St George Girls High School

Year 11

End of Preliminary Course Examination

2012

Mathematics

Time Allowed: 3 hours
(plus 5 minutes reading time)

Instructions

1. Attempt **all** questions.
2. All necessary working must be shown.
3. Marks will be deducted for careless work or poorly presented solutions.

Part A
Total marks (12)
Attempt Questions 1 – 12
Use the answer sheet provided

Part B
Total marks (84)
Attempt Questions 13 – 19
Start each question in a new booklet

Student Name: _____

Part A

Multiple-choice Answer Sheet - Questions 1 – 12

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample   2 + 4 =   (A) 2   (B) 6   (C) 8   (D) 9

A ○   B ●   C ○   D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ●   B ✖   C ○   D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

↙*correct*

A ✖   B ✖   C ○   D ○

1.	A	○	B	○	C	○	D	○
2.	A	○	B	○	C	○	D	○
3.	A	○	B	○	C	○	D	○
4.	A	○	B	○	C	○	D	○
5.	A	○	B	○	C	○	D	○
6.	A	○	B	○	C	○	D	○
7.	A	○	B	○	C	○	D	○
8.	A	○	B	○	C	○	D	○
9.	A	○	B	○	C	○	D	○
10.	A	○	B	○	C	○	D	○
11.	A	○	B	○	C	○	D	○
12.	A	○	B	○	C	○	D	○

Part A

Multiple Choice Questions 1 - 12
All questions are of equal value (1 mark each) - (12 Marks)
Select the alternative A, B, C or D that best answers the question. Fill in the response oval on the Answer Sheet.

1. What is $\dfrac{1+\sqrt{3}}{5-2\sqrt{3}}$ as a fraction with a rational denominator?

    (A) $\dfrac{-5-\sqrt{3}}{7}$

    (B) $\dfrac{-5+\sqrt{3}}{13}$

    (C) $\dfrac{11-7\sqrt{3}}{7}$

    (D) $\dfrac{11+7\sqrt{3}}{13}$

2. The formula $H = 5m(Y - X)$ is used to calculate the heat (H) required to raise the temperature of a steel rod, of mass m, from a temperature of X to a temperature of Y. Rearrange the formula to make X the subject.

    (A) $X = \dfrac{5m - H}{Y}$

    (B) $X = \dfrac{H - 5m}{Y}$

    (C) $X = \dfrac{H - 5mY}{5m}$

    (D) $X = \dfrac{5mY - H}{5m}$

3. What is the domain and range of the function $f(x) = \sqrt{9 - x^2}$?

    (A) Domain: $-3 \le x \le 3$, Range: $0 \le y \le 3$

    (B) Domain: $-3 \le x \le 3$, Range: $-3 \le y \le 3$

    (C) Domain: $0 \le x \le 9$, Range: $-9 \le y \le 9$

    (D) Domain: $0 \le x \le 9$, Range: $0 \le y \le 9$

Part A (cont'd)

4. What is the value of $\lim\limits_{x \to c} \dfrac{x^3 - c^3}{x - c}$?

    (A) Undefined

    (B) c^2

    (C) $3c^2$

    (D) c^3

5. Alex leaves point X and walks on a bearing of 230°.

    Brooke leaves point X and walks on a bearing of S70°E.
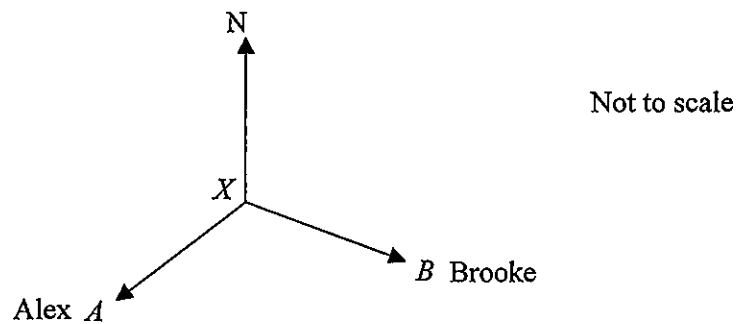
    Not to scale

    What is the angle AXB?

