(x)$?

(A) $y \leq -1, y \geq 1$

(B) all real y

(C) $y < -1, y \geq 1$

(D) $-1 \leq y \leq 1$

10 If $8^{x+3} \times 2^{x-2} = 2^x \times 4^{3x-1}$, what is the value of x?

(A) 0

(B) 2

(C) 3

(D) -3

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Question 11 (15 marks) **Use a SEPARATE writing booklet.**

(a) Evaluate $\sqrt{\dfrac{2.6^2+5.2^2}{1.3}}$ correct to 2 significant figures. **2**

(b) Expand and simplify $5(x+3)^2 - x(7-2x)$. **2**

(c) Factorise $3x^2+15x-72$. **2**

(d) Rationalise the denominator of the expression $\dfrac{4+\sqrt{6}}{\sqrt{2}}$. **2**

(e) The point $M(-4,1)$ is the midpoint of the points $A(2,6)$ and $B(k,-4)$. Find the value of k. **1**

(f) Determine if the function $f(x)=3x^4-4x^2$ is even, odd or neither. **2**

(g) Find the equation of the circle with radius 3 units and centre $(0,-4)$. **1**

(h) Shade the region defined by $y \geq x^2+1$, $x \geq 0$ and $y \leq 3$. Show all y-intercepts and points of intersection. **3**

End of Question 11

Question 12 (15 marks) **Use a SEPARATE writing booklet.**

(a) Differentiate the following functions with respect to x

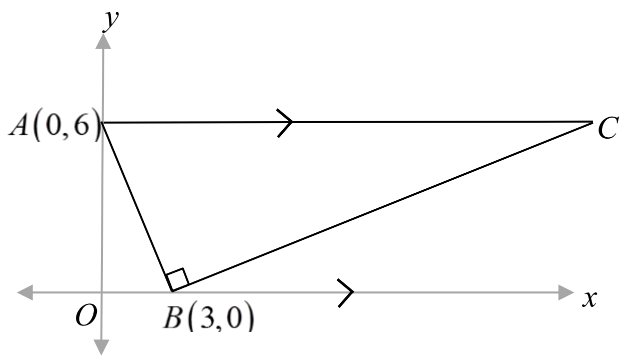
(i) $f(x) = 4x^2 - 6x - 9$ **1**

(ii) $f(x) = \dfrac{x}{\sqrt{x}}$ **2**

(iii) $f(x) = \dfrac{2x^3}{3 + x^2}$ **2**

(b) Differentiate from first principles $f(x) = 2x^2 - 3x + 7$ **2**

(c) The diagram shows the triangle ABC. The coordinates of A are $(0,6)$ and the coordinates of B are $(3,0)$. BC is perpendicular to AB. AC is parallel to the x-axis.



NOT TO SCALE

(i) Find the gradient of line BC. **1**

(ii) Show that the equation of the line BC is $x - 2y - 3 = 0$ **1**

(iii) Hence or otherwise, show that the coordinates of the point C are $(15,6)$. **1**

(iv) Find the length of BC. Express your answer in exact simplified form. **2**

(v) Find the size of $\angle ACB$, correct to the nearest degree. **2**

(vi) Find the area of triangle ABC. **1**

End of Question 12

Question 13 (15 marks) **Use a SEPARATE writing booklet.**

(a) If $f(x) = \begin{cases} 1 & x < 0 \\ x+2 & x \geq 0 \end{cases}$

(i) Evaluate $f(-4) + f\left(-\frac{1}{2}\right) + f(0)$ **2**

(ii) Sketch $y = f(x)$ for $-3 \leq x \leq 3$ **2**

(b) Given $\sin\theta = -\frac{4}{5}$ and $\tan\theta > 0$, find the exact value of

