SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2004

PRELIMINARY
HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 2 periods.
- Write using black or blue pen.
- Board approved calculators may be used.
- All *necessary* working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for messy or badly arranged work.
- Start each **NEW** section in a separate answer booklet.

Total Marks - 100 Marks

- Attempt Questions 1 - 5
- All questions are of *equal* value.

Examiner: *C. Kourtesis*

Question 1

Marks

(a) Express $\frac{2\pi}{9}$ radians in degrees — 1

(b) Solve $|4-x|=1$ — 1

(c) Factorize $3a^2-6a$ — 1

(d) Solve the inequality $5+3x<7$ — 1

(e) Simplify $49^{\frac{1}{2}}\times 81^{\frac{1}{4}}$ — 1

(f) Simplify $\log_6 216$ — 1

(g) Express with rational denominator $\frac{1}{\sqrt{6}+2}$ — 2

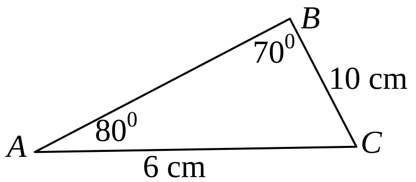
(h) On the same set of axes sketch the graphs of — 2

$y=\log_{10} x$ and $y=10^x$

(i) Factorize m^2-16 — 1

(j) Simplify $6-a-2(3-2a)$ — 2

(k) Find the area of the triangle ABC — 2



(l) If $f(\theta)=\tan 3\theta+\sin 3\theta$ find $f\left(\frac{\pi}{3}\right)$ in exact form — 2

(m) Factorize x^2-5x+6 — 1

(n) Simplify $t+\frac{2t-1}{3}$ — 2

Question 2

Marks

(a) The point $(3k, 2)$ lies on the line $2x - 3y = 10$ — 2
Find the value of k

(b) Solve the equation $\tan 2\theta = 1$ for $0 \le \theta \le 2\pi$ — 2

(c) Evaluate $\dfrac{2+\pi}{2-\pi}$ to 1 decimal place — 1

(d) Find the derivative of

(i) $4x^9 - 6x + 1$ — 1

(ii) $(5x+6)^{10}$ — 1

(iii) $\dfrac{x^3+10}{x^2}$ — 2

(e) Use the quotient rule to differentiate — 2

$$y = \frac{x}{3x+2}$$

(f) Find integers a and b such that — 2

$$(5-\sqrt{2})^2 = a + b\sqrt{2}$$

(g) For the parabola $(x-2)^2 = 4y$ find the coordinates of the

(i) vertex — 1

(ii) focus — 1

(h) Write down the roots of the equation — 1

$$(1-3x)(x+10) = 0$$

(i) Sketch the following graphs on separate number plane graphs, indicating any intercepts with the coordinate axes — 4

(i) $y = x + 1$ (iii) $y^2 = -x$

(ii) $y = \dfrac{4}{x}$ (iv) $y = 2(x+3)^2$

Question 3

Marks

(a) Given that $\log a = 0\cdot 86$ and $\log b = 0\cdot 42$ evaluate

(i) $\log\left(\frac{a}{b}\right)$ (ii) $\log\sqrt{ab}$ 3

(b) If α and β are the roots of $3x^2 - 2x + 1 = 0$
Evaluate:

(i) $\alpha + \beta$ 1

(ii) $\alpha\beta$ 1

(iii) $\frac{1}{\alpha^2\beta} + \frac{1}{\beta^2\alpha}$ 2

(c) The roots of the equation $mx^2 - x + n = 0$ are -2 and 3. 2
Find the values of m and n.

