SYDNEY BOYS HIGH
MOORE PARK, SURRY HILLS

SEPTEMBER 2008
YEARLY EXAMINATION
YEAR 11 Continuers

Mathematics Preliminary

General Instructions:

- Working time—90 minutes.
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may NOT be awarded for messy or badly arranged work.
- Answer in simplest exact form unless otherwise instructed.

Total marks—80 Marks

- Attempt all questions.
- Start each NEW section in a separate answer booklet.
- Hand in your answer booklets in 4 sections: A, B, C, D.

Examiner: Mr D. Hespe

Section A — Start a new booklet

Marks

Question 1 (20 marks)

(a) Evaluate $\dfrac{(3.26)^2}{16.39 - 12.51}$ correct to two decimal places. $\boxed{2}$

(b) Express 0.00035 in scientific notation. $\boxed{1}$

(c) (i) Find the reciprocal of $2\frac{3}{4}$. Answer as a fraction. $\boxed{1}$

(ii) Also express the answer as an exact decimal. $\boxed{1}$

(d) Simplify $\sqrt{147} - \sqrt{75}$. $\boxed{2}$

(e) Evaluate $\cot 3$ correct to 3 significant figures. $\boxed{2}$

(f) Express $\dfrac{2\sqrt{3}}{\sqrt{3} - 1}$ with a rational denominator in simplest form. $\boxed{2}$

(g) Factorise $2x^2 + 3x - 2$. $\boxed{2}$

(h) Solve $5t + 3 = 2(1 - t)$. $\boxed{2}$

(i) Find the slope of the normal to the line $2x - 3y + 6 = 0$. $\boxed{2}$

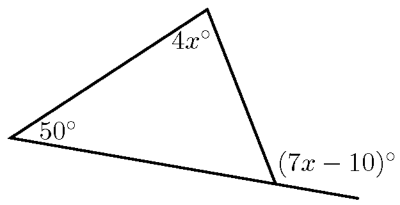
(j) Sketch the curve $y = \sqrt{4 - x}$ for $0 \leqslant x \leqslant 4$. $\boxed{3}$

Section B — Start a new booklet

Question 2 (20 marks)

(a) If $r = \sqrt{\dfrac{A}{4\pi}}$, find A (to the nearest integer) when $r = 6.25$. $\boxed{2}$

(b) Find the value of x, giving reasons. $\boxed{4}$



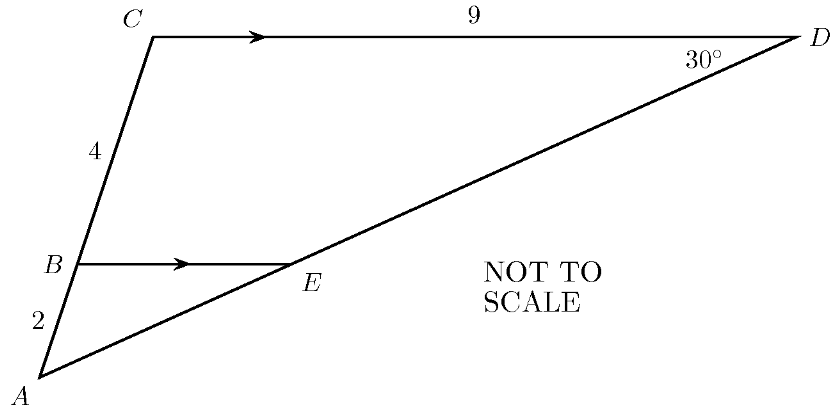
(c) Solve the pair of equations $\begin{aligned} 3x - 2y &= 9 \text{ and} \\ x + 4y &= -4. \end{aligned}$ $\boxed{3}$

(d) Factorise completely $x^4 - xy^3 + x^3y - y^4$. $\boxed{2}$

(e) Write down the domain and range of the function $y = 2 + |x - 3|$. $\boxed{2}$

(f) A triangle is isosceles with its vertex angle 20° more than a base angle. Find the measure of the base angle. $\boxed{2}$

(g)



In the diagram, ACD is a triangle where $AB = 2\,\text{cm}$, $BC = 4\,\text{cm}$, $CD = 9\,\text{cm}$, and $\angle CDE = 30°$. Also, BE is parallel to CD.