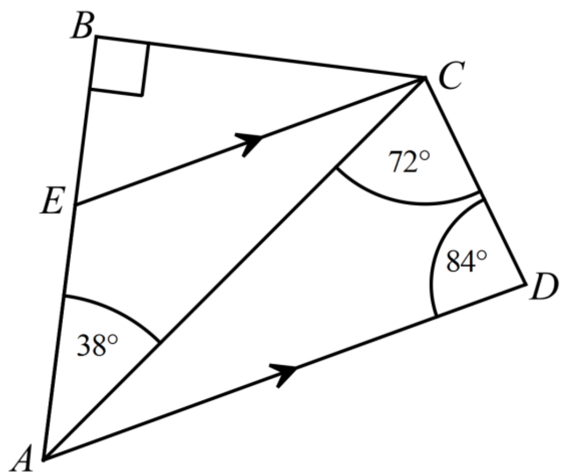
(i) $\cos\theta$ **1**

(ii) $\cot\theta$ **1**

(c) Prove $\dfrac{\sin A}{1+\cos A} + \dfrac{1+\cos A}{\sin A} = 2\operatorname{cosec} A$ **3**

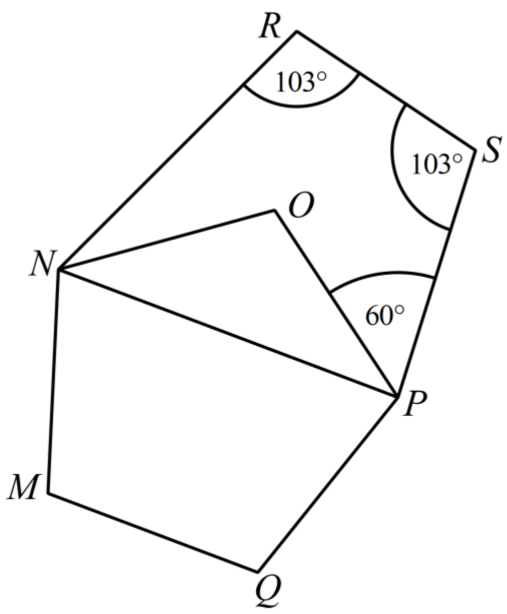
(d) Find the gradient of the normal to the curve $f(x) = (2x+3)^2$ at the point where $x = -1$. **2**

(e) *ABCD* is a quadrilateral, with the diagonal *AC* drawn as shown. *E* is a point on *AB*, such that *EC* || *AD*. Find the size of $\angle AEC$ and $\angle BCE$. **2**



Question 13 Continues on Page 9

(f) The polygon $MNOPQ$ is a regular pentagon $\angle OPS = 60°$, $\angle RSP = 103°$ and $\angle NRS = 103°$ as shown in the diagram.