(d) Find $\lim_{n\to 10} \frac{n^2 - 100}{n - 10}$ 1

(e) Solve $|3x+1| > 8$ 2

(f) If $\sqrt[x]{m} = n^3$ express x in terms of m and n 2

(g) The lines $4y = bx + 3$ and $y + 2x = 1$ are parallel. Find b. 2

(h) Solve the inequality $\theta^2 - 4\theta < 0$ 2

(i) Find the equation of the tangent to the curve 2
$y = x^2 - 2x$ at the point where $x = 1$

Question 4

Marks

(a) If $3x^2 + 4x + 5 \equiv A(x+1)^2 + B(x+1) + C$ find the value of the constants A, B and C. 2

(b) Given $P(x) = k - 3kx - x^2$ find the values of k for which $P(x) < 0$ for all x. 2

(c) Solve the equation $x^4 - 10x^2 + 9 = 0$ 2

(d) The vertex of the parabola $y = x^2 + bx + c$ is at the point (2, 3). Find the values of b and c. 2

(e) The locus of a point P is described by the equation 2

$$x^2 + y^2 - 4x + 2y = 0$$

Find the centre and radius.

(f) (i) Sketch the graph of $y = x^2 - 6$ and label all intercepts with the axes. 1

(ii) On the same set of axes carefully sketch the graph of $y = |x|$ 1

(iii) Find the x coordinates of the two points where the graphs intersect 1

(iv) Hence solve the inequality $x^2 - 6 \le |x|$ 1

(g) Sketch the region in the number plane which satisfies both the inequalities $y \le \sqrt{16 - x^2}$ and $x - y \ge 4$. …3

(h) If $\tan A + \sec A = 2$, evaluate $\cos A$ …3

Question 5

Marks

(a) Use the product rule to differentiate

$(x^2+5)(1-4x)$ 3

(b) Given the function $f(x)=\dfrac{1}{\sqrt{3-x^2}}$ find the:

(i) domain 2

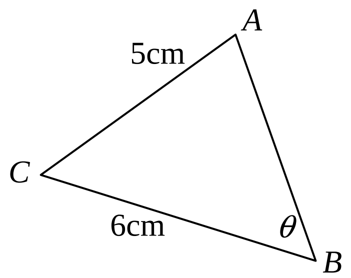
(ii) range

(c) Show that $h(x)=x^3-x$ is an odd function 2

(d) For the triangle ABC 3

Show that

$\dfrac{\sqrt{11}}{6}<\cos\theta<1$



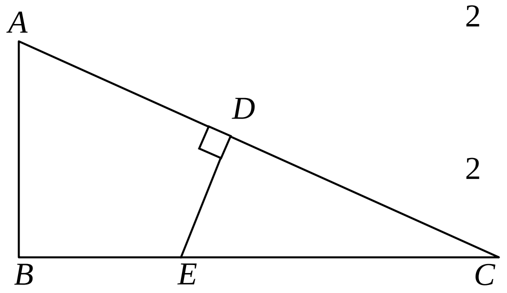
(e) ΔABC is right-angled at B and DE is perpendicular to AC (see diagram)

(i) Prove that ΔABC and ΔCDE are similar 2

(ii) Prove that $BC\times CE=AC\times CD$ 2

(iii) Prove that:

$DE^2=AD\times DC-BE\times EC$ 2



(f) If α and β are the roots of $x^2+5x+7=0$, form the equation whose roots are $(\alpha-\beta)^2$ and $(\alpha+\beta)^2$ …4

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Sample Solutions

Question	Marker
1	Mr Fuller
2	Mr Boros
3	Mr Dowdell
4	Mr Dunne
5	Ms Nesbitt

Question 1:

(a) $\frac{2\pi}{9}\times\frac{180}{\pi} = 40°$

(b) $|4-x|=1$

$4-x=1$ or $-(4-x)=1$

$-x=-3$ $\quad$ $-4+x=1$

$x=3$ $\quad$ $x=5$

(c) $3a^2-6a=3a(a-2)$

(d) $5+3x<7$

$3x<2$

$x<\frac{2}{3}$

(e) $49^{\frac{1}{2}}\times 81^{\frac{1}{4}} = 7\times 3$

$=21$

(f) $\log_6 216=\frac{\ln 216}{\ln 6}$ or let $\log_6 216 = x$

$=3$ $\quad$ $216=6^x$

$6^3=6^x$

$\therefore \log_6 216 = 3$

(g) $\frac{1}{\sqrt{6}+2}\times\frac{\sqrt{6}-2}{\sqrt{6}-2} = \frac{\sqrt{6}-2}{6-4}$

