SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

Yearly Examination 2011

Mathematics
2 Unit

General Instructions

- Working time – 90 Minutes
- Reading Time – 5 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators may be used.
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- All necessary working should be shown in every question.
- Answer in simplest exact form unless otherwise instructed.

Total Marks – 64

- All Questions may be attempted.
- Section A (Q1 & Q2) and Section B (Q3 & Q4) should be handed up in separate examination booklets.
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- Full Marks may not be awarded for careless or poorly set out work.

Examiner – *P.R.Bigelow*

Section A
[Start a new booklet]

Question 1. (17 Marks)

(a) Write down the exact values of: **2**

(i) $\sin 45^\circ$

(ii) $\cos\dfrac{7\pi}{6}$

(b) The point (4, *a*) lies on the line $4x - 3y - 7 = 0$. Find the value of *a*. **1**

(c) Solve the following quadratic equations: **4**

(i) $x^2 - x - 12 = 0$.

(ii) $2x^2 + 4x - 5 = 0$ (Leave answer in simplest surd form.)

(d) What is the domain of the function $y = \sqrt{2 - x}$? **1**

(e) State whether the following functions are ODD, EVEN, or NEITHER. **2**

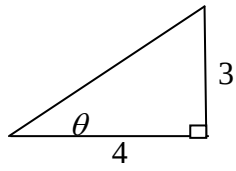
(i) $f(x) = x^2 + 12$

(ii) $f(x) = \dfrac{-x}{x^2 + 1}$

(f) Write down the value of: **2**

(i) $\cot\theta$

(ii) $\sin\theta$



Question 1 (Continued)

(g) If α and β are roots of $x^2 - 6x - 4 = 0$, find the values of: **5**

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$

(iv) $\alpha^2 + \beta^2$

Question 2. (16 Marks)

(a) What is the slope of the line $x+3y-5=0$? **1**

(b) Sketch the following showing essential features: **4**

(i) $y=x+2$ (iii) $y=2^x$

(ii) $xy=2$ (iv) $y=|x-2|$

(c) For the parabola $x^2=8(y-1)$, write down the: **4**

(i) co-ordinates of the vertex

(ii) focal length

(iii) co-ordinates of the focus

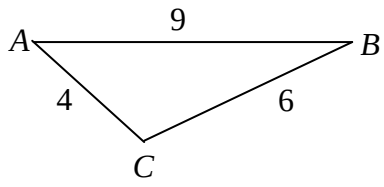
(iv) equation of the directrix.

(d) Find the equations of the following lines (giving your answer in general form). **4**

(i) through $(4,6)$ with slope -2.

(ii) perpendicular to $x-2y+5=0$ and passing through $(3,6)$.

(e) Find the size of the largest angle in the triangle (to the nearest minute). **3**



Section B
[Start a new booklet]

Question 3. (15 Marks)

(a) Find the value of θ (to the nearest degree) if **2**

$\cos\theta = -0.7$ and $180^\circ \le \theta \le 360^\circ$.

(b) Show that $(\sin\phi + \cos\phi)^2 + (\sin\phi - \cos\phi)^2 = 2$. **2**

(c) Differentiate $f(x) = x^2 - 6x + 1$ by first principles. **2**

(d) For the parabola $y = x^2 - 6x + 1$, find: **3**

(i) the equation of the axis

(ii) the range of values of y.

(e) Find the perpendicular distance of the point $(4,1)$ from the line $3x - 4y + 12 = 0$. **2**

(f) Sketch the regions of the number plane defined by: **4**

(i) $x^2 + y^2 \le 16$

(ii) $y > x^2 + 9$

Question 4 (16 Marks)

(a) Differentiate the following: **6**

(i) $y = 3 - x + x^2$

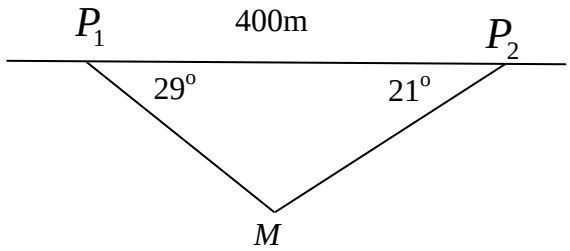
(ii) $y = \dfrac{\sqrt{x}}{\sqrt{\ }}$

(iii) $y = \dfrac{x+1}{x-2}$

(iv) $y = \left(2 - 3x^2\right)^5$

(b) Find the centre and radius of the circle $x^2 - 4x + y^2 + 6y = 0$. **2**

(c) **4**



P_1 and P_2 are points 400m apart on horizontal ground. Mine shafts are driven from P_1 and P_2 to meet underground at M.

