



SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

SEPTEMBER 2005

Yearly Examination

YEAR 11

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 90 minutes.
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for messy or badly arranged work.
- Start each **NEW** section in a separate answer booklet.

Total Marks - 80 Marks

- Attempt Questions 1 - 4
- All questions are of equal value.

Examiner: *P. Bigelow*

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Attempt Questions 1 - 4
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Answer each SECTION in a SEPARATE writing booklet.

Section A		Marks
Question 1 (20 marks)		
(a)	Express 0.0407 in scientific notation	1
(b)	If $\sqrt{54} = a\sqrt{6}$ find a	1
(c)	If $f(x) = 4x - 7$, find $f(a+1)$	1
(d)	Solve	
(i)	$\frac{2x}{3} = -4$	1
(ii)	$2x(x-4) = 0$	2
(iii)	$\frac{4}{y-2} = 3$	1
(e)	If $\cos A = 0.407$ find A , where A is an acute angle.	2
(f)	Evaluate $\frac{4 \cdot 2^3 - 6 \cdot 1^2}{\sqrt{4 \cdot 6} - 0.3}$ correct to 1 decimal place.	1
(g)	Express $\frac{2}{\sqrt{3}-1}$ with a rational denominator in <i>simplest</i> form.	2
(h)	Factorise fully:	
(i)	$2a^2 - 8$	2
(ii)	$2a^2 - 2a + ac - c$	2
(iii)	$4y^2 - 15y - 4$	2
(i)	Express $0.\dot{3}\dot{6}$ as a fraction in <i>simplest</i> form.	2

Section B (Use a SEPARATE writing booklet)**Marks****Question 2** (20 marks)

- (a) Find
- (i) $\lim_{x \rightarrow 4} \frac{x^2 + x - 20}{x - 4}$ 1
- (ii) $\lim_{x \rightarrow \infty} \frac{4 - x + 2x^2}{x^2 + 7x - 9}$ 1
- (b) Find x in the following:
- (i) $8^x = 2^{x-4}$ 1
- (ii) $\log_x 36 = 2$ 1
- (iii) $\log_3 \frac{\sqrt{3}}{9} = x$ 1
- (c) Differentiate the following
- (i) $y = (7 - 2x)^6$ 1
- (ii) $y = 7x^{-3} + 4x - 17$ 1
- (iii) $f(x) = x\sqrt{x-2}$ 2
- (iv) $f(x) = \frac{x+4}{x+5}$ 2
- (d) Write down a quadratic equation with roots 4 and -7 . 2
- (e) If $f(x) = \sqrt{x^2 + 16}$ find $f'(3)$. 2
- (f) Find the equation of the tangent to $y = x^2 + 3x + 4$ at the point where $x = 0$. 2
- (g) For the parabola $(x + 2)^2 = 4y - 8$, find the:
- (i) coordinates of the vertex; 1
- (ii) coordinates of the focus; 1
- (iii) equation of the directrix. 1

Section C (Use a SEPARATE writing booklet)

Marks

Question 3 (20 marks)

- (a) Solve the following pair of simultaneous equations 3
- $$\begin{aligned}2x - y - 7 &= 0 \\3x + 2y - 14 &= 0\end{aligned}$$
- (b) Expand and simplify $(a + 1)^2 - (a + 1)(a - 1)$ 3
- (c) Use the quadratic formula to solve $x^2 + 4x - 6 = 0$.
Express your answers in *simplest surd* form. 2
- (d) Sketch the following on separate number planes:
- (i) $3x - 2y + 12 = 0$; 1
- (ii) $y = \frac{4}{x}$; 1
- (iii) $y = x^2 - 4x + 3$. 1
- (e) Write down the exact values of:
- (i) $\tan 30^\circ$; 1
- (ii) $\cos 210^\circ$. 1
- (f) Find to the nearest degree, the angle of depression of a boat
300 metres out to sea, from the top of a vertical cliff of height
50 metres. 3
- (g) If $5^x = 7$ find x correct to three decimal places. 2
- (h) Solve $2 \sin x + 1 = 0$ for $0^\circ \leq x \leq 360^\circ$ 2

Section D (Use a SEPARATE writing booklet)

Marks

Question 4 (20 marks)

