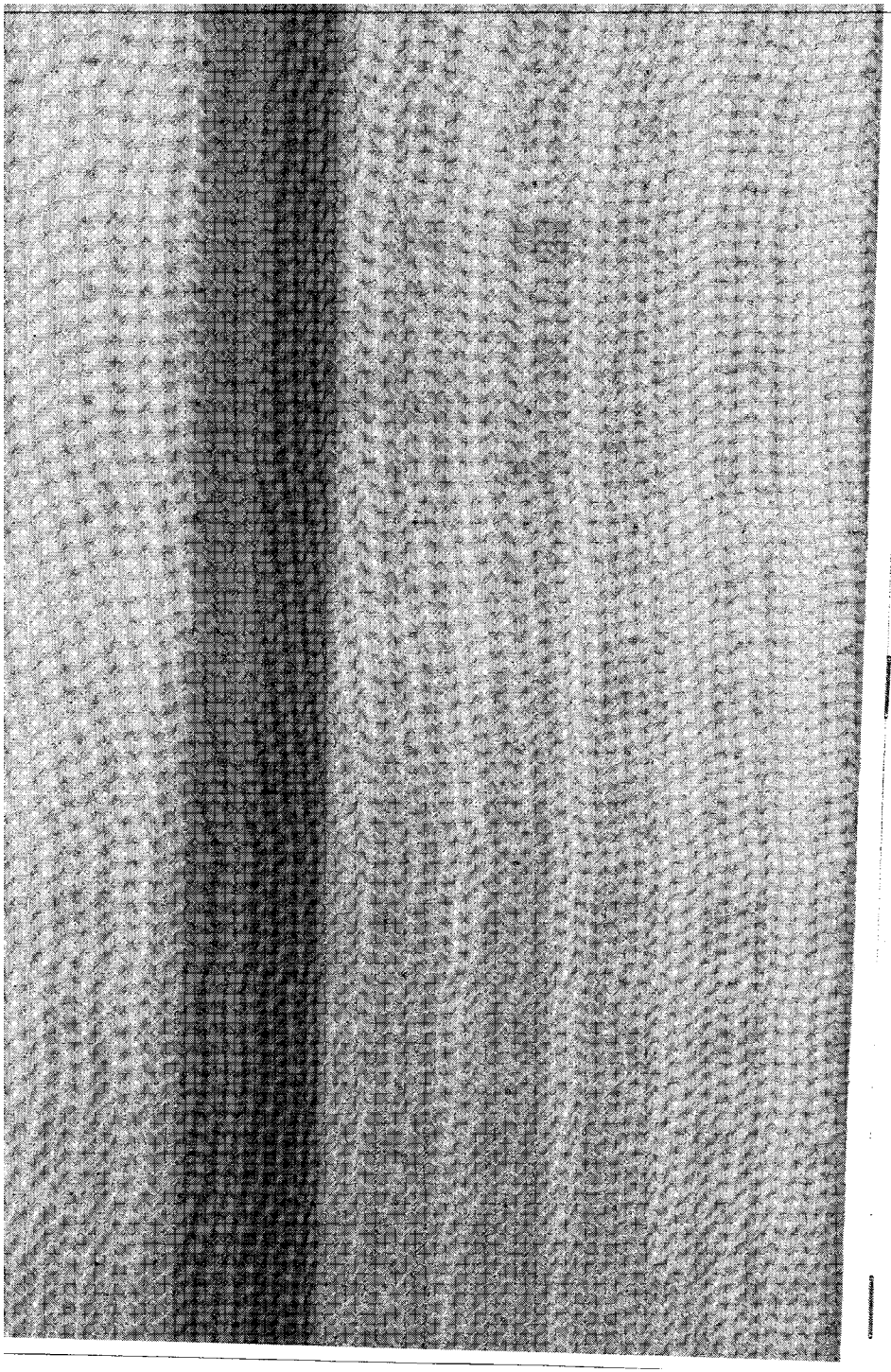


SYDNEY BOYS' HIGH SCHOOL

MOORE PARK, SURRY HILLS

Year 11 YEARLY EXAMINATIONS

SEPTEMBER 1998

MATHEMATICS

2/3 UNIT COMMON

Time allowed — 2 Hours (Plus 5 minutes reading time)
Examiner: P.S. Parker

DIRECTIONS TO CANDIDATES

- *ALL* questions may be attempted.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.
- Hand up your answers in 8 separate booklets. Start a new booklet for EACH question. Indicate your name, class and teacher on each booklet.
- If required, additional booklets may be obtained from the Examination Supervisor upon request.