(i) Find the size of $\angle BED$. Give a reason for your answer. $\boxed{1}$

(ii) Find the length of BE. Give reasons for your answer. $\boxed{4}$

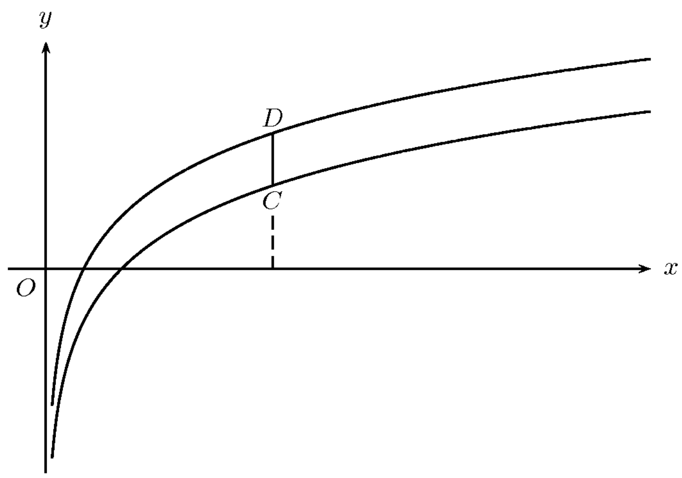
Section C — Start a new booklet

Question 3 (20 marks)

(a) Solve for x: $\log_7\left(\dfrac{x-4}{x-1}\right) = 2$. $\boxed{2}$

(b) Find the equation in general form of the line through the point (2, 0) and the intersection of the two lines $x + y + 1 = 0$, $2x - y = 3$. $\boxed{3}$

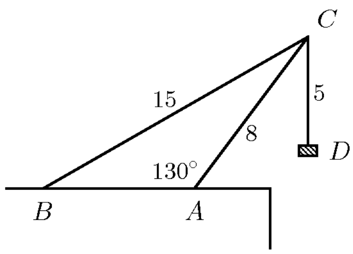
(c) $\boxed{2}$



A vertical line is drawn to cross the two graphs $y = \log_{10} x$ and $y = \log_{10} 2x$ at points C and D. Show that the distance CD is constant (that is, does not depend on the position where the vertical line is drawn).

(d) (i) In the shear-legs shown to the right, find the height of the load D above the horizontal level of AB. Note that all lengths are in metres and answer to 2 decimal places. $\boxed{2}$

(ii) Find also the length of AB correct to 2 decimal places. $\boxed{3}$



(e) Consider the parabola $12y = x^2 - 6x - 3$.

(i) Find the equation of its directrix. $\boxed{2}$

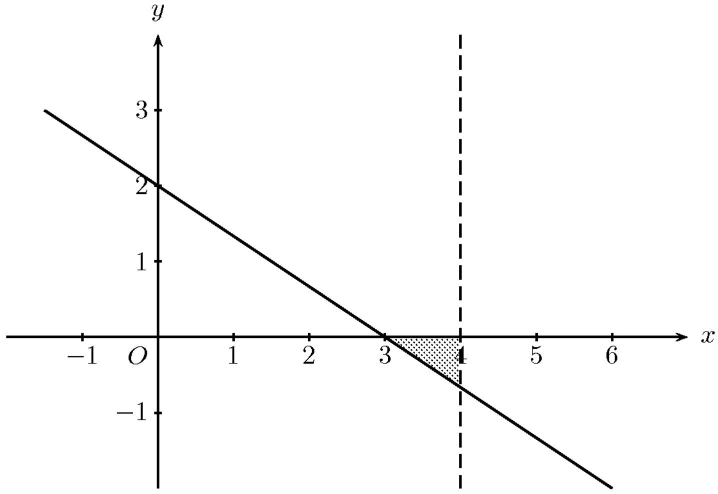
(ii) Find the coordinates of its focus. $\boxed{2}$

