SYDNEY BOYS HIGH SCHOOL

YEAR 11 YEARLY EXAMINATION

SEPTEMBER 1999

MATHEMATICS

2/3 UNIT COMMON

Time allowed — 2 Hours (Plus 5 minutes reading time)
Examiner: C Kourtesis

DIRECTIONS TO CANDIDATES

- *ALL* questions may be attempted
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.
- Hand up your answers in separate booklets. Start a new booklet for EACH question.
  Indicate your name, class and teacher on each booklet.
- If required, additional booklets may be obtained from the Examination Supervisor upon request.

Question 1 [10 marks]

(a) Calculate $\frac{16 \cdot 4}{8 \cdot 3 \times 1 \cdot 9}$ correct to two decimal places.

(b) Simplify $\frac{3 + 6m}{3}$

(c) Solve the equation

$$3x = 8 - x$$

(d) Simplify $\sqrt{12} + \sqrt{27}$

(e) Solve $2^{a-4} = 128$

(f) If $\tan A = 0 \cdot 14$ where angle A is acute, find angle A to the nearest minute.

(g) Simplify $(a^2 b) \times \frac{a}{b}$

(h) Solve $|2x| = 30$

Question 2 [12 marks]

(a) Simplify $\frac{a}{2} + \frac{4a}{5}$

(b) Evaluate $\tan (420^\circ) + \cos (135^\circ)$

(c) If $g(x) = x^2 + 1$

(i) Evaluate $g(4)$

(ii) For what values of x is $g(x) = 9$?

(d) On separate diagrams sketch the graphs of

(i) $y = 4^x$ (ii) $y = \sqrt{4 - x^2}$

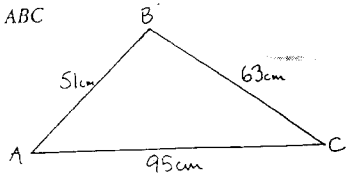
(e) Simplify

$$\frac{r^2 - 1}{2r + 2}$$

(f) Express $0 \cdot 1\dot{5}$ as a rational number

Question 3 [11 marks]

(a) In the triangle ABC

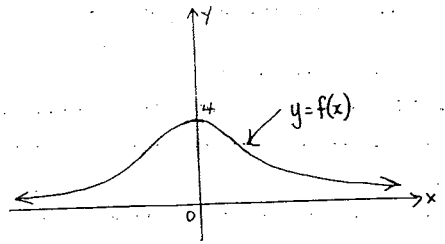


Find:

(i) The size of angle ABC (to the nearest minute)

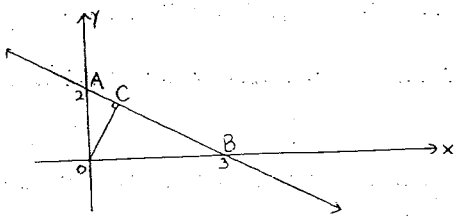
(ii) The area of the triangle ABC (to nearest cm^2)

(b)



State the domain and range of $y = f(x)$

(c)



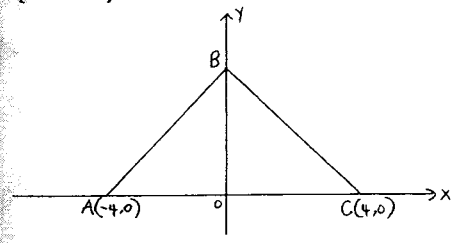
(i) Find the equation of the straight line passing through points A and B

(ii) Find the gradient of OC

(iii) Find the shortest distance from the origin O to the straight line AB

Question 4 [11 marks]

(a)



If the triangle ABC is equilateral find the coordinates of point B.

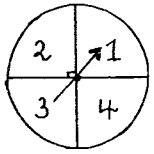
(b) Solve $\sin\theta = \frac{1}{2}$ for $0^\circ \le \theta \le 360^\circ$

(c) Factorize:

(i) $8a^3 + 1$

(ii) $xy + 3y - xt - 3t$

(d) The spinner shown below is used in a game



It is spun twice and the score recorded after each spin. Find the probability that:

(i) In each of the two spins the result is 3.

(ii) The sum of the two spins is 4.

