SYDNEY BOYS HIGH
MOORE PARK, SURRY HILLS

SEPTEMBER 2007
YEARLY EXAMINATION
YEAR 11 Continuers

Mathematics

General Instructions:

- Reading time—5 minutes.
- Working time—90 minutes.
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may NOT be awarded for messy or badly arranged work.

Total marks—100 Marks

- Attempt all questions.
- Start each NEW section in a separate answer booklet.
- Hand in your answer booklets in 3 sections: A, B, C.

Examiner: Mr C.Kourtesis

Section A — Start a new booklet

Marks

Question 1 (31 marks)

(a) If $L = gt^2$, find the value of L if $g = 10$, $t = 3$. 1

(b) Solve $2a + 1 = -7$. 1

(c) Factorize

(i) $3a + 15$; 1

(ii) $x^2 - A^2$. 1

(d) Simplify

(i) $3(2a - 4)$; 1

(ii) $\frac{n}{4} + \frac{n}{3}$; 1

(iii) $10a^2b \times 2b^3$. 1

(e) Solve

(i) $|x - 4| = 14$; 2

(ii) $-3x \geq 12$; 1

(iii) $4x^2 = 5$. 1

(f) Express $\frac{1}{\sqrt{10}}$ with a rational denominator. 1

(g) If $f(x) = \frac{4}{x}$, find $f\left(\frac{1}{a}\right)$. 1

(h) If $y = 7x^4 + x - 1$, find $\frac{dy}{dx}$. 2

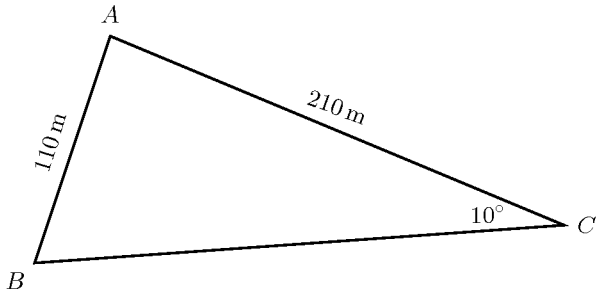
(i) Find the exact value of: $\tan 150° + \cos 45°$. 2

(j) The line $6x - ky = 4$ passes through the point $(2, -1)$. Find the value of k. [2]

(k) Solve the pair of simultaneous equations [2]

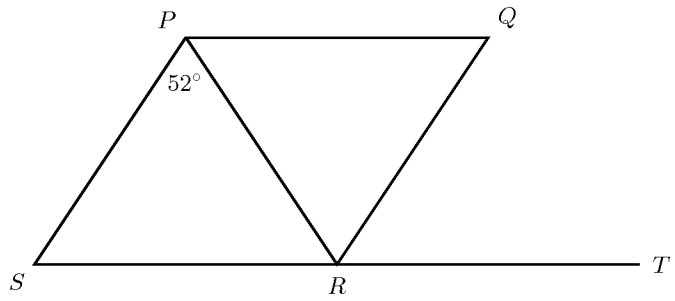
$$y = 2x + 1$$
$$x - 2y = 7$$

(l) [2]



Find the size of $A\widehat{B}C$ to the nearest degree.

(m)



$PQRS$ is a rhombus where $S\widehat{P}R = 52°$ and SR is produced to T.

(i) Find the size of $S\widehat{P}Q$. [1]

(ii) What is the size of $Q\widehat{R}T$? Give reasons. [2]

(n) If $\sqrt{45} + \sqrt{80} = a\sqrt{5}$, find the value of a. [2]

(o) Find

(i) $\log_2 8$; [1]

(ii) $\log_3 54 - \log_3 18$. [2]

Section B — Start a new booklet

Question 2 (35 marks)

(a) Express $0.\dot{2}\dot{4}$ as a rational number. [1]

(b) Factorize $x^3 - 27$. [1]

(c) Solve the equation [2]

$$\tan\theta = -\frac{1}{\sqrt{3}} \text{ for } 0° \leq \theta \leq 360°.$$

(d) The first three terms of an arithmetic series are 12, 17, and 22. Find:

(i) the thirty-fifth term of the series; [2]

(ii) the sum of the first thirty-five terms. [1]

(e) A geometric series has second term 6 and the ratio of the seventh term to the sixth term is 3.