(i) Find the size of $\angle ONP$. **1**

(ii) Find the size of $\angle ONR$. **1**

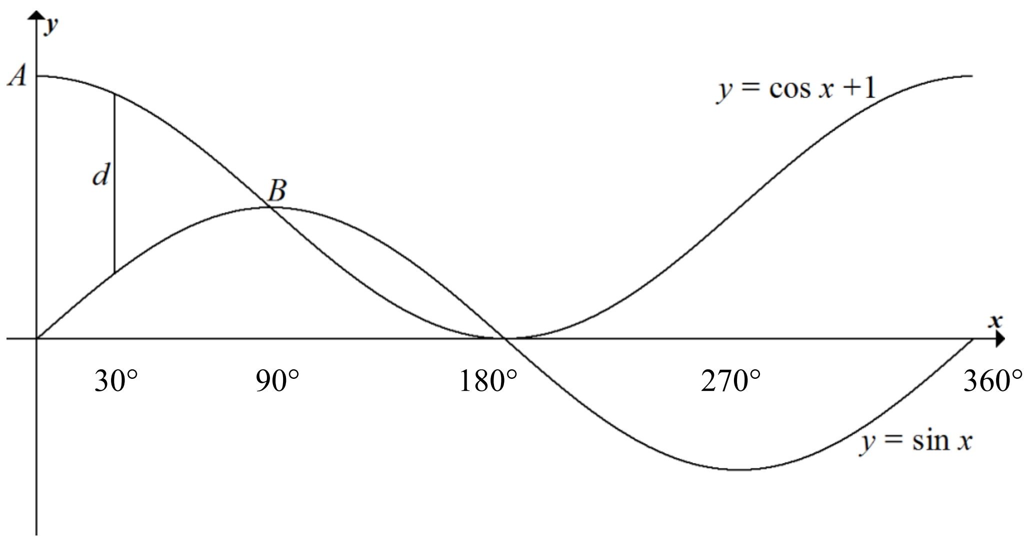
End of Question 13

Question 14 (15 marks) **Use a SEPARATE writing booklet.**

(a) Solve $\cos^2\theta = \frac{1}{4}$ where $0 \le \theta \le 360°$. **2**

(b) Solve for x, $\log_5\left(\frac{x+10}{x-2}\right) = 2$. **2**

(c) The diagram shows the graphs $y = \sin x$ and $y = \cos x + 1$ for $0° \le x \le 360°$.



(i) State the coordinates of the point A. **1**

(ii) Determine the y-ordinate of the point B. **1**

(iii) Find the coordinates of the first point of intersection of the two graphs for $x \ge 360°$. **1**

(iv) Let d be the vertical distance between the two graphs when $x = 30°$. Find the exact value of d. **1**

(v) Find the next value of x where the vertical distance between the two graphs equals d. **1**

Question 14 Continues on Page 11

(d) A yacht sails 640 metres from point P to point A on a bearing of $050°$. It then sails 960 metres from point A to point B on a bearing of $120°$.

(i) Draw a clear diagram showing the above information. **2**

(ii) Find the distance of point B from point P correct to the nearest metre. **2**

(iii) Find the bearing of point P from point B correct to the nearest degree. **2**

End of Examination

Y11 2U Prelim 2017 Solutions

Q1 $x + 4 \geq 0$

$x \geq -4$ C

Q2 $\frac{\sin\theta}{18} = \frac{\sin 40^\circ}{11}$

$\sin\theta = \frac{18\sin 40^\circ}{11}$ D

Q3 $m = \frac{y_2 - y_1}{x_2 - x_1}$ A$(3, y)$ B$(7, 2)$ $m = -3$

$-3 = \frac{y-2}{3-7}$

$-3 = \frac{y-2}{-4}$

$12 = y - 2$

$\therefore y = 14$ A

Q4 $|3x - 5| = 10$

$3x - 5 = 10$ $3x - 5 = -10$

$3x = 15$ $3x = -5$

$x = 5$ $x = \frac{-5}{3}$ C

Q5 $x^3 - x^2 - 5x = 0$

$x(x^2 - x - 5) = 0$

$x = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{1 \pm \sqrt{1 + 20}}{2}$

$= \frac{1 \pm \sqrt{21}}{2}$ B

$\therefore$ 3 solutions

Q6 $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore\ \theta = 45°, 135°$ A

Q7 $\lim_{h\to 0} \frac{4(x^2 + 2xh + h^2) - 4x^2}{h}$

$= \lim_{h\to 0} \frac{\cancel{4x^2} + 8xh + 4h^2 - \cancel{4x^2}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(8x + 4h)}{\cancel{h}}$

