SYDNEY BOYS HIGH
MOORE PARK, SURRY HILLS

2002
YEAR 11 PRELIMINARY EXAMINATION

Mathematics

General Instructions
- Reading time — 5 minutes
- Working time — 90 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question

Total marks — 80
- Attempt questions 1–5
- All questions are of equal value

Examiner: B Opferkuch

QUESTION 1. (16 marks) **Start a new booklet** **Marks**

(a) Calculate $\sqrt{\dfrac{19}{4\pi}}$ correct to three decimal places. **1**

(b) Simplify $\sqrt{48} + \sqrt{108}$. **2**

(c) Factorise $4x^2 - 1$. **1**

(d) Express $(27x^6)^{\frac{1}{3}}x^{-3}$ in the simplest form, without the use of negative indices. **2**

(e) Express the recurring decimal $0.\dot{6}\dot{7}$ as a fraction in its simplest form. **1**

(f) Write down, in surd form, the value of $\cos 210^0$. **1**

(g) Find the value of x, if $\log_x 125 = 3$. **2**

(h) Solve $|5x - 4| = 9$ **2**

(i) If $g(x) = \begin{cases} 2 - x^3 \text{ if } x > 0 \\ x^2 + 1 \text{ if } x \le 0 \end{cases}$ **2**

Find the value of $g(3) + g(-4) - g(0)$.

(j) Graph on a number line the solution of the inequality $-2 < 4 - 2x \le 5$ **2**

(k) Solve $2^{5-3x} = 1$ **2**

QUESTION 2. (16 marks) **Start a new booklet** **Marks**

(a) Factorise

(i) $x^2 - x - 6$ **1**

(ii) $3x^2 + 4x - 7$ **2**

(iii) $8a^3 - 27$ **2**

(b) Consider the function $f(x) = \sqrt{4 - x^2}$. **5**

(i) Show that the function is even.

(ii) What is the range of $f(x)$?

(iii) Sketch the curve.

(c) If α and β are the roots of $2x^2 - 5x + 1 = 0$, find the values of **4**

(i) $\alpha + \beta$

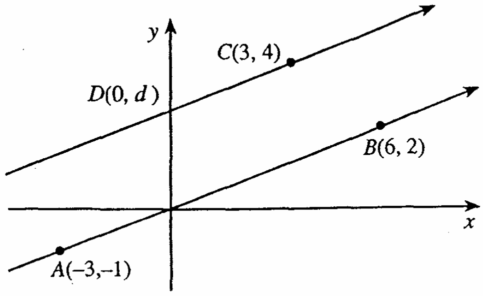
(ii) $\alpha\beta$

(iii) $\frac{2}{\alpha} + \frac{2}{\beta}$

(d) Simplify $\cot A \sec A \sin A$ **2**

QUESTION 3. (16 marks) **Start a new booklet** **Marks**

(a)



The diagram above shows the points A (-3,-1), B (6,2), C (3,4) and $D(0,d)$.

Copy the diagram to your answer booklet.
On your diagram indicate the given information.

(i) Find the gradient of AB. **1**

(ii) Find the value of d if CD and AB are parallel. **1**

(iii) Show that the equation of AB is $x-3y=0$. **2**

(iv) Find the perpendicular distance between C and the line AB. **2**

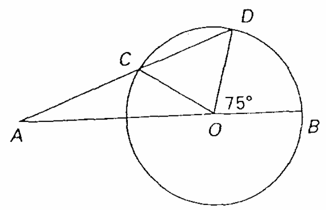
(v) Prove that $AB=3CD$. **3**

(vi) Find the area of the quadrilateral $ABCD$. **1**

(b) In triangle ABC, AB= 5cm, $BC=3$cm, and $<CAB=35^{0}11'$. **3**
Find the possible values for $<ACB$.

QUESTION 3. continued **Marks**

(c)



In this figure, AOB and ACD are straight lines.
O is the centre of circle radius r. $AC = DO$ and $<DOB = 75^0$.
If $<CAO = x$:

(i) Show that $<DCO = 2x$. **1**

(ii) Show that $<DOB = 3x = 75^0$ **2**

QUESTION 4. (16 marks) **Start a new booklet** **Marks**

(a) Differentiate the following with respect to x:

(i) $3x^4 - 4x + 7$ **1**

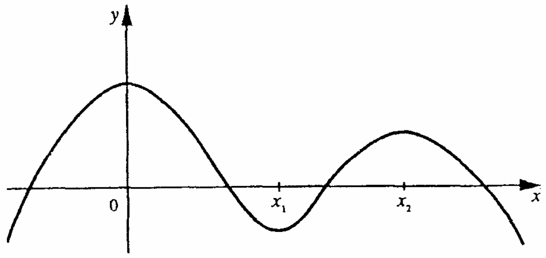
(ii) $\dfrac{2}{x^3}$ **1**

(iii) $x\sqrt{x}$ **1**

(b) (i) Use the product rule to differentiate $y = 5x^3(4x-3)^5$. **2**

(ii) Differentiate $y = \dfrac{x^2 - 2}{x+1}$ by using the quotient rule. **3**

(c) Sketch the gradient function of the following curve. **2**



(d) Find the equation of the normal to the parabola $y = \frac{1}{5}x^2$ at the point $(2, \frac{4}{5})$. **3**

(e) (i) Write down the discriminant of $3x^2 + 2x + k$. **1**

(ii) For what values of k does $3x^2 + 2x + k$ have real roots? **2**

QUESTION 5. (16 marks) **Start a new booklet** **Marks**

(a) The focus of a parabola is $S(0,4)$ and its directrix is the line $y = -2$. **4**

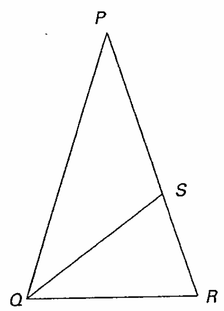
(i) Sketch the parabola and indicate the coordinates of the vertex V.

(ii) Write down the focal length of the parabola.

(iii) Find the equation of the parabola.

(b) Consider $\log_a b^2 + \log_a b^3 - \log_a b^4 = 1$. **2**
Find the relationship between a and b.

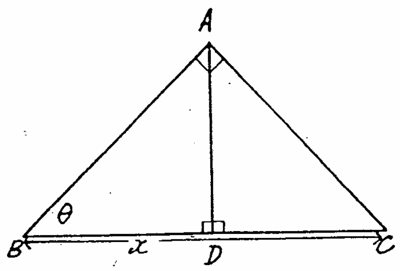
(c) In $\Delta PQR, < QPR = x, < PQR = 2x$ and QS bisects $<PQR$, meeting PR at S.



(i) Prove that $\Delta QSR \,|||\, \Delta PQR$. **3**

(ii) If $QR = 6$cm and $PQ = 10\frac{2}{3}$ cm, find PR. **3**

(d) The diagram shows a right angled triangle ABC. $\angle ABC = \theta$. The line AD is perpendicular to BC and $BC = x$ units. **4**



(i) show that $BA = x\cos\theta$

(ii) show that $BD = x\cos^2\theta$

(iii) and hence that $DC = x\sin^2\theta$

End of paper

SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

SEPTEMBER 2002

YEARLY EXAMINATION

YEAR 11

Mathematics

Sample Solutions

Question 1

(a) $\sqrt{\dfrac{19}{4\pi}} \approx 1\cdot 22962269\ldots$

$= 1\cdot 230 \qquad \text{(3 d.p.)}$

(b) $\sqrt{48} + \sqrt{108} = 4\sqrt{3} + 6\sqrt{3} = 10\sqrt{3}$

(c) $4x^2 - 1 = (2x-1)(2x+1)$

(d) $\left(27x^6\right)^{\frac{1}{3}} x^{-3} = 3 \times x^2 \times x^{-3} = 3x^{-1} = \dfrac{3}{x}$

(e) $0\cdot\dot{6}\dot{7} = \dfrac{67}{99}$

(f) $\cos 210^\circ = \cos(180+30)^\circ = -\cos 30^\circ = -\dfrac{\sqrt{3}}{2}$

(g) $\log_x 125 = 3 \Leftrightarrow x^3 = 125$

$\therefore x = \sqrt[3]{125} = 5$

(h) $|5x-4| = 9$

$\therefore 5x-4 = 9, 5x-4 = -9$

$\therefore x = 2\cdot 6, -1$

(i) $g(x) = \begin{cases} 2 - x^3 & \text{if } x > 0 \\ x^2 + 1 & \text{if } x \le 0 \end{cases}$

$\therefore g(3) + g(-4) - g(0) = \left(2 - 3^3\right) + \left[(-4)^2 + 1\right] - \left[0^2 + 1\right] = -9$

(j) $-2 < 4 - 2x \le 5 \Rightarrow -6 < -2x \le 1$

$\therefore 3 > x \ge -\dfrac{1}{2}$

(k) $2^{5-3x} = 1 = 2^0$

$\therefore 5 - 3x = 0$

$\therefore x = \dfrac{5}{3}$

Question 2

(a) (i) $x^2 - x - 6 = (x-3)(x+2)$

(ii) $3x^2 + 4x - 7$

$(3\times -7 = -21 \Rightarrow a+b=4, ab=-21)$

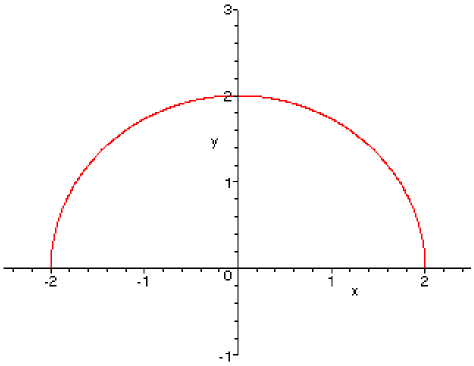
$3x^2+4x-7 = 3x^2+7x-3x-7 = x(3x+7)-(3x+7) = (x-1)(3x+7)$