(f) Find all real numbers x which satisfy the equation $4x^4 = 4x^2 + 3$. $\boxed{4}$

Section D — Start a new booklet

Question 4 (20 marks)

(a) Write down three inequalities to describe the shaded region given. $\boxed{3}$



(b) The coordinates $A(-1,\ 1)$, $B(1,\ 4)$, and $C(10,\ -2)$ are joined to form $\triangle ABC$.

(i) Draw a diagram on a number plane, marking on it the information supplied. $\boxed{1}$

(ii) Prove that $\triangle ABC$ is right-angled. $\boxed{2}$

(iii) Find the coordinates of M, the midpoint of the hypotenuse. $\boxed{1}$

(iv) Show that M is the centre of a circle that could be drawn through A, B, and C. $\boxed{3}$

(c) A triangle PQR has side lengths $p = 20$, $q = 15$, $r = 11$.

(i) Find the size of each angle. $\boxed{4}$

(ii) Hence or otherwise, find its area. $\boxed{2}$

(d) Prove that $\sin^2 x = 1 - \cfrac{1}{1 - \cfrac{1}{1 - \cfrac{1}{\sin^2 x}}}$ $\boxed{4}$

End of Paper

Question 1

a) $\dfrac{(3.26)^2}{16.39 - 12.51}$

$= 2.739072165$

$= 2.74$ (2dp) (1)

b) 0.00035

$= 3.5 \times 10^{-4}$ (1)

c) i) $2\frac{3}{4} = \frac{11}{4}$

reciprocal $= 4/11$ (1)

ii) $4/11 = 0.\dot{3}\dot{6}$ (1)

d) $\sqrt{147} - \sqrt{75}$

$= \sqrt{49} \times \sqrt{3} - \sqrt{25} \times \sqrt{3}$ (1)

$= 7\sqrt{3} - 5\sqrt{3}$

$= 2\sqrt{3}$ (1)

e) $\cot 3 = \dfrac{1}{\tan 3}$ (1)

$= 19.08113669$

$= 19.1$ (3 sig fig) (1)

f) $\dfrac{2\sqrt{3}}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \dfrac{6 + 2\sqrt{3}}{3-1}$ −1 for 1st mistake ½ for each extra.

$= \dfrac{6+2\sqrt{3}}{2}$

$= 3 + \sqrt{3}$ (2)

g) $2x^2 + 3x - 2$ -4

$= \dfrac{(2x+4)(2x-1)}{2}$

$= \dfrac{2(x+2)(2x-1)}{2}$

$= (x+2)(2x-1)$ (2)

h) $5t + 3 = 2(1-t)$

$5t + 3 = 2 - 2t$

$7t = -1$

$t = -1/7$

i) $2x - 3y + 6 = 0$

$3y = 2x + 6$

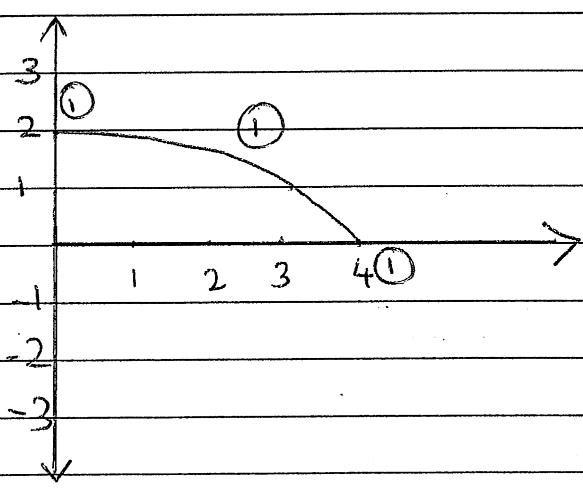
$y = \frac{2}{3}x + 2$

$\therefore$ slope $= 2/3$ (1)