(i) Find the common ratio, r. [1]

(ii) What is the first term, a? [1]

(iii) Calculate the sum of the first 12 terms. [1]

(f) Differentiate with respect to x:

(i) $(1 - 10x)^{11}$; [1]

(ii) $(2x + 1)(3 - x^2)$ using the product rule; [2]

(iii) $\dfrac{x}{x^2 + 4}$ using the quotient rule. [2]

(g) Let α and β be the roots of the equation $x^2 - 5x + 2 = 0$. Find the values of:

(i) $\alpha + \beta$ [1]

(ii) $\alpha\beta$ [1]

(iii) $\alpha^2 + \beta^2$ [2]

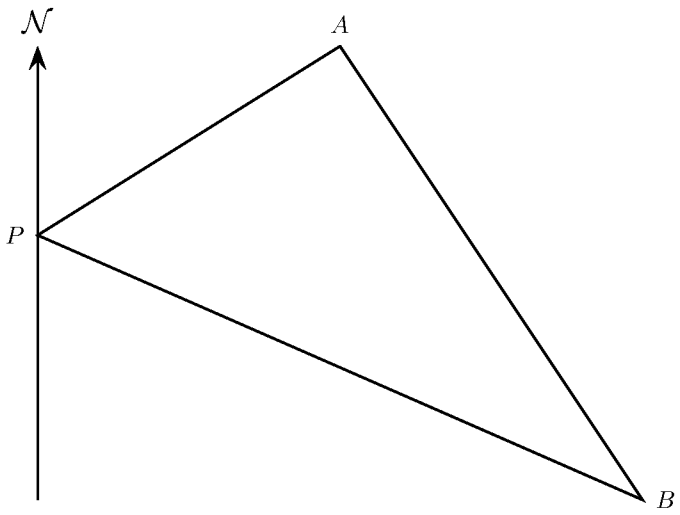
(h) A parabola P has equation $x^2 = 8(4 - y)$. Find:

(i) the coördinates of its focus; [2]

(ii) the equation of the directrix; [1]

(iii) the coördinates of all points of intersection of P with the coördinate axes. [1]

(i)



Ship A is 35 nautical miles from a port P and is on a bearing of 065°. Ship B is 50 nautical miles from P and is on a bearing of 115°.
Find:

(i) the size of $A\widehat{P}B$; [1]

(ii) the distance between the ships (answer to the nearest nautical mile). [2]

(j) A function $f(x)$ is defined by

$$f(x) = \begin{cases} 0 & \text{if} \quad x \leq -2 \\ -1 & \text{if} \quad -2 < x < 0 \\ x & \text{if} \quad x \geq 0 \end{cases}$$

(i) Find $f(-4) + f\left(-\frac{1}{2}\right) + f(0)$. [2]

(ii) Sketch the graph of $y = f(x)$. [1]

(k) In a certain school the senior students are divided into two classes A and B. Class A contains 11 female and 9 male students and class B contains 8 female and 12 male students. A certain occasion requires two speakers, one from each class. These are selected at random. Find the probabilities that:

(i) both speakers are male; [1]

(ii) both speakers are female; [1]

(iii) one speaker is male and one is female. [2]

(l) Find the domain and range of [2]

$$y = \frac{1}{\sqrt{9 - x^2}}.$$

Section C — Start a new booklet

Question 3 (34 marks)

(a) Solve

(i) $|4 - 5x| > 1$ [2]

(ii) $|x + 1| = 2x + 7$ [2]

(b) Solve the equation [3]

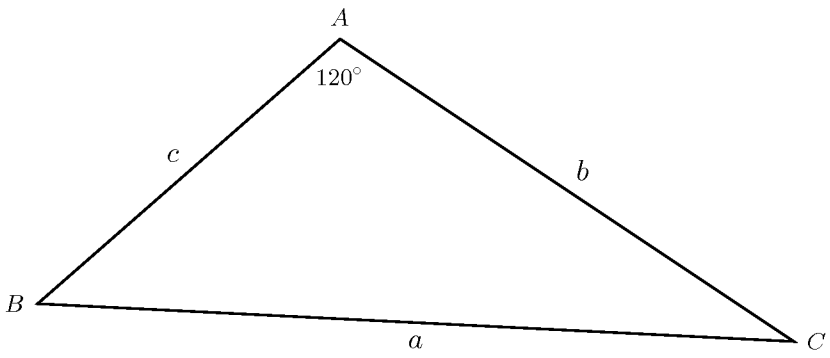
$$x^6 - 28x^3 + 27 = 0.$$

(c) Find the perpendicular distance from the point $(4, -3)$ to the line $5x = 2y + 9$. [2]