Question 1

(a) Factorise $3x^2 - 13x - 30$ — 2

(b) Find the value of A correct to 2 significant figures given $B = 7.4$ and $C = 8.5$ and $\frac{1}{A} = \frac{1}{B} + \frac{1}{C}$ — 2

(c) Find $\frac{dy}{dx}$ for — 3

(i) $y = 6x^2 + 5x - 17$

(ii) $y = (5x - 3)^7$

(d) Find rational numbers a and b such that $(1 + \sqrt{b})^2 = a + \sqrt{3}$ — 2

(e) If $x = 1$ find the value of $\log_3(x^4 + x^2 + 1)$ — 2

(f) Find the values of x for which $|x + 2| > 3$ — 2

Marks

Question 2 (Start a new booklet)

(a) What is the equation of the line parallel to $4y = 12x + 7$ and passing through $(0, 3)$? — 2

(b) The two straight lines $3y + ax = 5$ and $12y - ax = 1$ are perpendicular. Find the value of a. — 2

(c) What is the tangent of the angle that the straight line $3x + 4y + 5 = 0$ makes with the positive direction of the x axis? — 1

(d) Given $A(4, 0)$, $B(0, 4)$ and $C(12, 8)$

(i) Find the coordinates of D and E, the midpoints of AC and AB respectively. — 2

(ii) Find the gradient of AC. — 1

(iii) Show that the equation of the line, l, through D perpendicular to AC is given by $x + y - 12 = 0$ — 2

(iv) Write down the coordinates of X, the point of intersection of the lines l and $y = x$, the line perpendicular to AB through E. — 2

(v) Show that A, B and C are equidistant from X. — 1

(vi) Write down the equation of the circle, centre X which also passes through A, B and C — 1

Question 3 (Start a new booklet)

(a) Write down the exact value of:

(i) $\tan^2 30°$

(ii) $\sin\frac{5\pi}{4}$

(b) $\tan\alpha = \frac{3}{5}$, $180 \le \alpha \le 270$ then

(i) $\cos\alpha =$

(ii) $\operatorname{cosec}\alpha =$

(c) Ship A is 18·5 km from a port P on a bearing of 045°T and ship B is 26·4 km from P on a bearing of 105°T. Calculate:

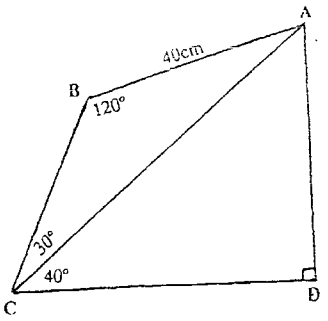
(i) the distance between the ships.

(ii) the bearing of A from B.

(d) $BADC$ is a quadrilateral with:

$\angle ADC = 90°$,
$\angle ABC = 120°$,
$\angle BCA = 30°$,
$\angle ACD = 40°$ and
$AB = 40$ cm

DIAGRAM NOT TO SCALE



(i) Show $AC = 40\sqrt{3}$ cm

(ii) Hence find the length of CD, correct to 1 decimal place.

