SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2010
Year 11 Yearly Examination

Mathematics
(2 Unit)

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- All necessary working should be shown in every question.
- All answers to be given in simplified exact form unless otherwise stated.
- Hand in your answers in 2 separate bundles:

  Section A (Question 1, Question 2 and Question 3), and

  Section B (Question 4, Question 5 and Question 6)

Total Marks – 70

- Attempt questions 1-6
- All questions are **NOT** of equal value.

Examiner: *P. Bigelow*

Section A – Start a new booklet.

Question 1 (10 marks). **Marks**

a) The point (2,*c*) lies on $x+2y+4=0$, find the value of *c*. 2

b) Find T_{10} of $5+9+13+17+...$ 1

c) If $f(y)=9-y^2$, find: 2

(i) $f(-2)$

(ii) $f(y+1)$

d) Solve $x^2+2x-8=0$. 1

e) State the domain of $f(x)=\sqrt{3-x}$. 1

f) Find *x* in the following: 2

(i) $\log_x 36=2$

(ii) $\log_8 128=x$

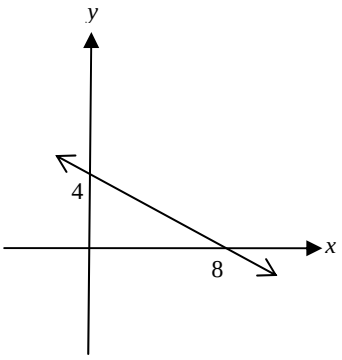
g) Form a quadratic equation with roots 2 and -5. 1

End of Question 1

Question 2 (10 Marks). **Marks**

a) Find the sum of the first 9 terms of $2-1+\frac{1}{2}-\frac{1}{4}+...$ 1

b) Write down the equation of the line represented in the diagram: 2



c) State whether the following functions are ODD, EVEN or NEITHER: 3

(i) $f(x)=\frac{4}{9+x^2}$

(ii) $f(x)=\frac{x}{9+x}$

(iii) $f(x)=\frac{-x}{9+x^2}$

d) Solve $2^{3x+2}=64$ 2

e) Use the quadratic formula to solve $2x^2+5x-1=0$. (Leave answers in surd form.) 2

End of Question 2

Question 3 (11 marks). **Marks**

a) By considering $0.\dot{5}\dot{4}$ as an infinite geometric series, express $0.\dot{5}\dot{4}$ as a fraction in simplest form. 2

b) Sketch the following lines on separate diagrams (showing essential features): 4

(i) $y = x + 2$

(ii) $3x - y - 6 = 0$

c) State whether the following quadratics are INDEFINITE, POSITIVE DEFINITE or NEGATIVE DEFINITE: 3

(i) $2x^2 + 3x + 7$

(ii) $6 - x - x^2$

(iii) $x^2 - 9x - 8$

d) Find $\lim_{x \to 4} \dfrac{x-4}{x^2-16}$. 2

End of Question 3

End of Section A

Section B – Start a new booklet.

Question 4 (13 marks). **Marks**

a) Sketch the following on separate diagrams (showing essential features): 6

(i) $xy = -4$

(ii) $y = 2^{-x}$

(iii) $y = \sqrt{4 - x^2}$

(iv) $x^2 + y^2 - 6y = 0$

b) If α and β are the roots of $2x^2 - 6x - 1 = 0$, find: 4

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$

(iv) $\alpha^2 + \beta^2$

c) If $f(x) = x^2 + 2x + 5$, find $f'(x)$ where $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. 3

End of Question 4

Question 5 (14 marks). **Marks**

a) Graph the region defined by the intersection of $y \geq x^2$ and $x + y \leq 2$. 2

b) Find $f'(x)$ in the following: 6

(i) $f(x) = 3x^2 - x + 1$

(ii) $f(x) = (1 - 5x)^6$

(iii) $f(x) = \frac{1}{\sqrt{x}}$

c) If $\log_a 3 = 0.477$ and $\log_a 2 = 0.301$. Find: 6

(i) $\log_a 6$

(ii) $\log_a 9$

(iii) $\log_a 1.5$

End of Question 5

Question 6 (14 marks). **Marks**

a) (i) Find the axis of symmetry of $y = 4 + x - x^2$. 2

(ii) Hence, or otherwise, find the maximum value of $4 + x - x^2$. 2

b) Find the equation of the two tangents to the curve $y = 3x^2 - 6x$ at the points where it crosses the x-axis. 4

c) Given $5^x = 13$, find x correct to two decimal places. 2

d) A couple wish to save for a deposit on a home. They need to save \$20,000 over a 5 year period. They deposit \$$P$ every month into an account which is paying 9%p.a., compounding monthly. 4

(i) Show that $20000 = P\left(1.0075 + 1.0075^2 + ... + 1.0075^{60}\right)$.