(iii) $8a^3 - 27 = (2a)^3 - 3^3 = (2a-3)\left[(2a)^2 + (2a)\times 3 + 3^2\right] = (2a-3)(4a^2+6a+9)$

(b) (i) $f(-x) = \sqrt{4-(-x)^2} = \sqrt{4-x^2} = f(x)$

(ii) $0 \le y \le 2$

(iii)



(c) $2x^2 - 5x + 1 = 0$

(i) $\alpha + \beta = \frac{5}{2}$

(ii) $\alpha\beta = \frac{1}{2}$

(iii) $\frac{2}{\alpha} + \frac{2}{\beta} = \frac{2(\alpha+\beta)}{\alpha\beta} = \frac{2\times\frac{5}{2}}{\frac{1}{2}} = 10$

(d) $\cot A \times \sec A \times \sin A = \frac{\cos A}{\sin A} \times \frac{1}{\cos A} \times \sin A = 1$

Question 3

(a) (i) $m_{AB}=\frac{-1-2}{-3-6}=\frac{1}{3}$

(ii) $AB \parallel CD \Rightarrow m_{AB}=m_{CD}$

$m_{CD}=\frac{d-4}{0-3}=-\frac{d-4}{3}$

$\therefore -\frac{d-4}{3}=\frac{1}{3}\Rightarrow -d+4=1$

$\therefore d=3$

(iii) $y-y_1=m_{AB}(x-x_1)$

$\therefore y-2=\frac{1}{3}(x-6)\Rightarrow 3y-6=x-6$

$\therefore x-3y=0$

(iv) $d_{\perp}=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$

$=\frac{|1\times 3+(-3)\times 4+0|}{\sqrt{1^2+(-3)^2}}=\frac{9}{\sqrt{10}}$

(v) $AB=\sqrt{(-3-6)^2+(-1-2)^2}=\sqrt{81+9}=\sqrt{90}=3\sqrt{10}$

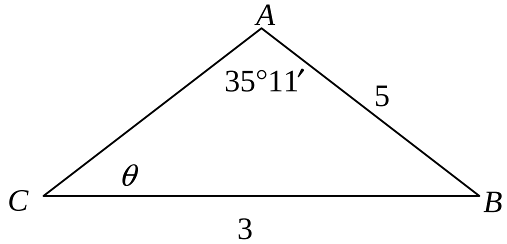
$CD=\sqrt{(0-3)^2+(3-4)^2}=\sqrt{9+1}=\sqrt{10}$

$\therefore AB=3CD$

(vi) $ABCD$ is a trapezium

Area = $\frac{1}{2}\times d_{\perp}\times(CD+AB)=\frac{1}{2}\times\frac{9}{\sqrt{10}}\times 4\sqrt{10}=18$ units squared

(b)



$\frac{\sin\theta}{5}=\frac{\sin 35°11'}{3}\Rightarrow \sin\theta=\frac{5\sin 35°11'}{3}\simeq 0\cdot 960324\ldots$

$\therefore \theta=73°48', 106°12'$

(c) (i) $OD=OC\Rightarrow AC=OC$. $\therefore \angle DCO=2x$ (Exterior angle theorem)

(ii) $\angle CDO=2x\Rightarrow \angle COD=180-4x$

$\therefore \angle DOB=180-(180-4x+x)=3x$ (Straight angle)

Question 4

(a) (i) $\frac{d}{dx}\left(3x^4-4x+7\right)=12x^3-4$

(ii) $\frac{d}{dx}\left(\frac{2}{x^3}\right)=\frac{d}{dx}\left(2x^{-3}\right)=-6x^{-4}=-\frac{6}{x^4}$

(iii) $\frac{d}{dx}\left(x\sqrt{x}\right)=\frac{d}{dx}\left(x^{\frac{3}{2}}\right)=\frac{3}{2}x^{\frac{1}{2}}=\frac{3\sqrt{x}}{2}$

(b) (i) $\frac{dy}{dx}=(4x-3)^5\times\frac{d}{dx}\left(5x^3\right)+5x^3\times\frac{d}{dx}\left((4x-3)^5\right)$