Marks

2

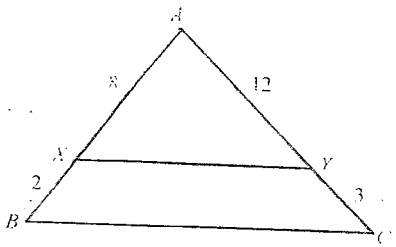
2

5

4

Question 4 (Start a new booklet)

(a)



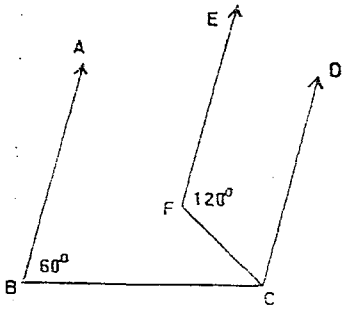
NOT TO SCALE

$AX = 8$, $BX = 2$, $AY = 12$ and $CY = 3$

(i) Show $\Delta ABC \parallel\parallel \Delta AXY$

(ii) Hence show $XY \parallel BC$

(b)



NOT TO SCALE

In the diagram:

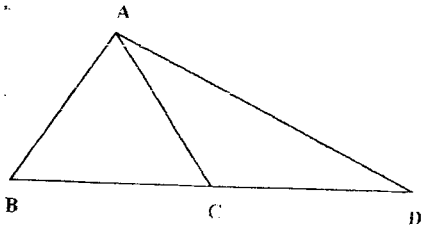
$AB \parallel EF \parallel DC$,
$\angle ABC = 60°$,
$\angle EFC = 120°$

(i) Copy the diagram onto your booklet

(ii) Find the size of $\angle BCF$, giving reasons

(c)

NOT TO SCALE



ABC is an equilateral triangle. BC is produced to D so that $BC = DC$

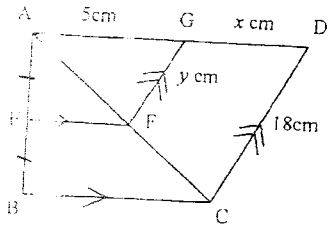
(i) Copy the diagram onto your booklet and mark on it all the given information.

(ii) Prove that $\angle BAD = 90°$

Question 4 Continued

(d)

NOT TO SCALE



In the diagram:

$AE \parallel FB$,
$EF \parallel BC$,
$FG \parallel CD$,
$AG = 5$ cm and
$CD = 18$ cm.

(i) Copy the diagram into your booklet.

(ii) Find the value of x, giving reasons.

(iii) Find the value of y, giving reasons.

Marks 3

Question 5 (Start a new booklet)

Marks

(a) Shade the region indicated by the following inequalities: 3
$x^2 + y^2 \leq 25$ and $x + y \leq 5$

(b) The function $f(x)$ is defined by 3

$$f(x) = \begin{cases} 3x & \text{if } x < -2 \\ x - 4 & \text{if } -2 \leq x \leq 5 \\ 1 & \text{if } x > 5 \end{cases}$$

(i) Evaluate $f(-3) - f(3) + f(6)$

(ii) What is the range of $y = f(x)$?

(c) Consider the parabola $4y = x^2 - 4x$ 4

(i) Show algebraically how the parabola can be expressed in the form $(x-2)^2 = 4(y+1)$

(ii) Write down the coordinates of the focus.

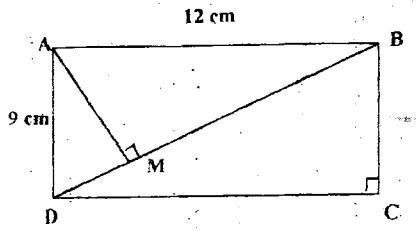
(iii) Find the equation of the directrix.

(d) (i) Find an expression for the discriminant of the quadratic function $x^2 + 6x + k + 8$ 3

(ii) For what value(s) of k is the line $y = 4x + k$ a tangent to the parabola $y = -8 - 2x - x^2$

Question 6 (Start a new booklet)

(a) $ABCD$ is a rectangle with $AB = 12$ cm, $AD = 9$ cm and AM is perpendicular to BD.



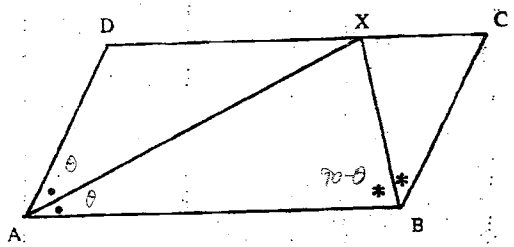
(i) Copy the diagram onto your booklet.