(ii) Find P to the nearest dollar.

End of Question 6

End of Section B.

End of Examination.

Solutions Yr 11 2 unit Yearly 2010.

(1) (a) $x + 2y + 4 = 0$

$x = 2$, $y = c$

$2 + 2c + 4 = 0$

$2c + 6 = 0$

$c = -3$. (2)

(b) $5 + 9 + 13 + 17 + \cdots$

Find T_{10}.

arithmetic $T_n = a + (n-1)d$

$T_{10} = 5 + 9 \times 4$

$= 41$ (1)

(c) $f(y) = 9 - y^2$

(i) $f(-2) = 9 - (-2)^2 = 9 - 4 = 5$ (1)

(ii) $f(y+1) = 9 - (y+1)^2 = 9 - (y^2 + 2y + 1) = 9 - y^2 - 2y -$

$= 8 - y^2 - 2y$ (

(d) $x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4, \quad x = 2.$ (1)

(e) $f(x) = \sqrt{3-x}$ $3 - x \geq 0$

$-x \geq -3$ (1)

$x \leq 3$

(f) (i) $x^2 = 36$ $x = 6$ (1)

(ii) $8^x = 128$

$2^{3x} = 2^7 \Rightarrow 3x = 7$ $x = \frac{7}{3}$ or $2\frac{1}{3}$ (1)

Q1 (a) $(x-2)(x+5)=0$

or $x^2+5x-2x-10=0$

$x^2+3x-10=0$ (1)

Q2 (a) $2-1+\frac{1}{2}-\frac{1}{4}+\cdots$

geometric

$a=2,\ n=9,\ r=-\frac{1}{2}$

$S_n=\frac{a(r^n-1)}{(r-1)}=\frac{2\left((-\frac{1}{2})^9-1\right)}{-\frac{1}{2}-1}=2\times\frac{2}{3}\left((-\frac{1}{2})^9-1\right)$

$=-\frac{4}{3}\left(-\frac{1}{512}-1\right)=1\frac{43}{128}$ (1)

(b) points are $(8,0)$ and $(0,4)$

$m=\frac{y_2-y_1}{x_2-x_1}=\frac{4-0}{0-8}=-\frac{1}{2}$

use $(y-y_1)=m(x-x_1)$ and $(8,0)$

$(y-0)=-\frac{1}{2}(x-8)$

$2y=-x+8$

$x+2y-8=0$ (2)

(c) (i) $f(x)=\frac{4}{9+x^2}$ $f(-x)=\frac{4}{9+(-x)^2}=\frac{4}{9+x^2}$ even (1)

(ii) $f(x)=\frac{x}{9+x}$, $f(-x)=\frac{-x}{9-x}=\frac{-x}{-(x-9)}=\frac{x}{x-9}$

neither (1)

2 (c) (iii) $f(x) = \frac{-x}{9+x^2}$.

$f(-x) = \frac{--x}{9+(-x)^2} = \frac{x}{9+x^2}$.

odd is $f(x) = -f(-x)$ so $-\left(\frac{x}{9+x^2}\right) = \frac{-x}{9+x^2}$.

odd. (1)

(d) $2^{3x+2} = 64$

$2^{3x+2} = 2^6$.