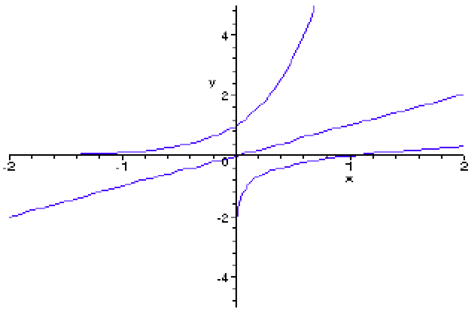
$=\frac{\sqrt{6}-2}{2}$

(h)

NB The line $y=x$ is a line of symmetry for the two graphs.

The y axis is a vertical asymptote for the log graph. The x axis is a horizontal asymptote for the exponential graph.

(1,10) would lie on the exponential graph and (10,1) would be on the log graph.



(i) $m^2-16 = (m-4)(m+4)$

(j) $6-a-2(3-2a)=6-a-6+4a$

$=3a$

(k) $\angle BCA = 30°$ $\quad$ ($\angle$ sum of a $\Delta = 180°$)

Area = $\frac{1}{2}\times 6\times 10\times \sin 30°$

= 15 square centimetres

(l) $f(\theta) = \tan 3\theta + \sin 3\theta$

$$f\left(\frac{\pi}{3}\right) = \tan 3\left(\frac{\pi}{3}\right) + \sin 3\left(\frac{\pi}{3}\right)$$

$$= \tan \pi + \sin \pi$$

$$= 0$$

(m) $x^2 - 5x + 6 = (x-3)(x-2)$

(n) $t + \frac{2t-1}{3} = \frac{3t+2t-1}{3}$

$$= \frac{5t-1}{3}$$

Q.2.

/20

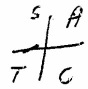
(a) $2\times 3k - 3\times 2 = 10$

$6k - 6 = 10$

$6k = 16$

$k = \frac{16}{6} = \frac{8}{3} = 2\frac{2}{3}$ (2)

(b) $\tan 2\theta = 1$



$2\theta = 45^\circ, 225^\circ, 45+360, 225+360$

$2\theta = 45, 225, 405, 585$

$\theta = 22\frac{1}{2}^\circ, 112\frac{1}{2}^\circ, 202\frac{1}{2}^\circ, 292\frac{1}{2}^\circ$

in radians $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$ (2)

(c) -4.5 (1)

(d) (i) $36x^8 - 6$ (1)

(ii) $10(5x+6)^9 \times 5 = 50(5x+6)^9$ (1)

(iii) $\frac{x^3}{x^2} + \frac{10}{x^2} = x + 10x^{-2}$

$y' = 1 + 10\times -2x^{-3}$

$= 1 - \frac{20}{x^3}$ or $\frac{x^3 - 20}{x^3}$ (2)

(e) $y' = \frac{(3x+2)\times 1 - x\times 3}{(3x+2)^2} = \frac{3x+2-3x}{(3x+2)^2} = \frac{2}{(3x+2)^2}$ (2)

(f) LHS/ $25 - 10\sqrt{2} + 2 = 27 - 10\sqrt{2}$

So $a = 27$ (1)

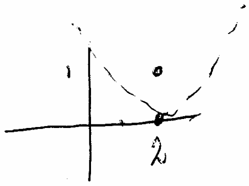
$b = -10$ (1)

(g) $(x-2)^2 = 4(y-0)$

matches with $(x-h)^2 = 4a(y-k)$

$V(2,0)$ (1)