    (A) 50°

    (B) 120°

    (C) 160°

    (D) 300°

6. What is the correct expression for AC in triangle ABC?

    (A) $\dfrac{15 \sin 80°}{\sin 40°}$

    (B) $\dfrac{15 \sin 80°}{\sin 60°}$

    (C) $\dfrac{15 \sin 40°}{\sin 60°}$

    (D) $\dfrac{\sin 40°}{15 \sin 80°}$
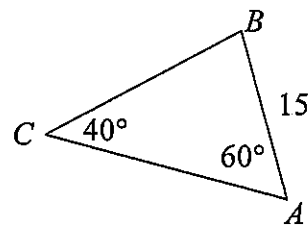
Part A (cont'd)

7. What is the value of x in the diagram?

    (A) $6\cos 42°$

    (B) $\dfrac{6}{\cos 42°}$

    (C) $6\sin 42°$

    (D) $\dfrac{6}{\sin 42°}$
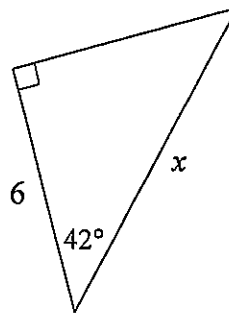
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8. What is the value of $\dfrac{a}{b}$ if the lines $ax + 2y = 6$ and $4y = bx - 9$ are parallel?

    (A) $\dfrac{1}{2}$

    (B) $-\dfrac{1}{2}$

    (C) -2

    (D) 2

9. Which of the following could be the graph of $y = \dfrac{2}{3}x - 2$?

    (A)

    (B)

    (C)

    (D)
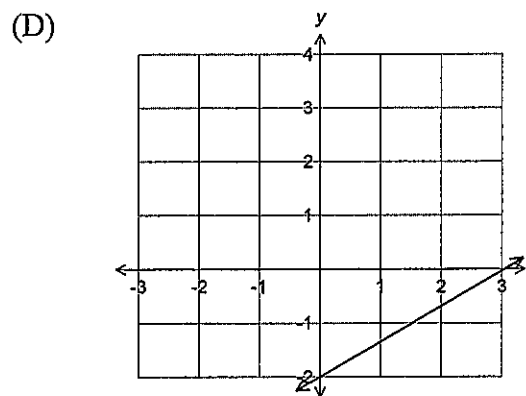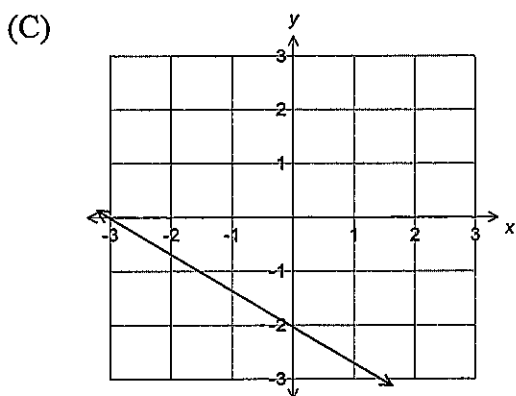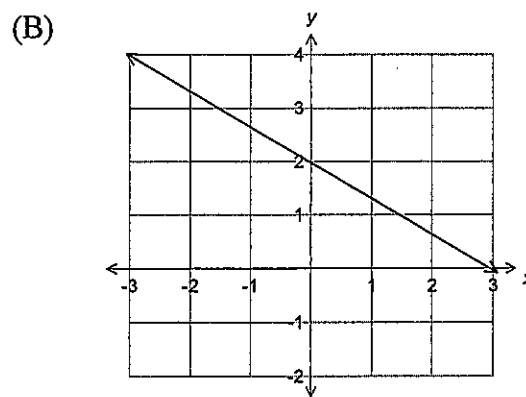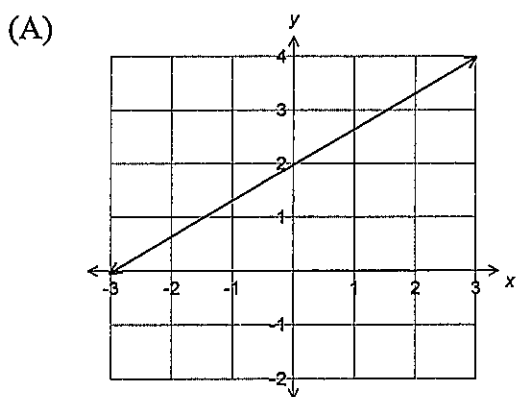
Part A (cont'd)