$3x+2 = 6$

$3x = 4 \quad x = 4/3$. (2)

(e) $2x^2 + 5x - 1 = 0$

$x = \frac{-5 \pm \sqrt{25 - 4 \times 2 \times -1}}{4}$

$= \frac{-5 \pm \sqrt{33}}{4}$ (2)

Q3. (a) $0.\dot{5}\dot{4} = 0.54 + 0.0054 + 0.000054 + \ldots$

geometric $a = 0.54$

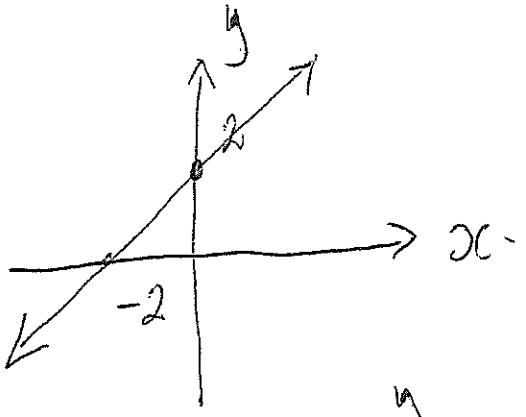
$r = 0.01$

$S_\infty = \frac{a}{1-r} = \frac{0.54}{1-0.01}$

$= \frac{0.54}{.99} = \frac{54}{99}$

(2) $= \frac{6}{11}$

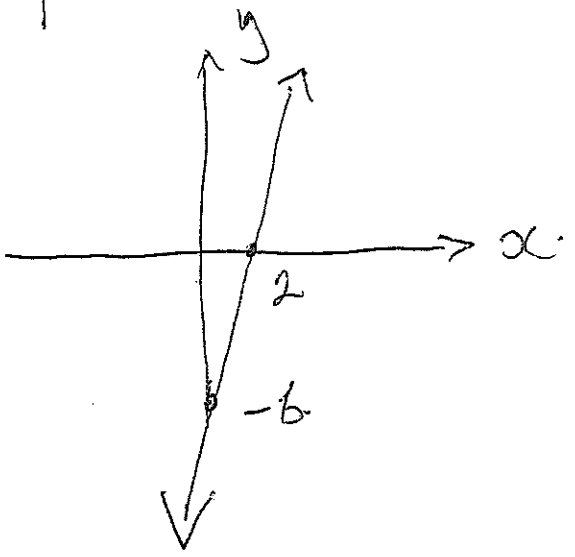
3 (b) (i) $y = x+2$.



(2)

(ii) $3x - y - 6 = 0$

$y = 3x - 6$



(2)

(c) $\Delta = b^2 - 4ac$

(i) $2x^2 + 3x + 7$ $\quad \Delta = 9 - 4 \times 2 \times 7 = -47$

concave up. So positive def (1)
no roots

(ii) $-x^2 - x + 6$ $\quad \Delta = 1 - 4 \times -1 \times 6$
$= 25$

concave down indefinite (1)
has roots

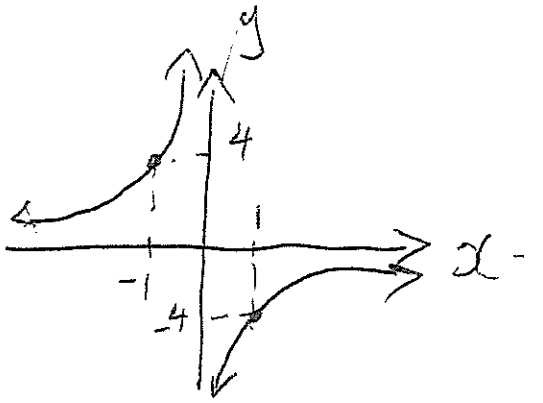
(iii) $x^2 - 9x - 8$ $\quad \Delta = 81 - 4 \times 1 \times -8$
$= 113.$

concave up indefinite (1)
has roots

(d) $\lim_{x \to 4} \dfrac{(x-4)}{(x-4)(x+4)} = \dfrac{1}{x+4} \to \dfrac{1}{8}$ (2)

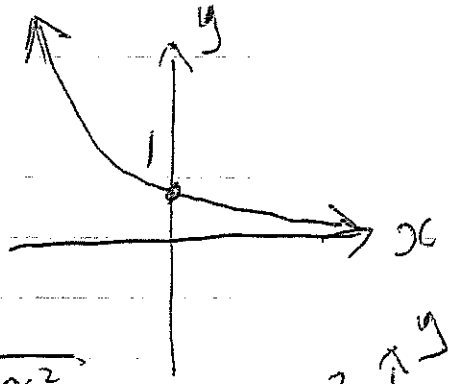
Q4 (a) (i) $xy = -4$

$y = -\frac{4}{x}$.