$= \lim_{h\to 0} 8x + 4h$

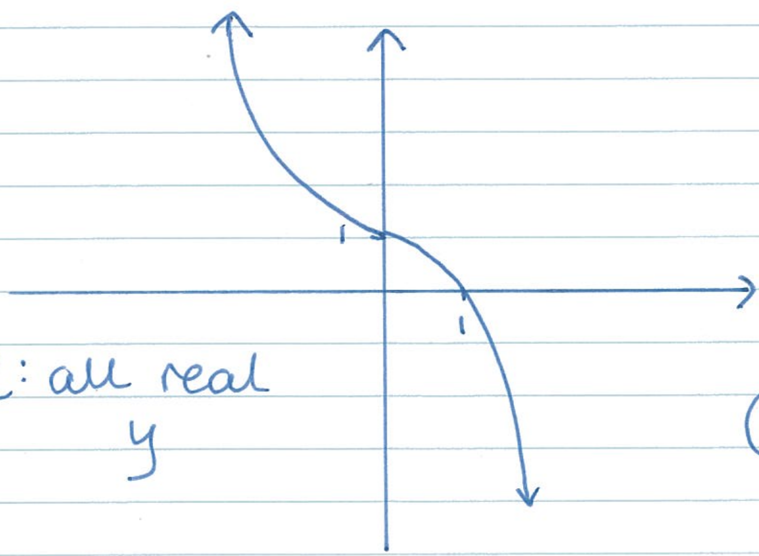
$= 8x$ D

Q8 $y' = 6x - 2$ $(1,2)$

$m = 6(1) - 2$

$m = 4$ A

Q9



R: all real y

B

Q10 $8^{x+3} \times 2^{x-2} = 2^{x} \times 4^{3x-1}$

$2^{3(x+3)} \times 2^{x-2} = 2^{x} \times 2^{2(3x-1)}$

$2^{3x+9+x-2} = 2^{x+6x-2}$

$2^{4x+7} = 2^{7x-2}$

$4x+7 = 7x-2$ C

$9 = 3x$

$\therefore x = 3$

SECTION II

Question 11

a) $= 5.0990\ldots$

$= 5.1$

b) $= 5(x^2+6x+9) - 7x + 2x^2$

$= 5x^2 + 30x + 45 - 7x + 2x^2$

$= 7x^2 + 23x + 45$

c) $= 3(x^2 + 5x - 24)$

$= 3(x+8)(x-3)$

d) $= \frac{4+\sqrt{6}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{4\sqrt{2} + \sqrt{12}}{2}$

$= \frac{4\sqrt{2} + 2\sqrt{3}}{2} = 2\sqrt{2} + \sqrt{3}$

e) $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$\therefore -4 = \frac{2+k}{2}$

$-8 = 2 + k$

$k = -10$

f) $f(x) = 3x^4 - 4x^2$

$$f(-x) = 3(-x)^4 - 4(-x)^2$$
$$= 3x^4 - 4x^2$$
$$= f(x)$$

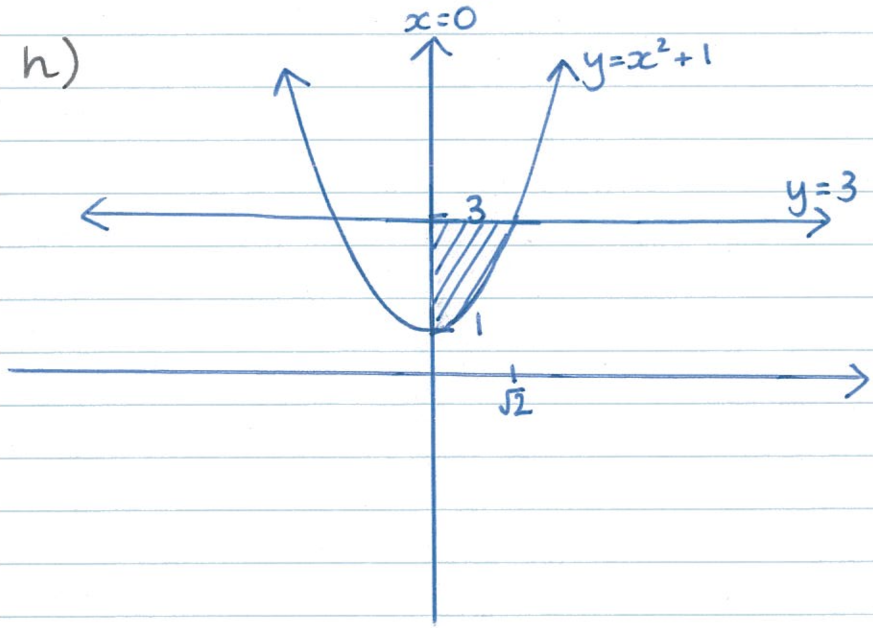
$\therefore$ even

g) $(x-h)^2 + (y-k)^2 = r^2$

$\therefore (x-0)^2 + (y+4)^2 = 9$

$$x^2 + (y+4)^2 = 9$$

h)