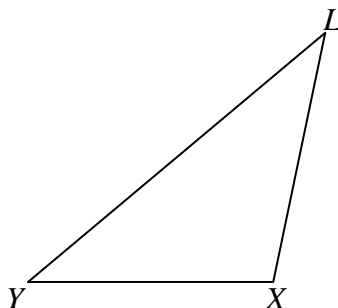
- (a) State the domain of the following:
- (i) $y = \frac{2}{x^2 - 1}$ 1
- (ii) $y = \sqrt{2 - x}$ 1
- (b) State whether the following are ODD, EVEN or NEITHER. Justify your answer. 6
- (i) $f(x) = 1 - x^2$
- (ii) $f(x) = \frac{x}{1 + x^2}$
- (iii) $f(x) = -3x - x^2$
- (iv) $f(x) = -3|x|$

- (c) Find the centre and radius of the circle 3

$$x^2 + y^2 - 4x + 6y - 3 = 0$$

- (d) (i) Graph the *intersection* of the following regions on a number plane: 3
- $$y \geq 2x + 6, \quad x \geq -3, \quad y < 8$$
- (ii) Find the area of this region. 1

- (e) A ship at X is 20 nautical miles from a lighthouse L , which is on a bearing of 025°T . The ship then sails due west to Y , from which the bearing of the lighthouse is 055°T .



Copy the diagram to your answer booklet.

- (i) Show that $\angle XLY = 30^\circ$. 2
- (ii) Show that $XY = 10 \operatorname{cosec} 35^\circ$ and hence find the distance sailed in nautical miles, correct to 1 decimal place. 3

End of paper



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Sample Solutions

Marker AW

QUESTION 12005 2U Mathematics - Solutions

a) $0.0407 = \underline{4.07 \times 10^{-2}}$

b) $\sqrt{54} = 3\sqrt{6} \therefore \underline{a=3}$

c) $f(a+1) = 4(a+1) - 7$
 $\underline{= 4a - 3}$

d) i) $\frac{2x}{3} = -4$
 $2x = -12$
 $\underline{x = -6}$

ii) $2x(x-4) = 0$
 $\underline{x=0 \text{ or } x=4}$

iii) $\frac{4}{y-2} = 3$
 $4 = 3(y-2)$
 $4 = 3y - 6$
 $\underline{y = \frac{10}{3} = 3\frac{1}{3}}$

e) $\cos A = 0.407$
 $\therefore \underline{A = 65^\circ 59'}$

f) $\frac{4 \cdot 2^3 - b \cdot 1^2}{\sqrt{4 \cdot 6} - 0.3} = \underline{20.0}$

g) $\frac{2}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{2\sqrt{3}+2}{3-1}$
 $\underline{= \sqrt{3}+1}$

h) i) $2a^3 - 8 = 2(a^3 - 4)$
 $\underline{= 2(a+2)(a-2)}$

i) ii) $2a^2 - 2a + ac - c$
 $= 2a(a-1) + c(a-1)$
 $\underline{= (2a+c)(a-1)}$

h) iii) $4y^2 - 15y + 4$

$= 4y^2 - 16y + y + 4$

$= 4y(y-4) + (y-4)$

$\underline{= (4y+1)(y-4)}$

i) let $x = 0.36\dot{3}6$ — ①

$100x = 36.36\dot{3}6$ — ②

$99x = 36$ — ② - ①

$\underline{x = \frac{36}{99} = \frac{12}{33}}$

QUESTION 2

a) i) $\lim_{x \rightarrow 4} \frac{x^2 + x - 20}{x - 4}$
 $= \lim_{x \rightarrow 4} \frac{(x+5)(x-4)}{(x-4)}$
 $= \underline{9}$ (1)

ii) $\lim_{x \rightarrow \infty} \frac{4 - x + 2x^2}{x^2 + 7x - 9}$
 $= \lim_{x \rightarrow \infty} \left[\frac{\frac{4}{x^2} - \frac{x}{x^2} + \frac{2x^2}{x^2}}{\frac{x^2}{x^2} - \frac{7x}{x^2} - \frac{9}{x^2}} \right]$
 $= \lim_{x \rightarrow \infty} \left[\frac{\frac{4}{x^2} - \frac{1}{x} + 2}{1 - \frac{7}{x} - \frac{9}{x^2}} \right]$ (1)