$\therefore$ slope of normal $= -3/2$ (1)

j) $y = \sqrt{4-x}$ for $0 \le x \le 4$

x	0	1	2	3	4
y	2	$\sqrt{3}$	$\sqrt{2}$	1	0



SECTION B

(a) $r = \sqrt{\frac{A}{4\pi}} = 6.25$

$\frac{A}{4\pi} = 6.25^2$

$A = 6.25^2 \times 4\pi = 490.87$

491 nearest integer

(b) $4x + 50 = 7x - 10$ (Ext. ∠ = sum of int. op. angles)

$60 = 3x$

$x = 20$

(c) $3x - 2y = 9$ ①

$x + 4y = -4$ ②

①×2 $6x - 4y = 18$

$7x = 14$

$x = 2$

② $2 + 4y = -4$

$y = -\frac{6}{4} = -1\frac{1}{2}$

$x = 2y = -1\frac{1}{2}$

(d) $x^4 - xy^3 + x^3y - y^4$

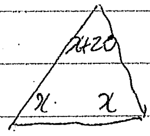
$x(x^3 - y^3) + y(x^3 - y^3)$

$(x+y)(x^3 - y^3)$

$(x+y)(x-y)(x^2 + xy + y^2)$

(e) Domain: All real x

Range $y \geq 2$

(f) 

$3x + 20 = 180$ (∠ sum of △)

$3x = 160$

$x = 53\frac{1}{3}^\circ$ = base angle

(g) $BED = 180 - 30 = 150^\circ$

(i) (co-int. to ∠CDE)

(ii) In △s ACD + ABE

∠CAD is common

∠BEA = ∠CDE

(corr ∠s CE∥BE)

∴ △ACD ||| △ABE (equiangular)

∴ $\frac{BE}{9} = \frac{2}{6}$ (corr sides)

$BE = 3u$

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3 (a) $\frac{x-4}{x-1} = \frac{7^2}{1}$

$x-4 = 49(x-1)$

$x-4 = 49x-49$

$-4+49 = 48x$

$45 = 48x$

$x = \frac{15}{16}$ ✓

2

(b) $x+y=-1$

$2x-y=3$ +

$3x = 2$

$x = \frac{2}{3}$

$y = -1\frac{2}{3}$

$(\frac{2}{3}, -1\frac{2}{3})$ $(2,0)$

$m = \frac{0+1\frac{2}{3}}{2-\frac{2}{3}} = \frac{1\frac{2}{3}}{1\frac{1}{3}} = 1\frac{1}{4} = \frac{5}{4}$

$(y-0) = \frac{5}{4}(x-2)$

$5x-4y-10=0$

OR.

$(x+y+1)+k(2x-y-3)=0$

$(2,0)$ $(2+0+1)+k(4-0-3)=0$

$3+k=0$

$k=-3$

$(x+y+1)-3(2x-y-3)=0$

$x+y+1-6x+3y+9=0$

$-5x+4y+10=0$

$5x-4y-10=0$ ✓

3

(c) $y=\log_{10}x \qquad y=\log_{10}2x$.

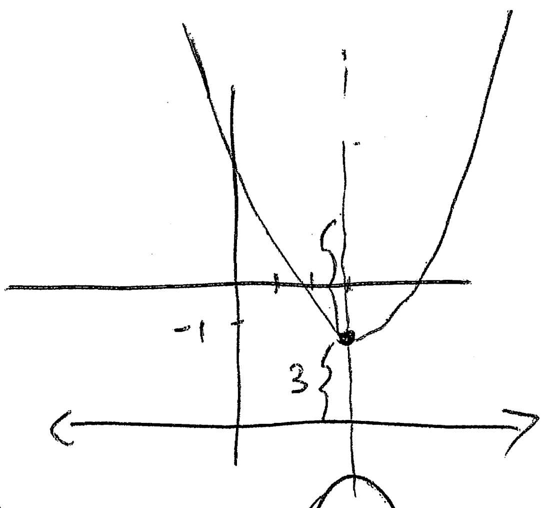
At $x=x$, $(x, \log_{10}x)$ and $(x, \log_{10}2x)$.