Question 12

a) i. $f'(x) = 8x - 6$

ii. $f(x) = x \cdot x^{-\frac{1}{2}}$
$= x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2} \cdot x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}}$

iii. $f'(x) = \frac{6x^2(3+x^2) - 2x^3(2x)}{(3+x^2)^2}$

$= \frac{18x^2 + 6x^4 - 4x^4}{(3+x^2)^2}$

$= \frac{18x^2 + 2x^4}{(3+x^2)^2}$

b) $f(x) = 2x^2 - 3x + 7$

$f(x+h) = 2(x+h)^2 - 3(x+h) + 7$
$= 2(x^2 + 2xh + h^2) - 3x - 3h + 7$
$= 2x^2 + 4xh + 2h^2 - 3x - 3h + 7$

$f'(x) = \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{- 3x} - 3h \cancel{+ 7} - (\cancel{2x^2} \cancel{- 3x} \cancel{+ 7})}{h}$

$= \lim_{h \to 0} \frac{4xh + 2h^2 - 3h}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h - 3)}{\cancel{h}}$

$= \lim_{h \to 0} 4x + 2h - 3$

$= 4x - 3$

c) i. $m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{0-6}{3-0}$

$= -2$

$AB \perp AC \therefore m_{AB} \times m_{AC} = -1$

$\therefore m_{AC} = \frac{1}{2}$

ii. $y - y_1 = m(x - x_1)$

$y - 0 = \frac{1}{2}(x-3)$

$2y = x - 3$

$\therefore x - 2y - 3 = 0$

iii. $y = 6, \quad x - 2y - 3 = 0$

$x - 12 - 3 = 0$

$x = 15$

$\therefore C(15, 6)$

iv. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(15-3)^2 + (6-0)^2}$

$= \sqrt{144 + 36}$

$= \sqrt{180}$

$= 6\sqrt{5}$

v. $d_{AB} = \sqrt{(3-0)^2 + (0-6)^2}$

$= \sqrt{45}$

$= 3\sqrt{5}$

$\tan\theta = \dfrac{AB}{BC}$

$\tan\theta = \dfrac{3\sqrt{5}}{6\sqrt{5}}$

$\tan\theta = \frac{1}{2}$

$\theta = 27°$

vi. $A = \frac{1}{2} bh$

$= \frac{1}{2} \times 3\sqrt{5} \times 6\sqrt{5}$

$= 45 u^2$

Question 13

a) i. $f(-4) = 1$

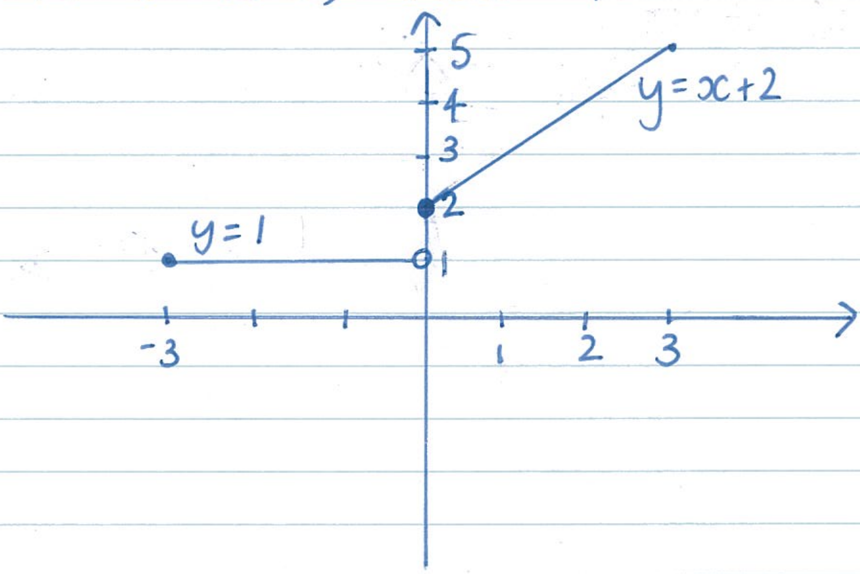