Find to the nearest metre

(i) the length of the shaft P_1M.

(ii) the vertical depth of M below the surface.

(d) For what values of k does $x^2 - (k+5)x + 9 = 0$ have **4**

(i) equal roots.

(ii) no roots.

This is the end of the paper.

'2011 2U YEARLY:

QUESTION ONE

A) i) $\sin 45^\circ = \frac{1}{\sqrt{2}}$ 1

ii) $\cos \frac{7\pi}{6} = \cos 210^\circ = +\cos(180+30) = -\cos 30 = \frac{-\sqrt{3}}{2}$ 1

b) $(4, a)$ $4x - 3y - 7 = 0$

SUB $x=4$ $y=a$ $4(4) - 3a - 7 = 0$

$-3a + 9 = 0$

$a = 3$ — (1)

c) i) $x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4$ or $x = -3$ — (2)

ii) $2x^2 - 4x - 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

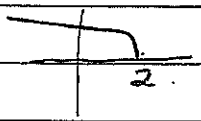
$$= \frac{4 \pm \sqrt{16 - 4(2)(-5)}}{4}$$

$$= \frac{4 \pm \sqrt{56}}{4}$$

$$= \frac{4 \pm 2\sqrt{14}}{4}$$

$$= \frac{2 \pm \sqrt{14}}{2} \checkmark$$ — (2)

d) $y = \sqrt{2 - x}$



— (1)

domain $x \in \mathbb{R} : x \leq 2$

e) i) $f(x) = x^2 - 12$ — (1)

$f(-x) = (-x)^2 - 12$

$= x^2 - 12$

Since $f(x) = f(-x)$

fn is even

ii) $f(x) = \frac{-x}{x^2+1}$ — (1)

$f(-x) = \frac{-(-x)}{(-x)^2+1} = \frac{x}{x^2+1}$

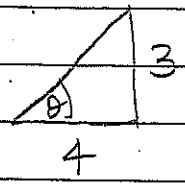
Since $f(x) = -f(x)$

fn is odd

2011 2U YEARLY

Q1

f):



i) $\cot\theta = \frac{adj}{opp} = \frac{4}{3}$ (1)

$3^2 + 4^2 = 5^2$

ii) $\sin\theta = \frac{opp}{hyp} = \frac{3}{5}$ (1)

g) $x^2 - 6x - 4 = 0$

i) $\alpha + \beta = \frac{-b}{a} = +6$ (1)

ii) $\alpha\beta = \frac{c}{a} = -4$ (1)

iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{6}{-4} = \frac{-3}{2}$ (1)

iv) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ (2)

$= (+6)^2 - 2(-4)$

$= 36 + 8$

$= 44$

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QUESTION TWO

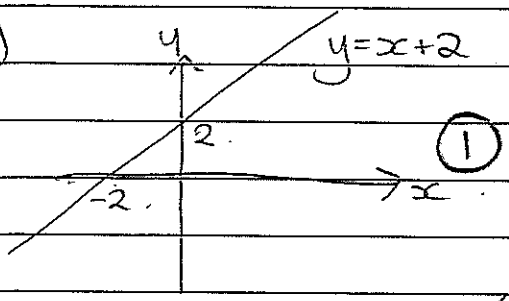
a) $x + 3y - 5 = 0$

$3y = -x + 5$

$y = -\frac{1}{3}x + \frac{5}{3}$

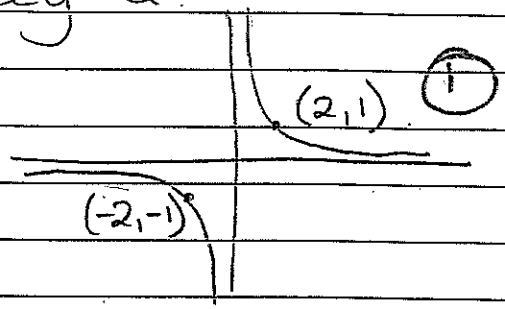
gradient $= -1/3$ (1)

b) i)