(ii) Find the length of BD.

(iii) Prove that $\Delta ABM \;|||\; \Delta DBA$.

(iv) Hence find the length of BM.

(b) $ABCD$ is a parallelogram. XA bisects $\angle DAB$ and XB bisects $\angle CBA$. X is a point on side DC.



(i) Copy the diagram onto your booklet.

(ii) Prove that ΔADX is isosceles.

(iii) Deduce that X is the midpoint of DC.

(iv) Find the size of $\angle AXB$, giving reasons.

Question 6 Continued **Marks**

(c) If α and β are the roots of $3x^2 + 4x - 3 = 0$, find the value of **3**

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$

(d) (i) If $3x + y - 5 = 0$ then find an expression for $y - 2$ in terms of x. **1**

(ii) The point $P(1, 2)$ lies on the line $3x + y - 5 = 0$. Find the coordinates of the two points Q and R on the line so that their distance from P is $\sqrt{10}$ units. **2**

Question 7 (Start a new booklet)

(a) Find the first derivative of the following

(i) $x\sqrt{1-x}$

(ii) $\dfrac{x}{1+x^2}$

(iii) $x^2\sqrt{x}$

(b) If $f(x)=6x^2+12x-20$, for what value(s) of x does $f'(x)=0$

(c) Write down the equation of the normal to the curve $y=x^2$ at $x=-2$

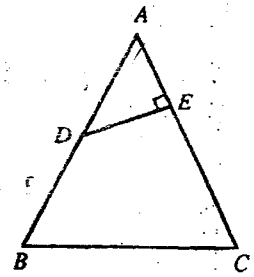
(d) The function $y=f(x)$ is defined such that $f(x+h)-f(x)=3x^2h+6xh^2+2h-h^3$.
Evaluate $f'(1)$ by first principles.

(e) (i) Find the x coordinates of the points of intersection of $y=\sin x$ and $y=\sqrt{3}\cos x$, for $0^\circ \le x \le 360^\circ$

(ii) Hence, by graphing $y=\sin x$ and $y=\sqrt{3}\cos x$ on the same diagram, for $0^\circ \le x \le 360^\circ$, solve $\sin x \le \sqrt{3}\cos x$, for $0^\circ \le x \le 360^\circ$

Question 8 (Start a new page) **Marks**

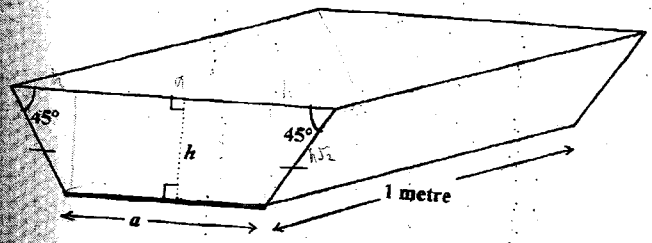
(a) In the diagram $AD = DB = 5$, $EC = 2AE = 8$ and $\angle AED = 90^\circ$. 2



(i) Copy the diagram into your booklet.

(ii) Find the length of BC.