NOTE
 $\lim_{x \rightarrow \infty} \frac{1}{x^2} = 0$
 $= \frac{2}{1} = \underline{2}$

b) i) $8^x = 2^{3x} = 2^{x-4}$
 $2^{3x} = 2^{x-4}$
 $3x = x - 4$
 $x = \underline{-2}$ (1)

ii) $\log_x 36 = 2$
 $x^2 = 36$
 $x = \underline{\pm 6}$ (1)

iii) $\log_3 \frac{\sqrt{3}}{9} = x$
 $\log_3 \sqrt{3} - \log_3 9 = x$
 $\frac{1}{2} - 2 = x$
 $x = \underline{-1\frac{1}{2}}$ (1)

c) i) $\frac{dy}{dx} = -12(7-2x)^5$

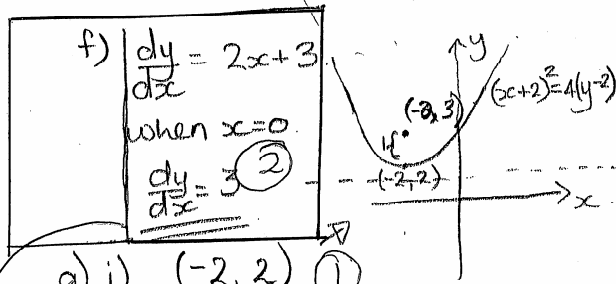
ii) $\frac{dy}{dx} = -21x^{-4} + 4$

iii) $f'(x) = u'v + uv'$
 $= (x-2)^{1/2} + x \left(\frac{1}{2}(x-2)^{-1/2} \right)$
 $= \sqrt{x-2} + \frac{x}{2\sqrt{x-2}}$ (2)

iv) $f'(x) = \frac{v'u' - uv''}{v^2}$
 $= \frac{(x+5) - (x+4)}{(x+5)^2}$
 $= \frac{1}{(x+5)^2}$ (2)

d) $(x-4)(x+7) = \underline{x^2 + 3x - 28}$ (2)

e) $f'(x) = \frac{1}{2}(x^2+16)^{-1/2} \cdot 2x$
 $= \frac{x}{\sqrt{x^2+16}}$ (2)
 $f'(3) = \frac{3}{5}$



g) i) $(-2, 2)$ (1)

ii) $(-2, 3)$ (1)

iii) $y = 1$ (1)

★ Gradient tangent = 3.
 when $x=0$, $y=4$.
 $4-4 = 3(x-0)$ $\& 3x - 4 + 4 = 0$

QUESTION 30

a) $2x - y - 7 = 0$ — ①
 $3x + 2y - 14 = 0$ — ②
 $2 \times ①$
 $4x - 2y - 14 = 0$ — ③

③ + ②

$7x - 28 = 0$

$x = +4$

$y = 2x - 7$

$y = +1$

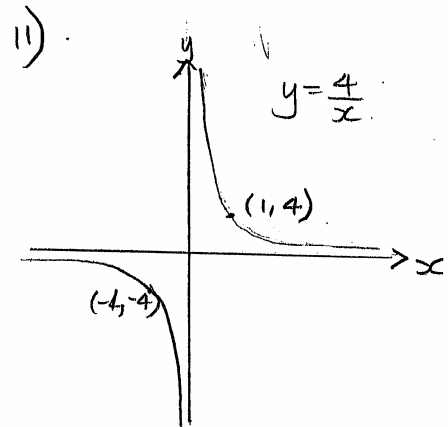
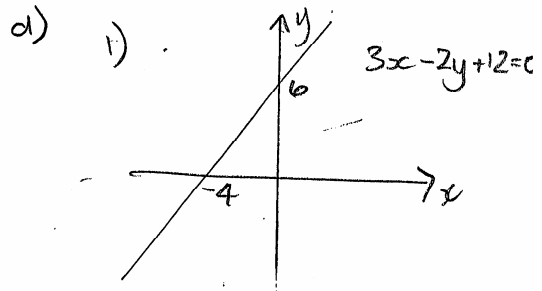
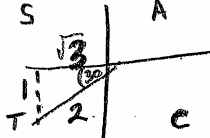
Solution is: $x=4, y=1$