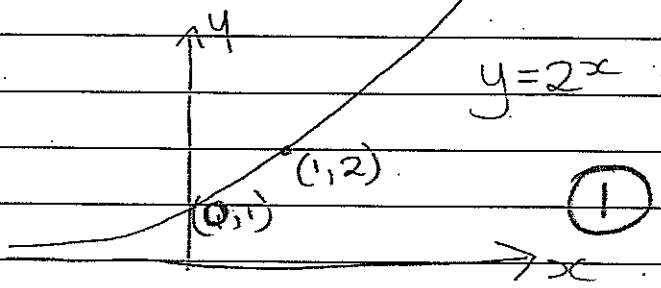
(1)

ii) $xy = 2$



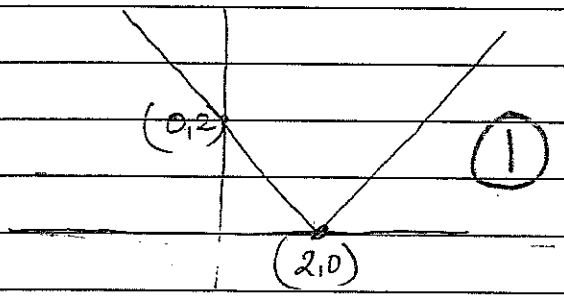
(1)

iii)



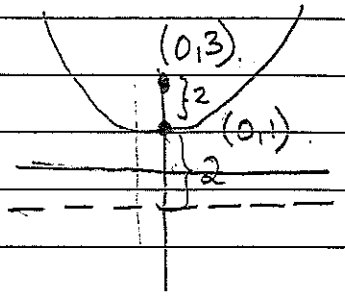
(1)

iv) $y = |x - 2|$



(1)

c) i) $x^2 = 8(y-1)$ (1)



vertex $(0, 1)$

ii) focal length $a = 2$ (1)

iii) focus $(0, 3)$ (1)

iv) $y = -1$ (1)

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QUESTION TWO.

d) i) $y - y_1 = m(x - x_1)$ $(4, 6)$ $m = -2$

$y - 6 = -2(x - 4)$

$y - 6 = -2x + 8$

$2x + y - 14 = 0$ ——— (2)

ii) $x - 2y + 5 = 0$

$-2y = -x - 5$

$y = \frac{1}{2}x + \frac{5}{2}$

grad $= \frac{1}{2}$ grad of perp $= -2$ $-2 \times \frac{1}{2} = -1$ ✓

$(3, 6)$

$y - 6 = -2(x - 3)$

$y - 6 = -2x + 6$

$2x + y - 12 = 0$ (2)

e) $\cos C = \frac{a^2 + b^2 - c^2}{2ab} =$

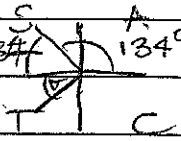
$= \frac{6^2 + 4^2 - 9^2}{2(6)(4)}$

$= 0.604$ (3dp)

$C = 127°10'$ ——— (3)

2011 2U YEARLY:
QUESTION THREE

a) $\cos\theta = -0.7$



$\theta = 134°25'$

$\theta = 226°$ —(2)

b) $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$ prove.

$LHS = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + (\sin^2\theta - 2\cos\theta\sin\theta + \cos^2\theta$

$= (\sin^2\theta + \cos^2\theta) + (\sin^2\theta + \cos^2\theta)$

$= 1 + 1$

$= 2$

$= RHS$ —(2)

b) $f(x) = x^2 - 6x + 1$

$f(x+h) = (x+h)^2 - 6(x+h) + 1$

$= x^2 + 2xh + h^2 - 6x - 6h + 1$

$= $

$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$

$= \lim_{h\to 0}\left[\frac{(x^2 + 2xh + h^2 - 6x - 6h + 1) - (x^2 - 6x + 1)}{h}\right]$

$= \lim_{h\to 0}\left(\frac{2xh + h^2 - 6h}{h}\right)$

$= \lim_{h\to 0}(2x + h - 6)$

$= 2x - 6$ —(2)