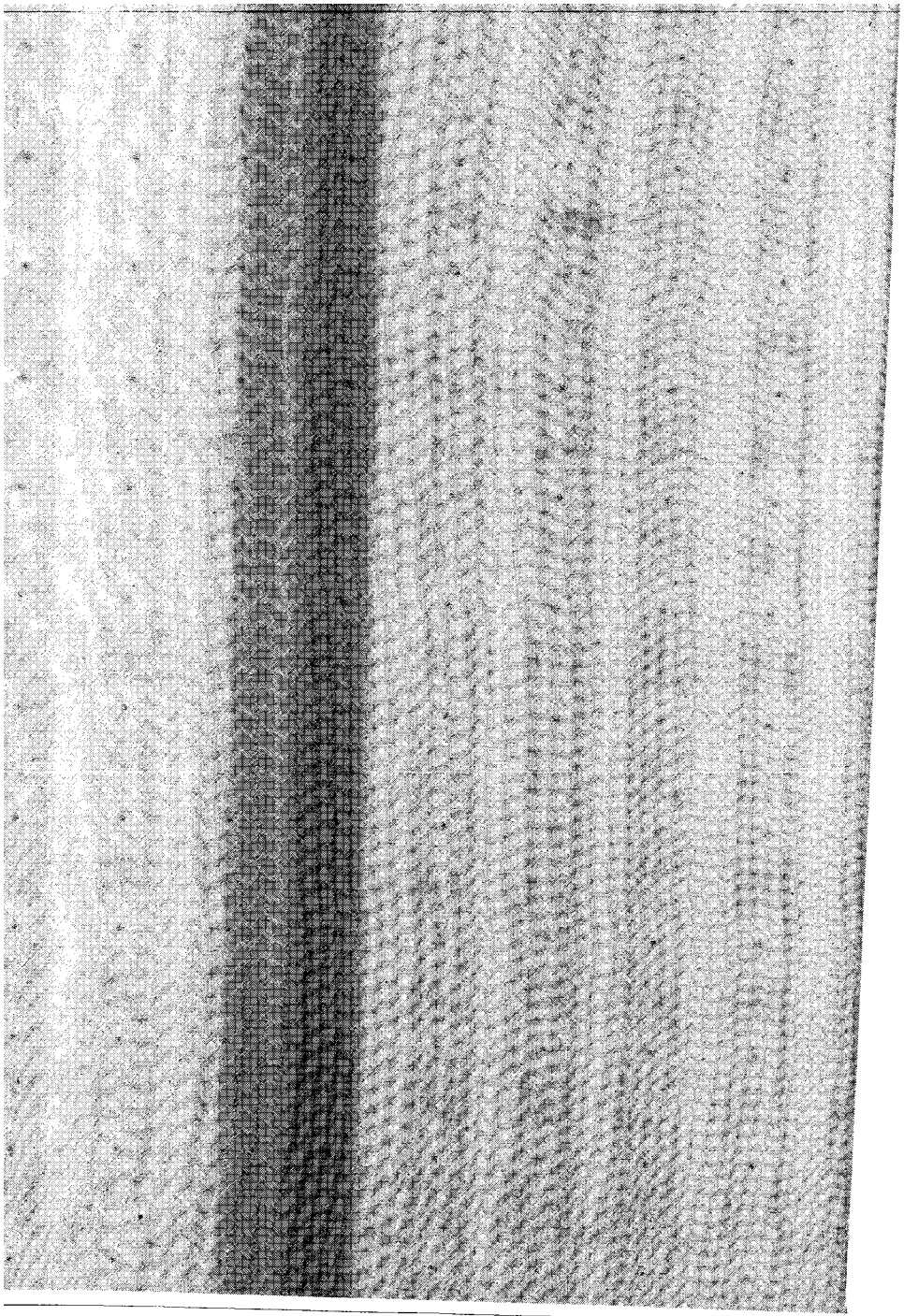
4



A trough of depth h metres and length 1 metre was constructed out of stainless steel sheeting. The cross section of the trough was an isosceles trapezium with the acute angles being 45° each. The width of the bottom of the trough was a metres. The area of the cross section measured 60 m^2.

(b) Show that $a=\dfrac{60}{h}-h$

(c) Show that the amount of stainless steel, A, in m^2, required to construct the trough was given by: $A=\dfrac{60}{h}-h+2h\sqrt{2}+120$



Question 8 Continued

(c) Show that the expression $2x^2 - 12x + 19$ is positive definite for all values of x. **2**

(d) (i) Show that the vertical distance, D, between the curves $y = x^2 - 4x + 8$ and $y = 5 - (x-4)^2$ at any x value, is given by $D = 2x^2 - 12x + 19$ **2**

(ii) Hence, find the coordinates so that the distance, D, is a minimum. Giving reasons for your answer. **3**

END OF THE PAPER