$f(-\frac{1}{2}) = 1$

$f(0) = 0+2$

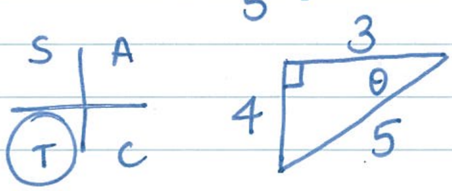
$= 2$

$\therefore f(-4) + f(-\frac{1}{2}) + f(0) = 4$

ii.



b) $\sin\theta = \frac{-4}{5}$, $\tan\theta > 0$



i. $\cos\theta = -\frac{3}{5}$

ii $\cot\theta = \frac{3}{4}$

c) $\dfrac{\sin A}{1+\cos A} + \dfrac{1+\cos A}{\sin A} = 2\operatorname{cosec} A$

$$\text{LHS} = \frac{\sin^2 A}{(1+\cos A)\sin A} + \frac{(1+\cos A)^2}{(1+\cos A)\sin A}$$

$$= \frac{\sin^2 A + 1 + 2\cos A + \cos^2 A}{(1+\cos A)\sin A}$$

$$= \frac{1 + 1 + 2\cos A}{(1+\cos A)\sin A}$$

$$= \frac{2\cancel{(1+\cos A)}}{\cancel{(1+\cos A)}\sin A}$$

$$= 2\operatorname{cosec} A$$

$$= \text{RHS}$$

d) $f'(x) = 2(2x+3) \times 2$

$f'(-1) = 2(2(-1)+3) \times 2$

$= 4$

$\therefore \; m_T = 4$

$m_T \times m_N = -1$

$\therefore \; m_N = \dfrac{-1}{4}$

e) $\angle CAD + 72° + 84° = 180°$ (angle sum of a triangle is 180°)

$\angle CAD = 24°$

$\angle CAD = \angle ACE$ (alternate angles on parallel lines are equal)

$\angle AEC + 38° + 24° = 180°$ (angle sum of a triangle is 180°) 240

$\angle AEC = 118°$

$\angle BEC + 118° = 180°$ (angles on a straight line are supplementary)

$\angle BEC = 62°$

$62° + 90° + \angle BCE = 180°$ (angle sum of a triangle is 180°)

$\angle BCE = 28°$

f) i. $\angle PON = \dfrac{180°(n-2)}{n}$ (interior angle of a regular polygon)

$= 108°$

$\angle ONP = \dfrac{180° - 108°}{2}$ (base angles of an isosceles triangle are equal)

$\angle ONP = 36$

ii. reflex $\angle PON = 360° - 108°$ (angles at a point add to 360°)

$= 252°$

$252° + 60° + 103° + 103° + \angle ONR = 540°$ (angle sum of a pentagon is 540°)