$$d = \sqrt{(\cancel{x-x})^2 + (\log_{10}2x - \log_{10}x)^2}$$
$$= \log_{10}2x - \log_{10}x$$
$$= \log_{10}2 + \cancel{\log_{10}x} - \cancel{\log_{10}x}$$
$$= \underline{\log_{10}2}$$

2

(e) $12y = x^2 - 6x - 3$.

$$12y + 3 = x^2 - 6x$$
$$12y+3+9 = x^2 - 6x + 9$$
$$12y + 12 = (x-3)^2$$
$$(x-3)^2 = 12(y+1)$$
$$(x-h)^2 = 4a(y-k)$$
$$a = 3,\ V(h,k) = (3,-1)$$



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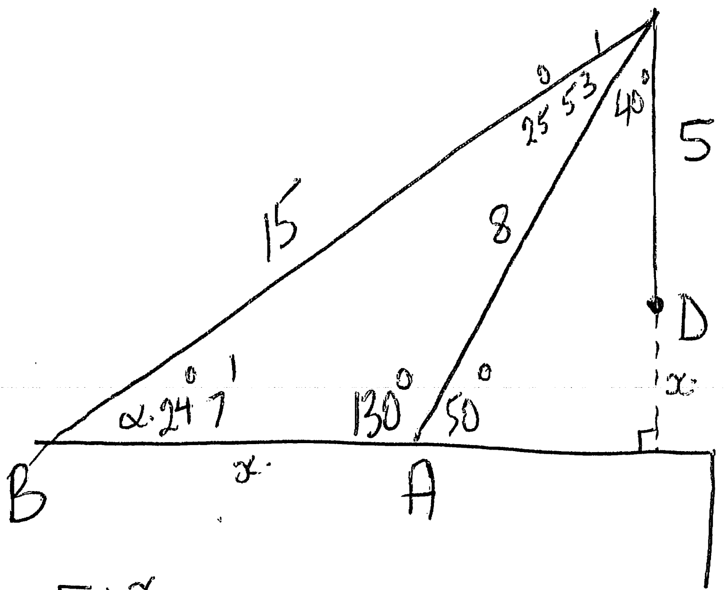
(i) $y=-4$ 2

(ii) $S(3,2)$. 2

(d)



(i)

$$\sin 50° = \frac{5+x}{8}$$

$$8\sin 50° = x+5$$

$$8\sin 50° - 5 = x$$

$$x \doteq 1.128$$

$$\doteq 1.13\text{ m (2DP)}$$

2

(ii)

$$\frac{\sin\alpha}{8} = \frac{\sin 130}{15}$$

$$\sin\alpha = \frac{8\times\sin 130}{15}$$

$$\alpha \doteq 24°7'$$

$$\frac{x}{\sin 25°53'} = \frac{15}{\sin 130°}$$

$$x = \frac{15\times\sin 25°53'}{\sin 130°}$$

$$= 8.5479\text{ m}$$

$$AB \doteq 8.55\text{ m (2DP)}$$

3

(f) $4x^4 = 4x^2 + 3$

$$4x^4 - 4x^2 - 3 = 0$$

let $u = x^2$, $\quad 4u^2 - 4u - 3 = 0$

$$(2u + 1)(2u - 3) = 0$$

$$u = -\frac{1}{2} \text{ and } u = \frac{3}{2}.$$

So $x^2 = -\frac{1}{2}$ $\qquad$ $x^2 = \frac{3}{2}$

no real solns. $\qquad$ $x = \pm\sqrt{\frac{3}{2}}$ $\qquad$ 4

SECTION D

Question 4 (3)