10. What is the value of $\frac{dy}{dx}$ if $y = 2\sqrt{x}$?

(A) $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$

(B) $\frac{dy}{dx} = \frac{2}{\sqrt{x}}$

(C) $\frac{dy}{dx} = \frac{\sqrt{x}}{2}$

(D) $\frac{dy}{dx} = 2$

11. What is the gradient of the curve $y = x^2 - x - 6$ at $(6,24)$?

(A) 11
(B) 12
(C) 23
(D) 24

12. What is the value of $f'(3)$ if $f(x) = 3x - x^3$?

(A) $f'(3) = -24$

(B) $f'(3) = -18$

(C) $f'(3) = 0$

(D) $f'(3) = 9$

Part B

<u>**Question 13 – Start a New Booklet – (12 Marks)**</u>   **Marks**

a)   Evaluate, correct to three significant figures.   2

$$\sqrt{\frac{1}{8.6} + 2.5^2}$$

b)   Factorise: $12x^2y^2 - 27y^2$ 2

c)   Simplify: $\sqrt{5} \times \sqrt{20}$ 1

d)   Solve: 2

$$\frac{2x}{3} - \frac{x+1}{4} = 2$$

e)   Solve the simultaneous equations   3

$$x - 3y = 11$$
$$2x + y = 8$$

f)   Simplify: 2

$$\frac{y^3 + 8}{2y^2 - 4y + 8}$$

___

Question 14 – Start a New Booklet – (12 Marks)   Marks

a)   Express in simplest exact form $\dfrac{ab^2}{c^3}$ where   2

$$a = \left(\frac{3}{2}\right)^3, \; b = \left(\frac{2}{3}\right)^2 \text{ and } c = \left(\frac{3}{2}\right)^{-2}$$

b)   Simplify:

   (i) $3^n \times 9^{n+1}$ 1

   (ii) $\dfrac{4^{n-1} \times 8^{2n}}{16^{2n-1}}$ 2

c)   If $\log_a 5 = p$ and $\log_a 2 = q$, find an expression in terms of p and q for   2