$\therefore \angle ONR = 22°$

c) i. A $(0,2)$

ii. $x = 90°$

$y = \sin(90°)$

$y = 1$

iii. $(450°, 1)$

iv. $y = \cos 30° + 1$

$y = \sin 30°$

$\therefore d = \cos 30° + 1 - \sin 30°$

$= \frac{\sqrt{3}}{2} + 1 - \frac{1}{2}$

$= \frac{\sqrt{3} + 2 - 1}{2}$

$= \frac{\sqrt{3} + 1}{2}$

v. $y = \cos 240° + 1 - \sin 240°$

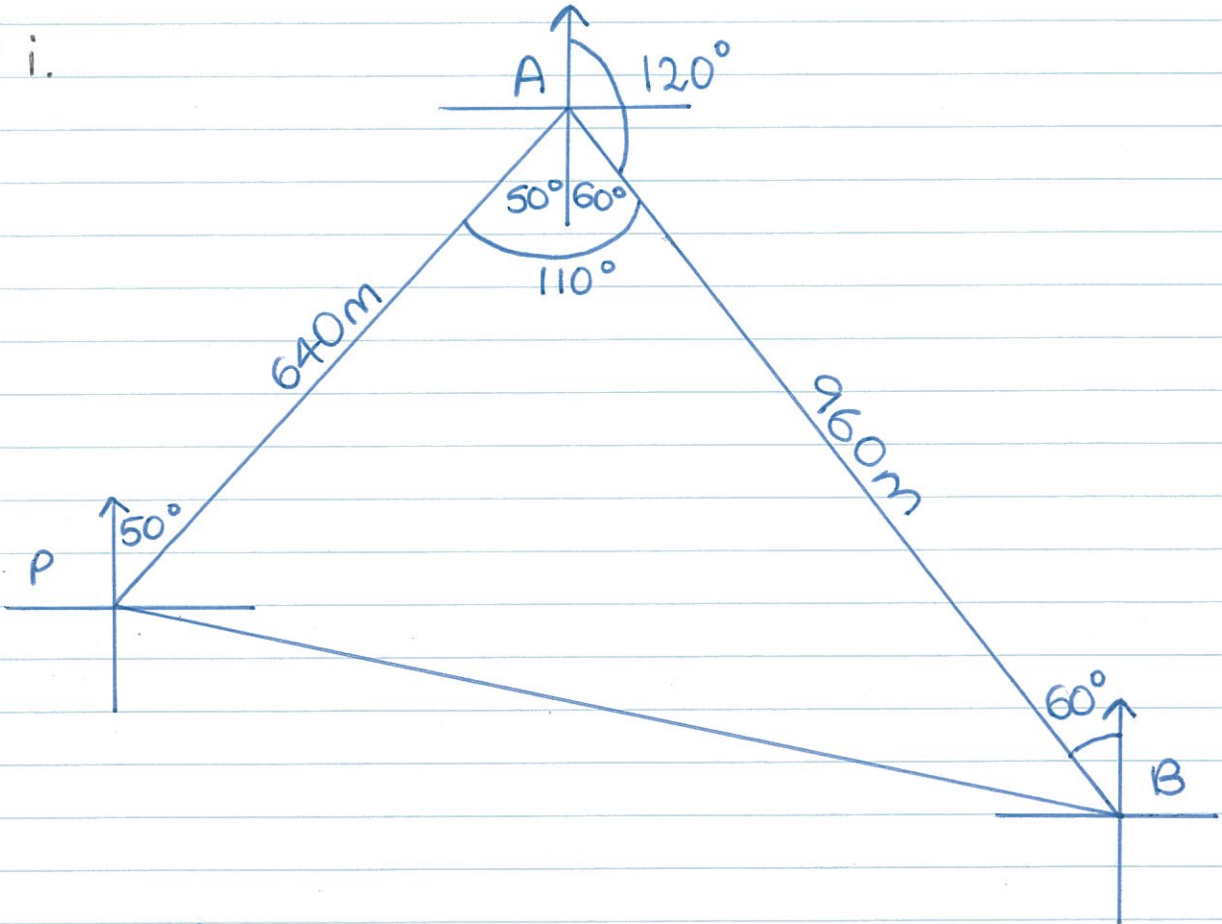
$= -\frac{1}{2} + 1 + \frac{\sqrt{3}}{2}$

$= \frac{-1 + 2 + \sqrt{3}}{2}$

$= \frac{1 + \sqrt{3}}{2}$

$\therefore$ next value of x is $240°$.

d) i.



ii. $(PB)^2 = 640^2 + 960^2 - 2 \times 640 \times 960 \cos 110°$

$PB = 1323.4...$
$PB = 1323m$

iii. $\frac{\sin \angle APB}{640} = \frac{\sin 110°}{1323}$

$\sin \angle ABP = \frac{640 \sin 110°}{1323}$

$\angle ABP = 27.0....°$
$= 27°$

$\therefore$ Bearing of P from B $= 360° - 27° - 60°$
$= 273°$