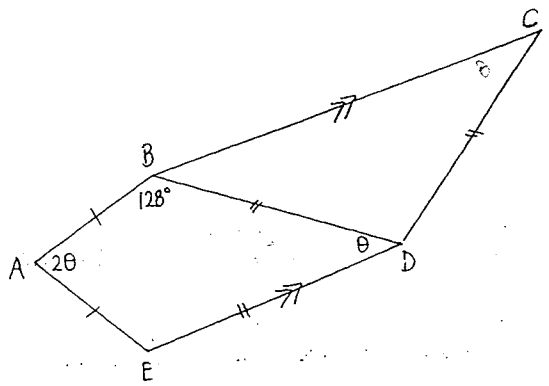
Question 5 [10 marks]

(a) On a number plane, sketch the region where

$y \le 4 - x^2$ and $x^2 + (y-4)^2 \le 16$

hold simultaneously.

(b)



In the diagram above $ABCDE$ is a quadrilateral and BCD is a triangle.

$BC \parallel ED$, $AB = AE$, $DE = BD = DC$, $\angle ABD = 128^o$

(i) Find the value of θ.

(ii) Determine the size of angle BCD giving reasons.

(c) A bag contains 4 red, 3 blue and 1 white ticket. Three tickets are randomly selected from the bag without replacement. Determine the probability that at least one is blue.

Question 6 [12 marks]

(a) Differentiate the following with respect to x

(I) $x^4 + 3x^2 + 2$

(ii) $\dfrac{1}{3\sqrt{x}}$

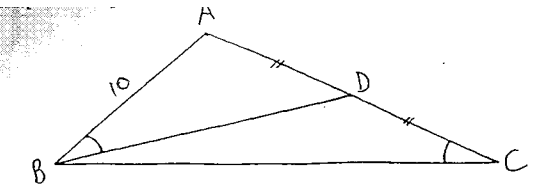
(b) Use the product rule to find $\dfrac{dy}{dx}$ if

$y = x(x-4)^7$

(c) If $f(t) = \dfrac{t}{t^3+1}$ use the quotient rule to find $f'(t)$.

(d) Find the gradient of the normal to the curve $y = x^3$ at the point where $x=2$

(a)



In the diagram $AB = 10cm$, $AD = DC$
and angle ABD = angle BCD

(i) Prove triangle ABD is similar to triangle ACB

(ii) Find the length of AD

(b) For the parabola

$$(x+1)^2 = 4(y-2)$$

write down the:

(i) equation of the axis of symmetry

(ii) equation of the tangent at the vertex

(iii) coordinates of the focus

(c) Given the quadratic equation

$$3x^2 - (k-3)x + (k-3) = 0 \qquad (k \text{ a constant})$$

(i) Show that the discriminant is given by

$$k^2 - 18k + 45$$

(ii) Find the values of k for which the quadratic equation has

(α) Two real roots

(β) No real roots

Question 8 [12 marks]

(a) If $\dfrac{7}{3-\sqrt{5}} = a + b\sqrt{5}$ (where a, b are rational)

find the values of a and b.

(b) A function is defined as

$$f(x) = \begin{cases} 6 - x^2 & \text{when } x \le 0 \\ 2^{-x} & \text{when } x > 0 \end{cases}$$

Evaluate:

(i) $f(3) + f(-4)$

(ii) $f(-a)$ when $a < 0$

(c) Sketch the graphs of the following

(i) $y = x - 1$

(ii) $|y| = x - 1$

(d) Find the equation of the locus of all points which are equidistant from the x and y axes.

(e) Prove that

$$\frac{1}{\cos\theta} + \tan\theta = \frac{\cos\theta}{1 - \sin\theta}$$

Question 9 **[10 marks]**

(a) Solve the inequality

$$|1 - 4x| < 4$$

(b) Find the domain of

$$y = \sqrt{x^2 - 9} + \sqrt{1 - x}$$

(c) Solve the equation

$$\cos\theta = \tan\theta \quad \text{for} \quad -180^\circ \leq \theta \leq 180^\circ$$

(Answer correct to nearest minute).

(d) A surveying party measures an east-west base line AB, where A lies to the west of B and AB=c metres in length. From A the bearing of the base of a chimney C is α° and from B the bearing of the chimney is β°.

(i) Draw a neat diagram of the above.

(ii) Show that the perpendicular distance of the base of the chimney C from AB is given by

$$\frac{c}{\tan\alpha - \tan\beta}$$