   $\log_a \left(\dfrac{8}{5}\right)$

d)   Evaluate correct to one decimal place $\log_2 9$ 2

e)   On separate diagrams draw a neat sketch, showing essential features of
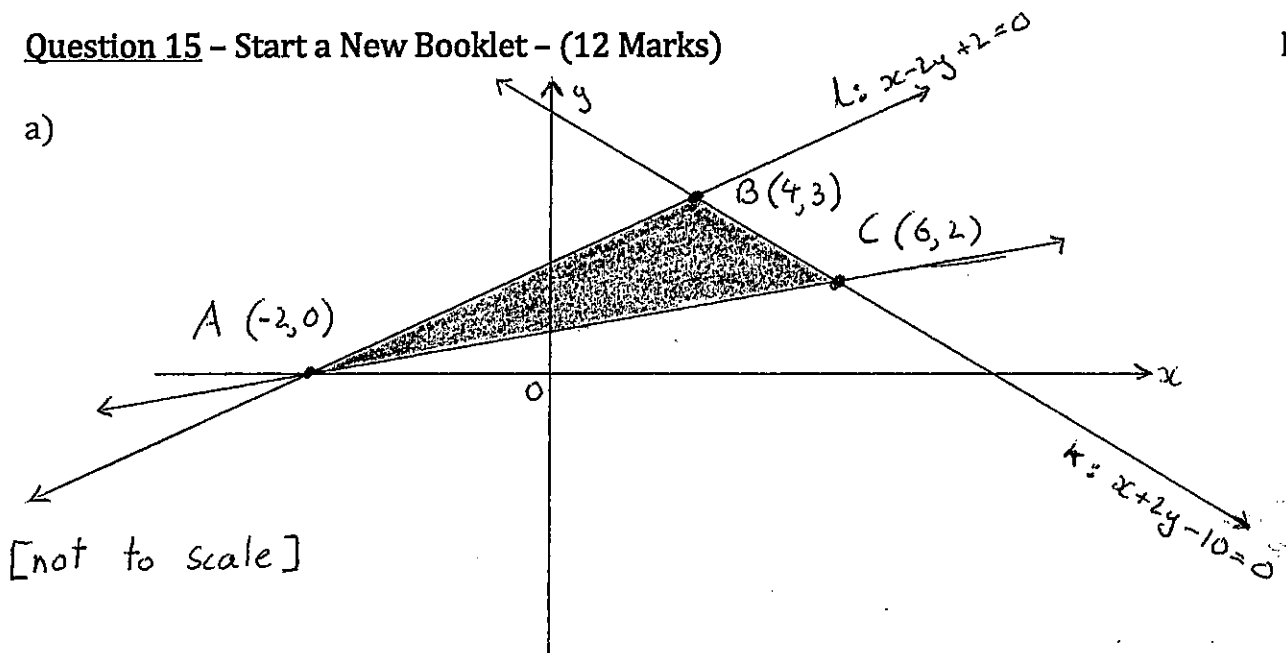
   (i) $y = 2^x$ 1

   (ii) $y = 1 - 2^x$ 2

<u>Question 15 – Start a New Booklet – (12 Marks)</u>   Marks

a)

[not to scale]

The equations of the lines $l: x - 2y + 2 = 0$ and $k: x + 2y - 10 = 0$ are given

(i)   Find the exact distance between $A(-2, 0)$ and $C(6, 2)$ 1

(ii) Show that the equation of the line through the points A and C is
      $x - 4y + 2 = 0$ 2

(iii) Find the perpendicular distance from B to the line AC 2

(iv) Find the area of $\triangle ABC$ in exact form.   2

(v)   State the three inequalities which simultaneously describe the interior of the
      shaded triangle ABC.   2

(vii) Find the coordinates of D such that the quadrilateral $ABCD$ is a
      parallelogram.   1

b)   Find the equation of the line which is perpendicular to the line with equation
     $3x - y + 5 = 0$ and passing through the point $M(-1, 4)$ 2

Question 16 – Start a New Booklet – (12 Marks)   Marks

a)   Simplify:

   (i) $\sec\theta.\cos\theta$ 1

   (ii) $\cos^3\theta + \sin^2\theta.\cos\theta$ 1

b)   Find the exact value of x, given   2
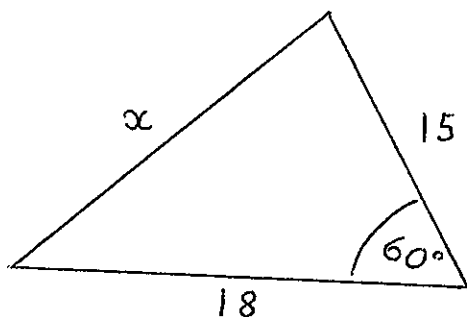
c)   If $3\sin\theta + 2 = 0$, find θ to the nearest minute where $0° \leq \theta° \leq 360°$ 2

d)   If $\sec\theta = -\dfrac{5}{2}$ and $\sin\theta < 0$ give the exact value of