$a = 1$



$F(2,1)$ (1)

(h) let $1-3x=0$

$3x = 1$

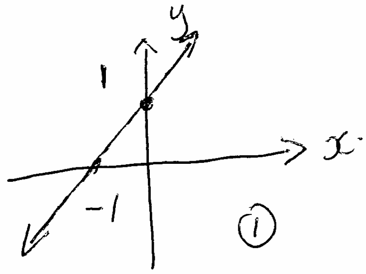
$x = \frac{1}{3}$

let $x+10=0$

$x=-10$

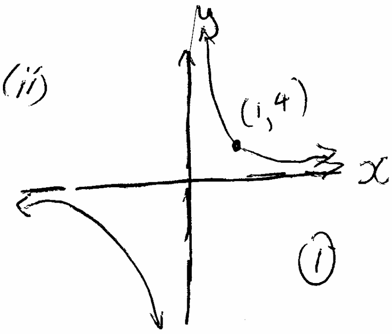
(1)

(i) (1)



(1)

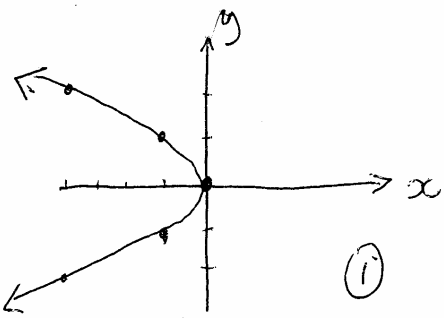
(ii)



(1)

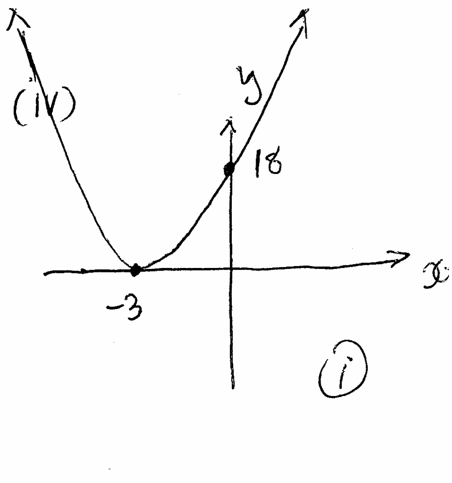
(iii) $y^2 = -x$

x	0	-1	-1	-4	-4	-9	-9			
y	0	1	-1	2	-2	3	-3			



(1)

(iv)



(1)

Q 3

(a) (i) $\log\left(\frac{a}{b}\right) = \log a - \log b$

$= 0.86 - 0.42$

$= 0.44$ (1)

(ii) $\log\sqrt{ab} = \frac{1}{2}\log(ab)$

$= \frac{1}{2}(\log a + \log b)$

$= \frac{1}{2}(0.86 + 0.42)$

$= 0.64$ (2)

(b) (i) $\alpha + \beta = -\frac{-2}{3}$

$= \frac{2}{3}$ (1)

(ii) $\alpha\beta = \frac{1}{3}$ (1)

(iii) $\frac{1}{\alpha^2\beta} + \frac{1}{\beta^2\alpha} = \frac{\beta+\alpha}{\alpha^2\beta^2}$

$= \frac{\left(\frac{2}{3}\right)}{\left(\frac{1}{3}\right)^2}$

$= \frac{2}{3} \times \frac{9}{1}$

$= 6$ (2)

(c) $mx^2 - x + n$

$-2 + 3 = \frac{1}{m}$

$\therefore 1 = \frac{1}{m}$

$\therefore m = 1$

$-2 \times 3 = \frac{n}{m}$

$\therefore n = -6$ (2)

(d) $\lim_{n \to 10} \frac{(n-10)(n+10)}{n-10}$

$= \lim_{n \to 10} (n+10)$ (1)