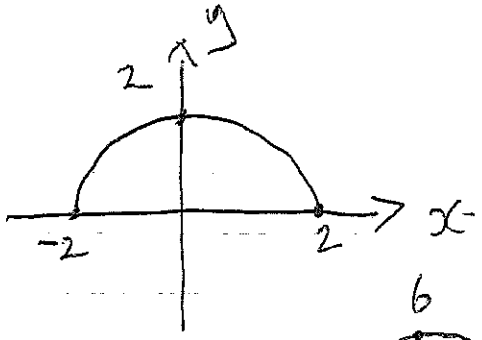
(1)

(ii) $y = 2^{-x}$



(1)

(iii) $y = \sqrt{4-x^2}$



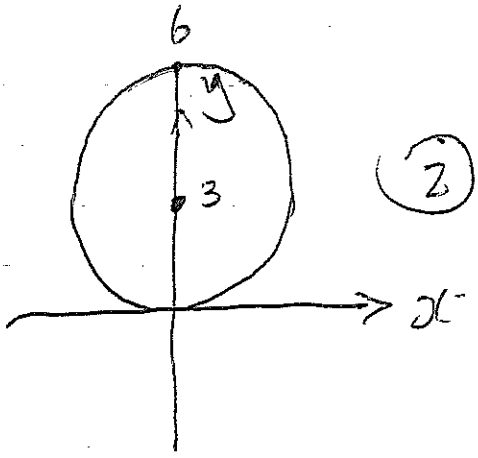
(2)

(iv) $x^2 + y^2 - 6y = 0$

$\left(\frac{-b}{2}\right)^2$

$x^2 + y^2 - 6y + 9 = 9$

$x^2 + (y-3)^2 = 9$



(2)

circle centre $(0, 3)$ $r = 3$.

(b) $2x^2 - 6x - 1 = 0$

(i) $\alpha + \beta = -\frac{b}{a} = -\frac{-6}{2} = 3$ (1)

(ii) $\alpha\beta = \frac{c}{a} = -\frac{1}{2}$ (1)

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{3}{-\frac{1}{2}} = -6$ (1)

(iv) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 3^2 - 2\times\frac{-1}{2} = 9 + 1 = 10$ (1)

4 (c) $f(x) = x^2 + 2x + 5$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 2(x+h) + 5 - (x^2 + 2x + 5)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 2x + 2h + 5 - x^2 - 2x - 5}{h}$$

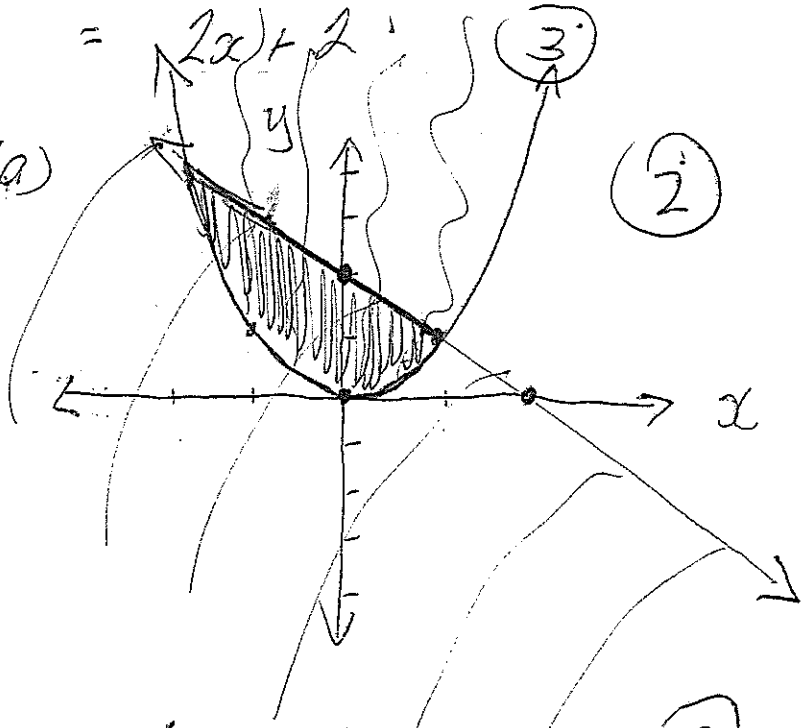
$$= \lim_{h \to 0} \frac{2xh + h^2 + 2h}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h + 2)}{h}$$

$$= 2x + 2$$

(3)

Q5 (a)



(2) $y \geq x^2$ and $x + y \leq 2$.