   (i) $\sin\theta$ 1

   (ii) $\cot\theta$ 1

Question 16 (cont'd)   **Marks**

e) A bushwalker walked from point A on a bearing of 117° for 50 km to a point B. From point B the walker changed course to a bearing of 215° and walked a further 40 km to point C.
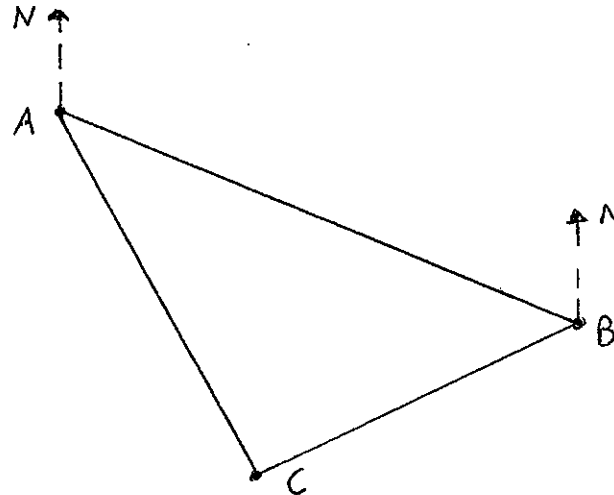
[not to scale]

(i) Find the exact size of $\angle ABC$ 1

(ii) Find the size of angle $\angle CAB$ if $AC = 59.5$ km. [correct to 1 nearest minute]   2

(iii) What is the bearing of A from C, if C is $S12°E$ of A?   1

Question 17 – Start a New Booklet – (12 Marks)   Marks

a) In a certain series, the first term is 17 and the eighth term is -4

    (i) find the common difference.   2

    (ii) find the value of the fifth term.   1

b) A geometric series has a first term of 24 and a limiting sum of 128

    (i) find its common ratio   2

    (ii) find the 3rd term of this series   2

c) Find   2

$$\sum_{n=1}^{6} (5n^2 - 3n)$$

d) What is the least number of terms of the series $6 + 12 + 24+...$ required to be taken to give a sum greater than 2000.   3

Question 18 – Start a New Booklet – (12 Marks)   Marks

a) Draw a neat sketch showing all essential features of the following, on separate
    number planes

(i) $f(x) = \begin{cases} x^2 & \text{for } -4 \leq x \leq 1 \\ x & \text{for } 1 < x \leq 4 \end{cases}$ 2

(ii) $y = -\sqrt{9 - x^2}$ 2

(iii) $y = |x - 2|$ 2

b) If $g(x) = 3x - 5$, simplify the expression   2

$$\frac{g(x + h) - g(x)}{h}$$

c) (i) Show that $y = \frac{x^2 - 1}{h}$ is an ODD function.   1

(ii) Show that as $x \to \infty$, $y \to x$ 1

(iii) Show that when $x = 1$ then $y = 0$ and when $x = 2$ then $y = \frac{3}{2}$ 1

(iv) Describe what geometrical feature would occur in a sketch of this curve
      where $x = 0$ 1

Question 19 – Start a New Booklet – (12 Marks)

<div align="right">Marks</div>

a) Differentiate

(i) $y = 5x^2 - 2x + 1$

<div align="right">1</div>

(ii) $f(x) = (5 - 3x)^4$

<div align="right">2</div>

(iii) $g(x) = x^2(2 - x)$

<div align="right">2</div>

(iv) $y = \dfrac{2x}{x+3}$

<div align="right">2</div>

b) Find the equation of the tangent to the curve $y = x^3 - 3x^2 + 2x - 1$ at $x = 1$

<div align="right">3</div>

c) If the first derivative of $f(x)$ is given by $f'(x) = x^2 - 2x$

<div align="right">2</div>

Describe how you would find x-values of the points on the curve where there are horizontal tangents.

Find these x-values.

<div align="center">End of Paper</div>