$=15x^2(4x-3)^5+5x^3\times5(4x-3)^4\times4$

$=5x^2(4x-3)^4\left[3(4x-3)+20x\right]=5x^2(32x-9)(4x-3)^4$

(ii) $\frac{dy}{dx}=\frac{(x+1)\times\frac{d}{dx}\left(x^2-2\right)-\left(x^2-2\right)\times\frac{d}{dx}(x+1)}{(x+1)^2}$

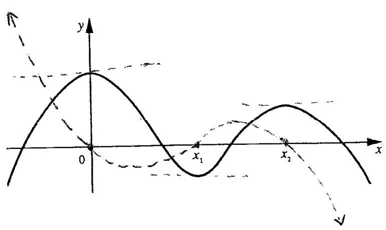
$=\frac{(x+1)\times2x-\left(x^2-2\right)}{(x+1)^2}=\frac{x^2+2x+2}{(x+1)^2}$

(c) At $x=0,x_1,x_2$ the tangents are horizontal so $y'=0$

For $x<0$, $x_1<x<x_2$ y is increasing so $y'>0$

For $0<x<x_1, x>x_2$ y is decreasing so $y'<0$

A possible graph could be



(d) $y=\frac{x^2}{5}\Rightarrow\frac{dy}{dx}=\frac{2x}{5}\qquad\therefore\left.\frac{dy}{dx}\right|_{x=2}=\frac{4}{5}$

So the tangent has a gradient of $4/5$, so the normal has gradient $-5/4$

So the normal has equation $y-\frac{4}{5}=-\frac{5}{4}(x-2)$

$20y-16=-25(x-2)\qquad\therefore 25x+20y-66=0$

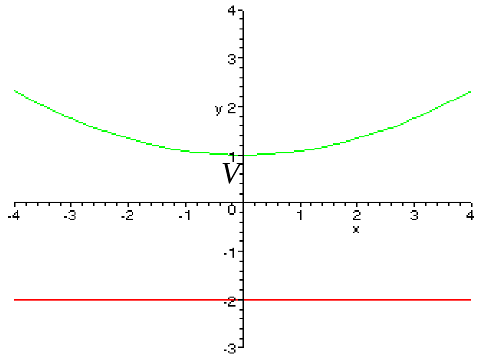
(e) (i) $\Delta=(2)^2-4\times3\times k=4-12k=4(1-3k)$

(ii) Real roots if $\Delta\ge0\Rightarrow4(1-3k)\ge0\qquad\therefore k\le1/3$

Question 5

(a) (i) Vertex is midway between focus and directrix

$V(0,1)$ and so $a=3$



(ii) Focal length = $|a|=3$

(iii) $(x-h)^2 = 4a(y-k) \Rightarrow x^2 = 4\times 3\times(y-1)$

$x^2 = 12(y-1)$

(b) $\log_a b^2 + \log_a b^3 - \log_a b^4 = 1$

$\therefore \log_a\left(\dfrac{b^2\times b^3}{b^4}\right) = 1 \Rightarrow \log_a b = 1$

$\therefore a^1 = b$

$\therefore a = b$

(c) (i) $\because QS$ bisects $\angle PQR \Rightarrow \angle RQS = \angle SQP = x$

$\angle QSR = \angle SQP + \angle SPQ = x + x = 2x$

So $\Delta QSR \,|||\, \Delta PQR$ (equiangular)

$[\angle PQR = \angle QSR = 2x, \angle QPR = \angle SQR = x]$

(ii) Let $a = PR, b = SR, c = QS$

$\dfrac{PR}{QR} = \dfrac{QR}{SR} \Rightarrow \dfrac{a}{6} = \dfrac{6}{b} \Rightarrow b = \dfrac{36}{a}$

$\dfrac{PR}{QR} = \dfrac{PQ}{QS} \Rightarrow \dfrac{a}{6} = \dfrac{10\frac{2}{3}}{c} \Rightarrow c = \dfrac{64}{a}$

Now QS = SP (ΔQSP is isosceles) and $PR = SP + SR$

So $PR = a = b + c$

$\therefore a = \dfrac{36}{a} + \dfrac{64}{a} \Rightarrow a^2 = 100$

$\therefore a = PR = 10$

(d) (i) In ΔBAC $\cos\theta = \dfrac{AB}{x} \Rightarrow AB = x\cos\theta$

(ii) In ΔABD $\cos\theta = \dfrac{BD}{AB} \Rightarrow DB = AB\cos\theta = (x\cos\theta)\cos\theta = x\cos^2\theta$

(iii) $DC = BC - BD = x - x\cos^2\theta = x(1-\cos^2\theta) = x\sin^2\theta$

$\because \sin^2\theta + \cos^2\theta = 1$