$= 20$

(e) $|3x+1| > 8$

$\therefore 3x+1 > 8$ OR $3x+1 < -8$

$\therefore 3x > 7$ $\quad 3x < -9$

$x > 2\frac{1}{3}$ $\quad x < -3.$ (2)

(f) $m^{\frac{1}{x}} = n^3$

$\therefore \frac{1}{x}\log m = 3\log n.$

$\therefore \frac{1}{x} = \frac{3\log n}{\log m}$ (2)

$\therefore x = \frac{\log m}{3\log n}$ $\left(= \frac{1}{3}\log_n m\right)$

(g) $m_1 = \frac{b}{4}$ $\quad m_2 = -2$

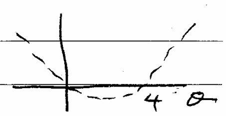
$\therefore \frac{b}{4} = -2$ (2)

$\therefore b = -8$

(h) $\theta^2 - 4\theta < 0$

$\theta(\theta - 4) < 0$

$\therefore 0 < \theta < 4$



(2)

(i) $y = x^2 - 2x$

$y' = 2x - 2$

When $x = 1$ $\quad y = -1$

$y' = 0.$ (2)

$\therefore$ Eqn of tangent is $y = -1.$

20

FOUR

a) $3x^2+4x+5 \equiv A(x^2+2x+1)+Bx+B+C$

$= Ax^2+(2A+B)x+A+B+C$

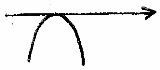
Hence $A=3$

$2A+B=4 \Rightarrow B=-2$

$A+B+C=5 \Rightarrow C=4$

b) $P(x)= k-3kx-x^2$

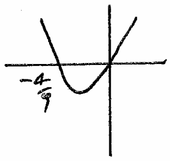
$=-x^2-3kx+k$

If $P(x)\leq 0$ then 

$b^2-4ac \leq 0$

$9k^2+4k\leq 0$

$k(9k+4)\leq 0$



$-\frac{4}{9} \leq k \leq 0$

c) $x^4-10x^2+9=0$

$(x^2-1)(x^2-9)=0$

$x^2=1,\ 9$

$x=\pm 1,\ \pm 3$

d) $y=x^2+bx+c$

$y'=2x+b$

$2x+b=0$ when $x=2$

$b=-4$

Then $y=x^2-4x+c$

Since $(3,3)$ lies on curve

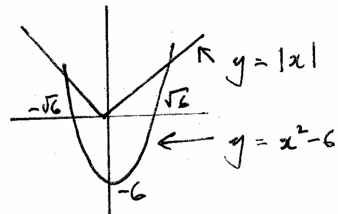
$3=4-8+c$

$c=7$

e) $x^2-4x+4+y^2+2y+1=5$

$(x-2)^2+(y+1)^2=5$

Circle centre $(2,-1)$ radius $\sqrt{5}$

f) 

Points of intersection

When $y=x$

$x=x^2-6$

$x^2-x-6=0$

$(x-3)(x+2)=0$

$x=3$ ONLY

for right hand branch of $y=|x|$

Hence $(3,3)$

When $y=-x$ (left hand branch)

$-x=x^2-6$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3$ ONLY

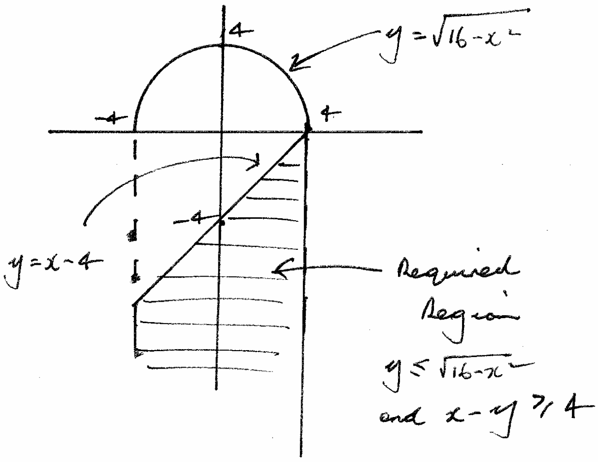
Hence $(-3,3)$

From sketch

$x^2-6\leq |x|$

When $-3\leq x\leq 3$

g)



h)