(a) Eq^n of oblique line

$$\frac{x}{3} + \frac{y}{2} = 1$$

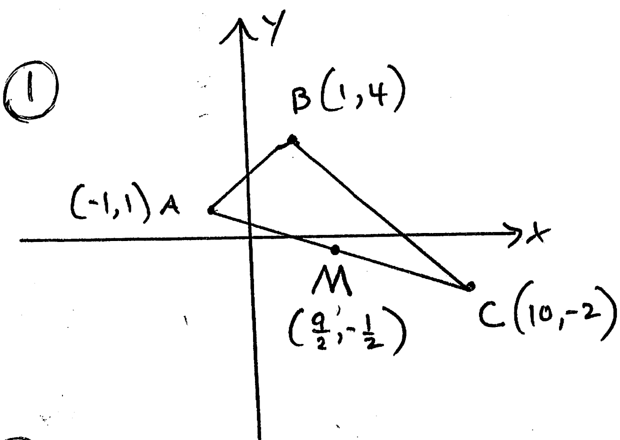
ie $2x + 3y = 6$

$\therefore$ Region defined by:

$$\boxed{2x + 3y \geq 6 \text{ and } x < 4 \text{ and } y \leq 0}$$

(b)

(i) (1)



(ii) (2) $M_{AB} = \frac{4-1}{1--1} = \frac{3}{2}$

$M_{BC} = \frac{4--2}{1-10} = -\frac{2}{3}$

Since $M_{AB} \times M_{BC} = -1$

$\Rightarrow AB \perp BC$

$\therefore \angle ABC = 90°$

(or use Pyth. Th.) and show

$(AC)^2 = (AB)^2 + (BC)^2$

(1) (b) (iii) ctd

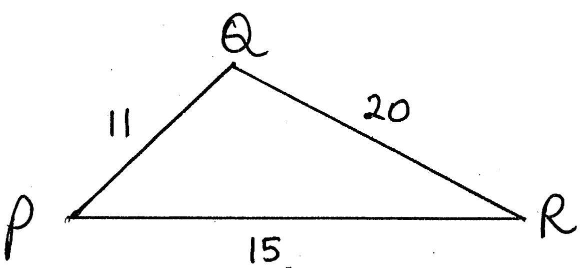
$$M\left(\frac{10+-1}{2}, \frac{-2+1}{2}\right) \equiv \left(\frac{9}{2}, -\frac{1}{2}\right)$$

(iv) (3) If $AM = BM = MC \Rightarrow A, B, C$ lie on circle with above equal radii, centre M.

$$AM = \sqrt{\left(\frac{9}{2}--1\right)^2 + \left(-\frac{1}{2}-1\right)^2} = \sqrt{\frac{130}{4}} = MC$$

$$BM = \sqrt{\left(\frac{9}{2}-1\right)^2 + \left(-\frac{1}{2}-4\right)^2} = \sqrt{\frac{130}{4}}$$

(c) (4) (i)



$$\cos P = \frac{11^2 + 15^2 - 20^2}{2.11.15} \Rightarrow P = 99°25'$$

$$\cos Q = \frac{11^2 + 20^2 - 15^2}{2.11.20} \Rightarrow Q = 47°43'$$

$\therefore R = 32°52'$

(ii) (2) Area $\Delta PQR = \frac{1}{2}.11.20 \sin Q$

$= 81.38$ units^2

(d) 4

$$RHS = 1 - \cfrac{1}{1 - \cfrac{1}{1 - \cfrac{1}{S^2}}}$$

$S = \sin x$

$$= 1 - \cfrac{1}{1 - \cfrac{1}{1 - \operatorname{cosec}^2 x}}$$

since

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$$= 1 - \cfrac{1}{1 - \cfrac{1}{-\cot^2 x}}$$

$$= 1 - \frac{1}{1 + \tan^2 x}$$

$$= 1 - \frac{1}{\sec^2 x}$$

since

$1 + \tan^2 x = \sec^2 x$

$$= 1 - \cos^2 x$$

$$= \sin^2 x$$

since

$\sin^2 x + \cos^2 x = 1$

$= LHS$