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QUESTION THREE

d) $y = x^2 - 6x + 1$

i) $x = \frac{-b}{2a} = \frac{6}{2} = 3$ — (1)

ii) y value when $x = 3$

$y = (3)^2 - 6(3) + 1$

$y = 9 - (18) + 1$

$= -9 + 1$

$= -8$

Range $y \in \mathbb{R}$: $y \geq -8$ — (2)

e) $(4, 1)$ $\quad 3x - 4y + 12 = 0$ $\quad$ $a \quad b \quad c$

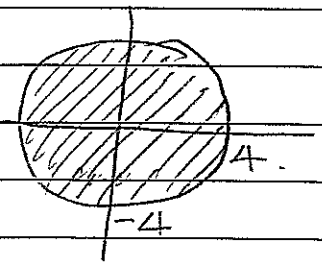
$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$d = \frac{3(4) - 4(1) + 12}{\sqrt{3^2 + (-4)^2}}$

$= \frac{12 - 4 + 12}{5}$

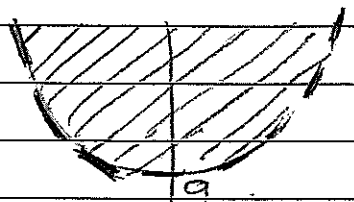
$= 4$ units — (2)

f) i) $x^2 + y^2 \leq 16$



(2)

ii) $y > x^2 + 9$



(2)

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QUESTION FOUR

a) i) $y = 3 - x + x^2$

$dy/dx = -1 + 2x$ —— (1)

ii) $y = \sqrt{x} = x^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}}$ —— (1)

iii) $y = \frac{x+1}{x-2}$ $u = x+1$ $u' = 1$ $v = x-2$ $v' = 1$

$\frac{dy}{dx} = \frac{vu' - uv'}{v^2} = \frac{(x-2)(1) - (x+1)(1)}{(x-2)^2}$

$= \frac{x - 2 - x - 1}{(x-2)^2}$

$= \frac{-3}{(x-2)^2}$ —— (2)

iv) $y = (2-3x^2)^5$

$\frac{dy}{dx} = 5(2-3x^2)^4 \times -6x$

$= -30x(2-3x^2)^4$ —— (2)

b) $x^2 - 4x + y^2 + 6y = 0$

$x^2 - 4x + 4 + y^2 + 6y + 9 = 13$

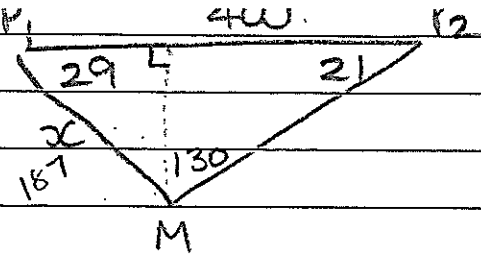
$(x-2)^2 + (y+3)^2 = 13$

Centre $= (2, -3)$ radius $\sqrt{13}$ —— (2)

2011 2U YEARLY

Q4

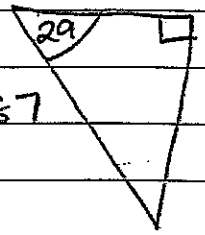
c)



P, M ⇒ $\frac{x}{\sin 21} = \frac{400}{\sin 130}$

$x = \frac{400(\sin 21)}{\sin 130}$ — (2)

$= 187$ m (nearest m)



$\sin 29 = \frac{x}{187}$

$x = 187 \sin 29$

$x = 90.72$

$= 91$ m — (2)

d) $x^2 - (k+5)x + 9 = 0$

Equal roots $b^2 - 4ac = 0$

$(-(k+5))^2 - 4(1)(9) = 0$

$k^2 + 10k + 25 - 36 = 0$

$k^2 + 10k - 11 = 0$

$(k+11)(k-1) = 0$

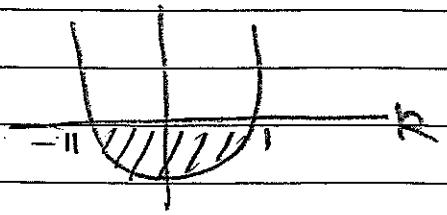
$k = -11$ or $+1$

No roots:

$b^2 - 4ac < 0$

$(-(k+5))^2 - 4(1)(9) < 0$

$k^2 + 10k - 11 < 0$



$-11 < k < 1$