(d) In the Cartesian plane, indicate by shading the region whose points simultaneously satisfy $y \geq 1$ and $y \leq \sqrt{4 - x^2}$. [3]

(e) If $n^2 \equiv an(n-1) + bn + c$, find the values of a, b, and c. [3]

(f) [3]



In triangle ABC, prove that

$$\cos C = \frac{2b + c}{2a}.$$

(g) Two marksmen, A and B, fire simultaneously at a target. If A is twice as likely to hit the target as B, and if the probability that the target does get hit is $1/2$, find the probability of A hitting the target. [3]

(h) If $\cot\theta = -\dfrac{5}{12}$ and $\cos\theta > 0$, find $\sin\theta$. 3

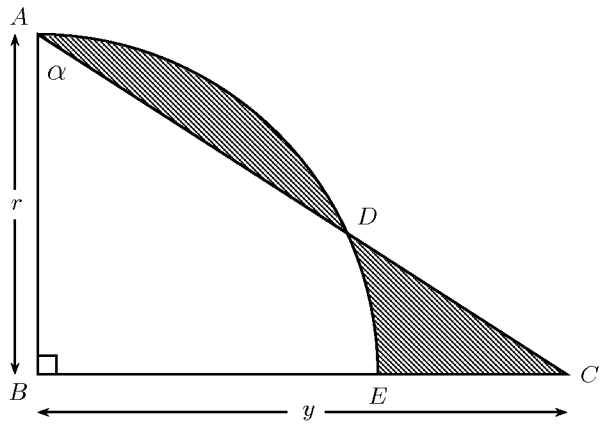
(i) The n^{th} term of the series $3 + \dfrac{4}{3} + \dfrac{11}{18} + \ldots$ is $A\left(\dfrac{1}{3}\right)^n + B\left(\dfrac{1}{2}\right)^n$. Find

(i) the values of A and B; 3

(ii) the fourth term. 1

(j) Find the equation of the parabola with axis $y = 0$, vertex $(0, 0)$, and the line $x - y + 3 = 0$ as a tangent to the curve. 3

(k) 3



$ADEB$ is a sector with centre B. $\triangle ABC$ is right-angled at B.
The two shaded regions have the same area.
Show that $\tan\alpha = \dfrac{\pi}{2}$.

End of Paper

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}dx = \ln x, \quad x > 0$$

$$\int e^{ax}dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos axdx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin axdx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 axdx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan axdx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Question 1:

a) $L = 10 \times 3^2$
$= 10 \times 9$
$= 90$

b) $2a = -8$
$a = -4$

c) i $3(a+5)$

ii $(x+A)(x-A)$

d) i $6a - 12$

ii $\frac{3n+4n}{12} = \frac{7n}{12}$

iii $20a^2b^4$

e) i $x - 4 = 14$
$x = 18$
$-x + 4 = 14$
$-x = 10$
$x = -10$

ii $x = 18, -10$

ii $-3x \geq 12$
$x \leq -4$

iii $x^2 = 5/4$

$x = \pm \frac{\sqrt{5}}{2}$

f) $\frac{\sqrt{10}}{10}$

g) $f'(a) = 4/4a$
$= 4a$

h) $\frac{dy}{dx} = 28x^3 + 1$

i) $\frac{-1}{\sqrt{3}} + \frac{1}{\sqrt{2}}$

j) $12 + k = 4$
$k = -8$

k) $y = 2x + 1$ ①
$x - 2y = 7$ ②

① into ②
$x - 2(2x+1) = 7$
$x - 4x - 2 = 7$
$-3x = 9$
$\boxed{x = -3}$

sub into ①
$y = 2x - 3 + 1$
$\boxed{y = -5}$

check in ②
$-3 - 2x - 5 = -3 + 10$
$= 7$ ✓

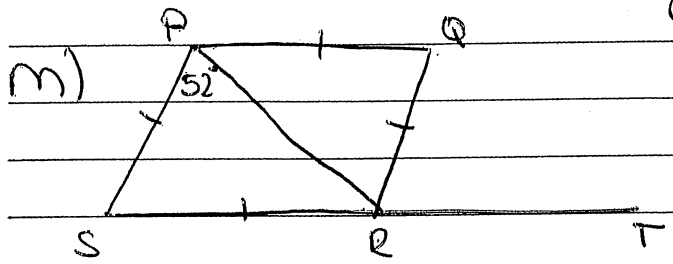
l) $\frac{\sin \angle ABC}{210} = \frac{\sin 10}{110}$

$\sin \angle ABC = \frac{210 \sin 10}{110}$

$\angle ABC = 19.36046155$

$= 19°$ (nearest degree)

m)



$PS = PQ = QR = RS$ (sides of a rhombus)

$\therefore \triangle PRS$ is isosceles

$\therefore \angle PRS = 52°$ (base $\angle$'s isos $\triangle$)

$\therefore \angle PSR = 76°$ ($\angle$ sum $\triangle$)

$\therefore \angle PQR = 76°$ (opp $\angle$'s in rhombus)

$\therefore \angle QPR$ & $\angle QRP = 52°$ (base $\angle$'s is∞ $\triangle$)

($\triangle SPR \equiv \triangle QPR$ SAS test)

$\therefore \angle SPQ = 104°$

ii) $\angle SRQ = 104°$ (Opp $\angle$'s of rhombus)

$\therefore \angle QRT = 76°$ ($\angle$ sum st line)

n) $\sqrt{9} \times \sqrt{5} + \sqrt{16} \times \sqrt{5}$

$= 3\sqrt{5} + 4\sqrt{5}$

$= 7\sqrt{5}$

$\therefore a = 7$

o) i $\log_2 8 = 3$

ii $\log_3 54 - \log_3 18$

$= \log_3 \frac{54}{18}$

$= \log_3 3 = 1$

Section B

2 (a) $0.\dot{2}\dot{4} = \frac{24}{99} = \frac{8}{33}$ (1)

(b) $(x-3)(x^2+3x+9)$ (1)

(c) $\tan\theta = -\frac{1}{\sqrt{3}}$ quad 2 $180-\theta$, quad 4 $360-\theta$

now $\tan 30^\circ = \frac{1}{\sqrt{3}}$.

So answers are $180-30 = 150^\circ$ (1)

$360-30 = 330^\circ$ (1)

(d) arithmetic 12, 17, 22

$a = 12$

$d = 5$

(i) $T_{35} = a+(n-1)d$

$= 12 + 34\times 5$

$= 182.$ (2)

(ii) $S_{35} = \frac{n}{2}(a+L)$

$= \frac{35}{2}(12+182)$

$= 3395$ (1)

(e) geometric $T_2 = ar = 6$

$\frac{T_7}{T_6} = \frac{ar^6}{ar^5} = 3 \quad r = 3.$

(i) $r = 3$ (1)

(ii) $3\times a = 6 \quad a = 2$ (1)

(iii) $S_n = \frac{a(r^n-1)}{r-1} = \frac{2(3^{12}-1)}{3-1} = 531440$ (1)

(f) (i) $\frac{d}{dx}(1-10x)^{11}$

$= 11(1-10x)^{10}\times -10$

$= -110(1-10x)^{10}$ (1)

(ii) $\frac{d}{dx}(2x+1)(3-x^2)$

$= (2x+1)\times -2x + (3-x^2)\times 2$

$= -4x^2-2x+6-2x^2$

$= -6x^2-2x+6.$ (2)

(iii) $\frac{d}{dx}\left(\frac{x}{x^2+4}\right)$

$= \frac{(x^2+4)\times 1 - x\times 2x}{(x^2+4)^2}$

$= \frac{x^2+4-2x^2}{(x^2+4)^2}$

$= \frac{4-x^2}{(4+x^2)^2}.$ (2)

(9) $x^2 - 5x + 2 = 0$

$a = 1 \quad b = -5 \quad c = 2$

(i) $\alpha + \beta = \frac{-b}{a} = \frac{5}{1} = 5$ (1)

(ii) $\alpha\beta = \frac{c}{a} = \frac{2}{1} = 2$. (1)

(iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

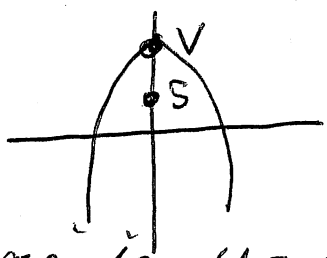
$= 25 - 4 = 21$ (2)

(b) $x^2 = 8(4-y)$

$(x-0)^2 = -8(y-4)$

$(x-0)^2 = -4\times 2(y-4)$. $\quad (x-h)^2 = -4a(y-k)$

(i) vertex is $(0,4)$ $a = 2$, down curve.



focus $(0,2)$ (2)

(ii) directrix is $y = 6$ (1)

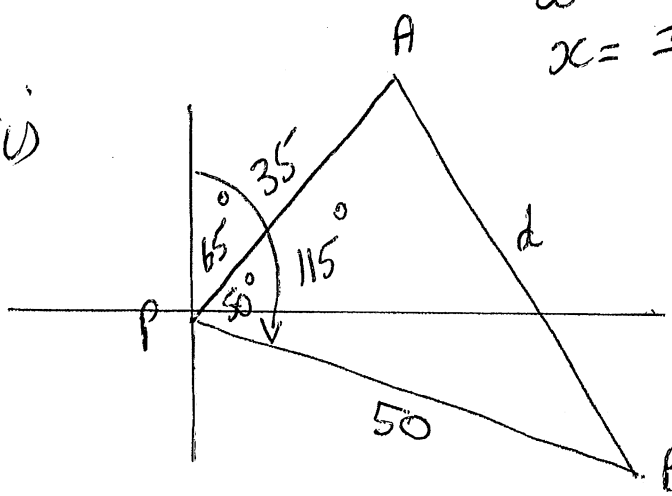
cuts y axis at $(0,4)$

cuts x axis when $x^2 = 8(4-0)$

$x^2 = 32$

$x = \pm\sqrt{32} = \pm 4\sqrt{2}$ (1)

(c)



(i) $A\hat{P}B = 50^\circ$ (1)

(ii) $d^2 = 35^2 + 50^2 - 2\times 35\times 50\times \cos 50^\circ$

$d \doteq 38.41$

$= 38$ mm (2)

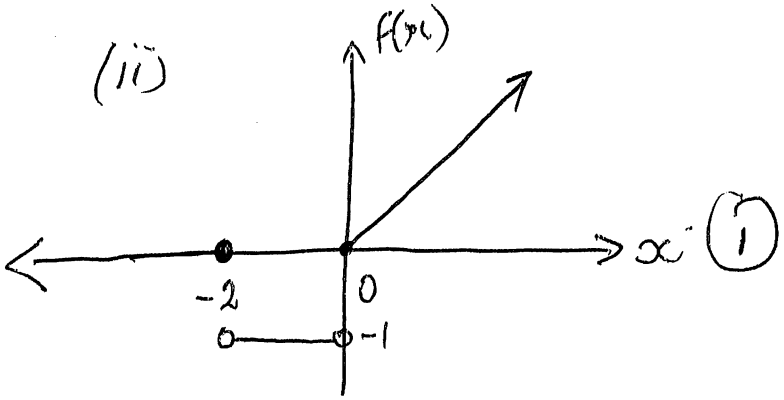
(j) (i) $f(-4) = 0$

$f(-\frac{1}{2}) = -1$

$f(0) = 0$

-1 (2)

(ii)



(1)

(k)

A	B
11 F	8 F
9 m	12 m.

(i) $P(mm) = \frac{9}{20} \times \frac{12}{20} = \frac{27}{100}$ (1)

(ii) $P(FF) = \frac{11}{20} \times \frac{8}{20} = \frac{11}{50}$ (1)

(iii) $\underset{m}{\frac{9}{20}} \times \underset{F}{\frac{8}{20}} + \underset{F}{\frac{11}{20}} \times \underset{m}{\frac{12}{20}} = \frac{61}{100}$ (2)

(L) $y = \frac{1}{\sqrt{9-x^2}}$.