$\tan A + \sec A = 2$

$\frac{\sin A}{\cos A} + \frac{1}{\cos A} = 2$

$\frac{\sin A + 1}{\cos A} = 2$

$1 + \sin A = 2\cos A$

$\sin A = 2\cos A - 1$

$\sqrt{1-\cos^2 A} = 2\cos A - 1$

$1-\cos^2 A = 4\cos^2 A - 4\cos A + 1$

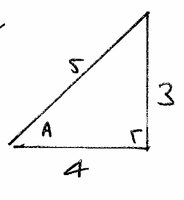
$5\cos^2 A - 4\cos A = 0$

$\cos A\,(5\cos A - 4) = 0$

Likely solutions are

$\cos A = 0, \frac{4}{5}$

Check



If $\cos A = \frac{4}{5}$

$\tan A + \sec A = \frac{3}{4} + \frac{5}{4}$

$= 2$

$\cos A = \frac{4}{5}$ is a solutions

If $\cos A = 0$

$A = 90^\circ$

$\tan A + \sec A \neq 2$

Since $\tan 90^\circ$ is undefined

$\cos A = 0$ is not a solution.

QUESTION 5

(a) $f(x) = (1-4x) \times 2x + (x^2+5)x - 4$

$= 2x - 8x^2 - 4x^2 - 20$

$= -12x^2 + 2x - 20$

(b) Domain: $-\sqrt{3} < x < \sqrt{3}$ Range $y \geq \frac{1}{\sqrt{3}}$

(c) $h(-x) = -x^3 + x = -h(x)$

$\therefore$ $h(x)$ is an odd function

(d) as 5 cm is not the longest side, $\theta < 90°$

$5^2 = 6^2 + x^2 - 2 \times 6 \times x \cos\theta$ (Cos rule)

$x^2 - 12x\cos\theta + 11 = 0$

Solution if $\Delta \geq 0$

$\frac{12^2\cos^2\theta - 4\times 11}{2} \geq 0$

$72\cos^2\theta - 22 \geq 0$

$\cos^2\theta \geq \frac{11}{36}$

$\cos\theta \geq \frac{\sqrt{11}}{6}$ or $\cos\theta \leq -\frac{\sqrt{11}}{6}$

$\frac{\sqrt{11}}{6} \leq \cos\theta$ θ in 1st quadrant

$\cos\theta < 1$ for all values of θ in 1st quadrant

$\therefore$ $\frac{\sqrt{11}}{6} \leq \cos\theta < 1$

(e) (i) In $\triangle ABC$ and $\triangle CDE$ $\angle ACB$ is common

$\angle ABC = \angle EDC = 90°$ (given)

$\therefore$ $\triangle ABC$ is similar to $\triangle DEC$ (equiangular)

(ii) In $\triangle ABC$ and $\triangle CDE$ $\frac{BC}{DC} = \frac{AC}{CE}$ (Corresponding sides)

$\therefore$ $BC \times CE = AC \times CD$

(iii) RHS $= AD \times DC - BE \times EC$

$= (AC - DC)DC - (BC - EC)EC$

$= AC.CD - DC^2 - BC.EC + EC^2$

$= CE^2 - DC^2 + AC.CD - BC.EC$

$= DE^2$ as required ($AC.CD = BC.EC$ from (ii))

(f) $\alpha + \beta = -\frac{b}{a} = -5$, $\alpha\beta = \frac{c}{a} = 7$

$(\alpha+\beta)^2 = 25$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$= 25 - 28 = -3$

required equation $[x - (\alpha-\beta)^2][x - (\alpha+\beta)^2]$

$= (x+3)(x-25)$

$= x^2 - 22x - 75$