(b) (i) $f'(x) = 6x - 1$ (2)

(ii) $f'(x) = 6(1-5x)^5 \times -5 = -30(1-5x)^5$ (2)

(iii) $f(x) = x^{-\frac{1}{2}}$, $f'(x) = -\frac{1}{2}x^{-1\frac{1}{2}} = \frac{-1}{2x\sqrt{x}}$ (2)

Q5 (c) (i) $\log_a 6 = \log_a(3\times 2)$

$= \log_a 3 + \log_a 2$

$= 0.477 + 0.301$

$= 0.778$ (2)

(ii) $\log_a 9 = \log_a 3^2$

$= 2\log_a 3 = 2 \times 0.477$

$= 0.954$ (2)

(iii) $\log_a \frac{3}{2} = \log_a 3 - \log_a 2$

$= 0.477 - 0.301$

$= 0.176$ (2)

Q6. (a) (i) $y = -x^2 + x + 4$

axis $x = -\frac{b}{2a} = \frac{-1}{2\times -1} = \frac{1}{2}$. (2)

max y value is $x = \frac{1}{2}$ into $y = -x^2 + x + 4$

$= -(\frac{1}{2})^2 + \frac{1}{2} + 4$

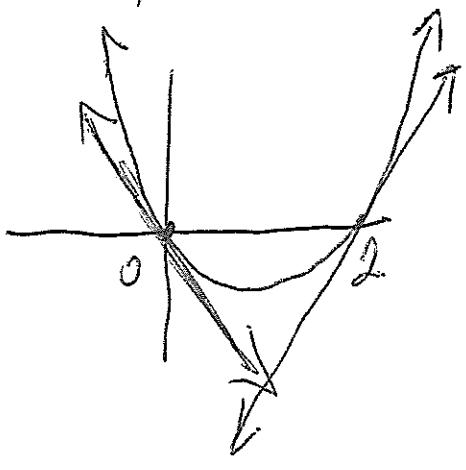
$= 4\frac{1}{4}$ (2)

(b) $y = 3x^2 - 6x$

$= 3x(x-2)$

when $3x(x-2) = 0$

$x = 0$ and $x = 2$.



6 (b) cont

$y = 3x^2 - 6x$

$y' = 6x - 6$

at $x=0$ $m = 0-6 = -6$

at $x=2$ $m = 6\times 2 - 6 = 6$.

So $(0,0)$ $m=-6$ and $(2,0)$ $m=6$.

$(y-0) = -6(x-0)$

$y = -6x$ (2)

$(y-0) = 6(x-2)$

$y = 6x - 12$ (2)

(c) $5^x = 13$.

$\log_{10} 5^x = \log_{10} 13$

$x \log_{10} 5 = \log_{10} 13$

$x = \dfrac{\log_{10} 13}{\log_{10} 5}$

$\doteq 1.59$ (2)

(d)

(6) (d) $20 000

5 years.

$P every month.

9% p.a $\Rightarrow$ 0.75% per month.

1st month $P + 0.75\%P = P(1+0.75\%)^{60}$

$= P(1.0075)^{60}$

2nd month $P(1.0075)^{59}$

last month $P(1.0075)^{1}$

So $P(1.0075)^{1} + P(1.0075)^{2} + \cdots + P(1.0075)^{60} = 2000$ (2)

So $P = \dfrac{20,000}{1.0075 + 1.0075^{2} + \cdots + 1.0075^{60}}$

denominator $\dfrac{r^{n}-a}{r-1} = \dfrac{1.0075^{60} \times 1.0075 - 1.0075}{1.0075 - 1}$

$= \dfrac{1.0075^{61} - 1.0075}{.0075}$

$= 75.98981795\ldots$

$P = \$263.19$

$\doteq \$263.$ (2)