Domain $-3 < x < 3$ (1)

Range if $x=0$, $y = \frac{1}{3}$ Range $y \geq \frac{1}{3}$ (1)

(3)

(a). (2)

$|4-5x| > 1$

(i) $4-5x > 1$ or $4-5x < -1$.

$5x < 3$ | $5x > 5$

$x < \frac{3}{5}$ | $x > 1$.

(ii) $|x+1| = 2x+7$ (2)

$x+1 = 2x+7$ or $x+1 = -2x-7$

~~$x = -6$~~ | $3x = -8$

| $x = -8/3$

(b) $x^6 - 28x^3 + 27 = 0$

$y^2 - 28y + 27 = 0$

$(y-27)(y-1) = 0$ (3)

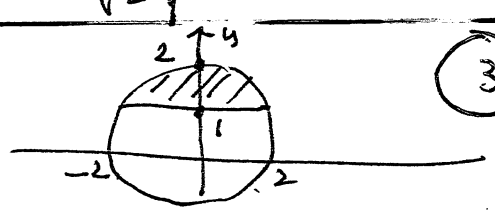
$\therefore x^3 = 27$, $x = 3$

$x^3 = 1$, $x = 1$

(c) $d = \frac{|5\times 4 - 2\times(-3) - 9|}{\sqrt{29}}$ (2)

$= \frac{17}{\sqrt{29}} \doteq 3.16$

(d) (3)

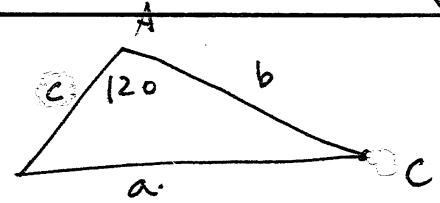


(e) $u^2 = au^2 + (b-a)u + c$.

$\therefore a = 1$, $b - 1 = 0 \therefore b = 1$.

$c = 0$. (3)

(f). (3)



$-\frac{1}{2} = \frac{b^2+c^2-a^2}{2bc}$.

$a^2 = b^2 + c^2 + bc$. —— (1)

or $c = \frac{a^2+b^2-c^2}{2ab}$ —— (2)

Subst. (1) into (2)

or $c = \frac{2b+c}{2a}$.

(g) (3)

$P(A)P(B) + P(A)P(\bar{B}) + P(\bar{A})P(B) = \frac{1}{2}$

Let $P(A) = p$, $P(B) = \frac{p}{2}$

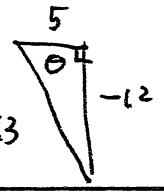
$\frac{p^2}{2} + p(1-\frac{p}{2}) + \frac{p}{2}(1-p) = \frac{1}{2}$

$\therefore p^2 + 2p - p^2 + p - p^2 = 1$

$p^2 - 3p + 1 = 0$

$p = \frac{3-\sqrt{5}}{2}$ $\because p \le 1$

(h) (3)



$\sin\theta = -\frac{12}{13}$.

$= -0.923$

(i) $3 + \frac{4}{3} + \frac{11}{18} + \cdots$

$= A(\frac{1}{3})^n + B(\frac{1}{2})^n$ (3)

$n = 1$, $\frac{A}{3} + \frac{B}{2} = 3$. —— (1)

$n = 2$ $\frac{A}{9} + \frac{B}{4} = \frac{47}{36}$. —— (2)

Solve (1) & (2) $A = 3$, $B = 4$.

(ii) $T_4 = 3(\frac{1}{3})^4 + 4(\frac{1}{2})^4$ (1)

$= \frac{31}{108}$.

(j). $y^2 = 4ax$ $x = y - 3$. (3)

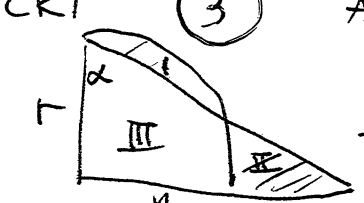
$y^2 - 4ay + 12 = 0$

$\Delta = 0$, $16a^2 - 48 = 0$

$a = \sqrt{3}$.

$\therefore y^2 = 4\sqrt{3}x$ —— *

(k) (3)



$A(I) = A(II)$

$y = r\tan\alpha$ (1)

$\frac{\pi r^2}{4}$

$I + III = II + III = \frac{r^2}{2}\tan\alpha$ (2)

$I - II = (1) - (2)$