b) $(a+1)^2 - (a+1)(a-1)$
 $= a^2 + 2a + 1 - a^2 + 1$
 $= 2a + 2$
 $= 2(a+1)$

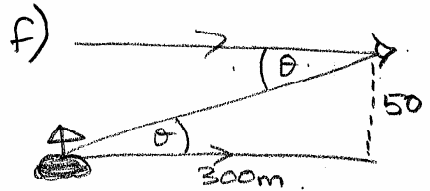
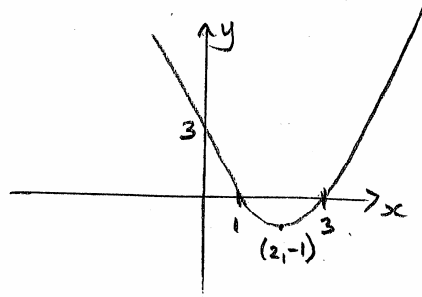
c) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $= \frac{-4 \pm \sqrt{16 + 24}}{2}$
 $= \frac{-2 \pm \sqrt{10}}{1}$

e) i) $\tan 30^\circ = \frac{1}{\sqrt{3}}$

ii) $\cos 210^\circ = \frac{-\sqrt{3}}{2}$



iii) $y = x^2 - 4x + 3$
 $y = (x+1)(x-3)$



$\tan \theta = \frac{50}{300} = \frac{5}{3} = \frac{1}{b}$

$\theta = 9^\circ 27' = 9^\circ$ (nearest degree)

QUESTION 3 (CONT)

$$g) 5^x = 7$$

$$\log_5 7 = x$$

$$\therefore x = \frac{\log_{10} 7}{\log_{10} 5}$$

$$= 1.209 \text{ (3 dp)}$$

$$h) 2 \sin x + 1 = 0$$

$$\sin x = -\frac{1}{2}$$

$$x = 210^\circ, 330^\circ$$

QUESTION 4

- a) i) $x \in \mathbb{R} : x \neq 1$
 ii) $x \in \mathbb{R} : x < 2$

b) i) $f(x) = 1 - x^2$
 $f(-x) = 1 - (-x)^2$
 $= 1 - x^2$
 $= f(x)$
 ∴ EVEN FUNCTION

ii) $f(x) = \frac{x}{1+x^2}$
 $f(-x) = \frac{-x}{1+(-x)^2}$
 $= \frac{-x}{1+x^2}$
 $= -f(x)$

∴ ODD FUNCTION

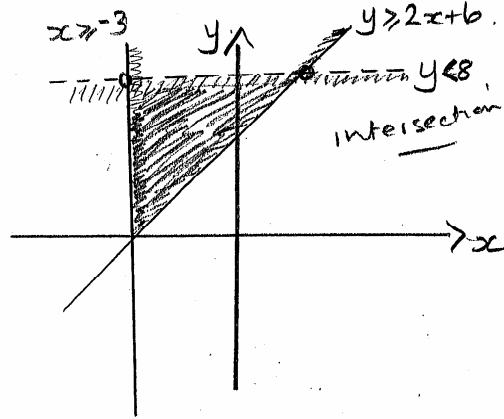
iii) $f(x) = -3x - x^2$
 $f(-x) = 3x - x^2$
 ∴ NEITHER

iv) $f(x) = -3|x|$
 $f(-x) = -3|-x|$
 $= -3|x|$
 $= f(x)$

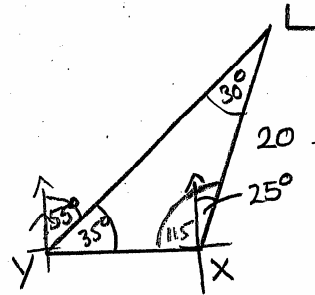
∴ EVEN FUNCTION

c) $x^2 + y^2 - 4x + by - 3 = 0$
 $x^2 - 4x + (2)^2 + y^2 + by + 3^2 = 3 + 4 + 9$
 $(x-2)^2 + (y+3)^2 = 16$
 Centre of Circle $(2, -3)$
 Radius = 4

d)



e)



i) $90^\circ - 55^\circ = 35^\circ$ (complement \angle s)

ii) $\frac{XY}{\sin \angle YLX} = \frac{YL}{\sin \angle LYX}$

$\frac{XY}{\sin 30^\circ} = \frac{20}{\sin 35^\circ}$

$XY = \frac{1}{2}(20) \operatorname{cosec} 35^\circ$
 $= 10 \operatorname{cosec} 35^\circ$

$XY = 17.4$ nautical miles