St George Girls High School

Year 11

End of Preliminary Course Examination

2011

Mathematics

Time Allowed: 3 hours
(plus 5 minutes reading time)

Instructions
1. Attempt all 8 questions.
2. All necessary working must be shown.
3. Begin each question on a **new page**.
4. Marks will be deducted for careless work or poorly presented solutions.

Question 1 (14 marks) – Start a new page

Marks

a) Expand and simplify $5x - 3(2x - 7)$

1

b) Factorise

(i) $4m - 16m^2$

1

(ii) $x^2 - 3x - 18$

1

(iii) $8 - 27x^3$

1

(iv) $x^3 + 5x^2 - 4x - 20$

2

c) Simplify

(i) $\dfrac{4x+8}{x} \times \dfrac{x^2}{x^2-4}$

2

(ii) $\dfrac{5}{x+2} - \dfrac{3}{x+3}$

2

d) Express $0.0\dot{5}$ as a simplified fraction.

2

e) Solve by "completing the square":

2

$$x^2 + 6x - 10 = 0$$

Question 2 (14 marks) – Start a new page   Marks

a)   Simplify:

   (i) $\sqrt{99}$ 1

   (ii) $\sqrt{54} - \sqrt{18} + \sqrt{50} - \sqrt{24}$ 2

b)   Solve for x:

   (i) $x^2 = 28 - 3x$ 2

   (ii) $\dfrac{2x-7}{x+3} = \dfrac{4}{3}$ 2

c)   Solve the simultaneous equations   2

$$2x - 5y = -2$$
$$3x + 5y = 17$$

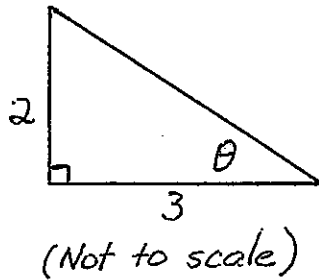d)   Solve $|3x - 2| \le 4$ and graph your solution on a number line.   3

e)   Show that $\dfrac{4}{3-\sqrt{3}} - \dfrac{2}{\sqrt{3}}$ is rational   2

Question 3 (14 marks) – Start a new page   Marks

a)

(Not to scale)

Write down the exact value of $\sec\theta$ 1

b)   If $\cos\theta = -0.6$ and $0° \leq \theta \leq 360°$ find all values of θ to the nearest minute.   2

c)   If $\tan\theta = -\dfrac{4}{9}$ and $\sin\theta < 0$ write down the exact value of:

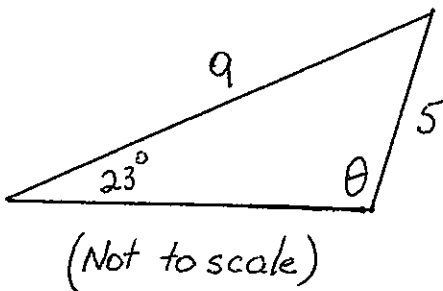(i) $\cos\theta$ 1

(ii) $\operatorname{cosec}\theta$ 1

d)   What is the exact value of $\sin 315°$?   1

e)   Prove that $\dfrac{1}{1+\cos\theta} + \dfrac{1}{1-\cos\theta} = 2\operatorname{cosec}^2\theta$ 2
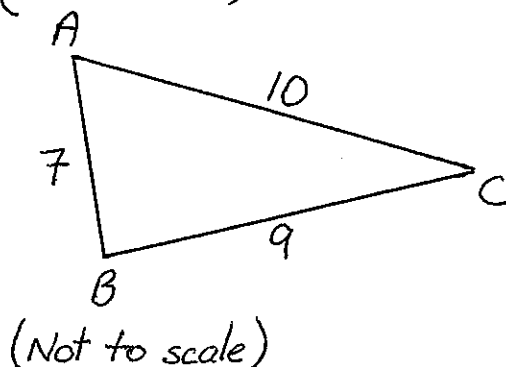
f)

(Not to scale)

Find θ to the nearest degree.   3

g)

(Not to scale)

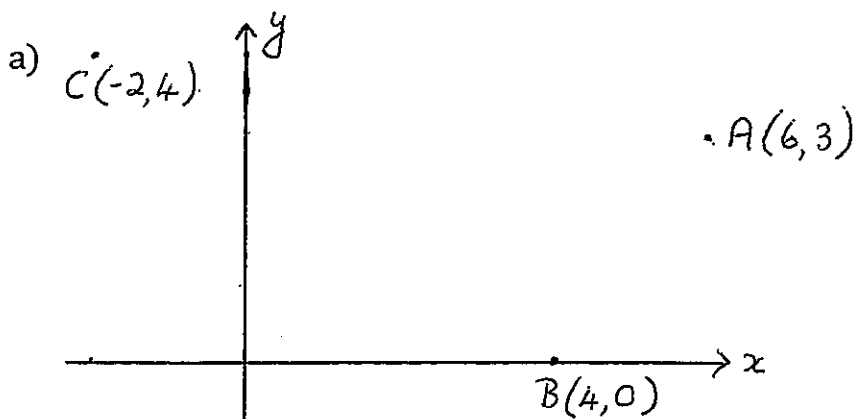Find, to the nearest minute, the size of the largest angle in $\triangle ABC$ 3

Question 4 (14 marks) – Start a new page   Marks

a)

$A(6,3)$, $B(4,0)$ and $C(-2,4)$ are shown on the diagram.

(i)   Find the gradient of AB .   1

(ii)   Show that $AB \perp BC$ 2

(iii) Find the coordinates of M the midpoint of AC.   1

(iv)   If $ABCD$ is a rectangle, find the coordinates of D.   2

b)   Find the angle of inclination of the line $y = -3x + 5$.   2
     (Answer to the nearest degree)

c)   Find the perpendicular distance from the point $(5,2)$ to the line $3x - 4y = 1$ 2

d)   Show that the points $P(1,-1)$, $Q(2,1)$ and $R(5,\ 7)$ are collinear.   2

e)   Find the value of k if the lines $2x + ky + 7 = 0$ and $3x - 2y + 8 = 0$ are
     parallel.   2

Question 5 (14 marks) – Start a new page

a)   Simplify $2^{3x+2} \times 4^{-x}$

1

b)   Solve $9^{x+1} = 27$

2

c)   If $\log_x 3 = m$ and $\log_x 2 = n$ find

(i) $\log_x 12$

1

(ii) $\log_x 64$

1

(iii) $\log_x (3x^2)$

1

d)   Solve for x, giving answer correct to 3 decimal places

(i) $2^x = 36$

2

(ii) $3^{x-2} < 10\,000$

2

d)   On separate diagrams sketch the graphs of the following functions, showing all important features.

(i) $y = 3^x$

1

(ii) $y = 4 - 2^x$

3

Question 6 (14 marks) – Start a new page   Marks

a)   For the arithmetic series $7 + 10 + 13 + \cdots$ find:

   (i)   an expression for T_n, the nth term.   2

   (ii)   the number of terms less than 1000.   2

b)   A geometric series has $T_3 = 108$ and $T_6 = 32$. Find the first term, a, and common ratio, r.   2

c)   Find the value of x, given that $\log_3 54$, $\log_3 x$, $\log_3 6$ are successive terms of an arithmetic series.   2

d)   For the arithmetic series $26 + 22 + 18 + \cdots$ find the number of terms if the sum is 90.   4

e)   Find   2

$$\sum_{k=1}^{\infty} 5 \times \left(\frac{2}{3}\right)^k$$

Question 7 (14 marks) – Start a new page   **Marks**

a)

(−2,−1)
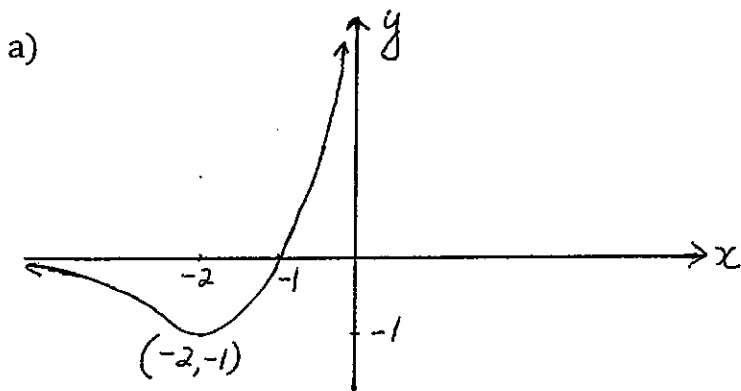
Copy and complete the graph of $y = f(x)$ given that $f(x)$ is an odd function.   2

b) Draw neat sketches of each of the following graphs, showing all relevant features.
    Each of your graphs should be at least $\frac{1}{4}$ of a page.

   (i) $x^2 + (y - 1)^2 = 1$ 2

   (ii) $y = |2x - 3|$ 2

   (iii) $y = \dfrac{1}{x-2}$ 2

   (iv) $y = \begin{cases} x + 2 & \text{for } x < 0 \\ 1 - x^2 & \text{for } x \geq 0 \end{cases}$ 2

c) Show that the function $g(x) = \dfrac{x^2}{x^4+4}$ is even.   2

d) Sketch the region defined by $y \leq \sqrt{1 - x^2}$ 2

Question 8 (14 marks) – Start a new page   **Marks**

a)   Differentiate with respect to x

    (i) $y = 3x^4 - x + 7$ 1

    (ii) $f(x) = (8x + 5)^6$ 2

    (iii) $g(x) = \dfrac{5}{x^3}$ 1

    (iv) $y = \dfrac{x^2}{\sqrt[3]{x}}$ 2

b)   Find the equation of the tangent to the curve $y = x^3 - 2x^2 + 5$ at the point where $x = 2$ 4

c)   If $y = x\sqrt{x + 1}$ find:

    (i) $\dfrac{dy}{dx}$ 2

    (ii)   the coordinates of the point at which $\dfrac{dy}{dx} = 0$ 2

End of Paper