



**SYDNEY BOYS HIGH  
SCHOOL**  
MOORE PARK, SURRY HILLS

**2012**  
**YEAR 11 Mathematics**  
**Yearly**

# Mathematics

## General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Each Section is to be returned in a separate bundle.
- Marks may **NOT** be awarded for messy or badly arranged work.
- All necessary working should be shown in every question.
- Answer must be given in simplest exact form.

## Total Marks – 70

- Attempt questions 1-15

Examiner: *P. Bigelow*

**Section I (10 marks)**

**Answer this section on the Multiple Choice Answer Sheet**

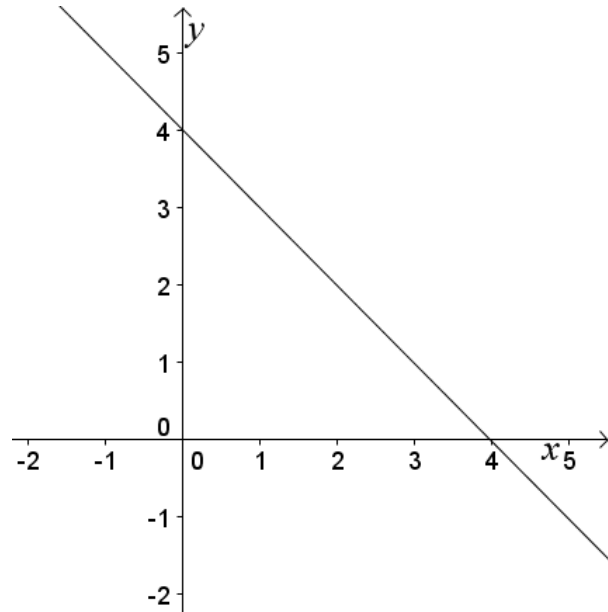
(1) The equation of the line is:

(A)  $x - y + 4 = 0$

(B)  $x + y - 4 = 0$

(C)  $y = -x - 4$

(D)  $x - y - 4 = 0$



(2)  $x^2 - 4x + 6$  has a minimum value of:

(A) 4

(B) 2

(C) 6

(D) 8

(3)  $3^x \times 2^x$  is equal to:

(A)  $6^x$

(B)  $6^{2x}$

(C)  $5^x$

(D)  $5^{2x}$

(4) The full solution to  $x^2 = 2x$  is:

(A)  $x = 2$

(B)  $x = -2, 0$

(C)  $x = 0, 2$

(D)  $x = -2$

(5) The solution to  $\log_5 x = 4$  is:

(A) 625

(B) 25

(C) 125

(D) 20

(6) If  $g(x) = 6 - x - x^2$  then  $g(-2)$  is:

(A) 6

(B) 4

(C) 0

(D) 12

(7) If  $25^{2-x} = 125$  then  $x$  is equal to:

(A) 1

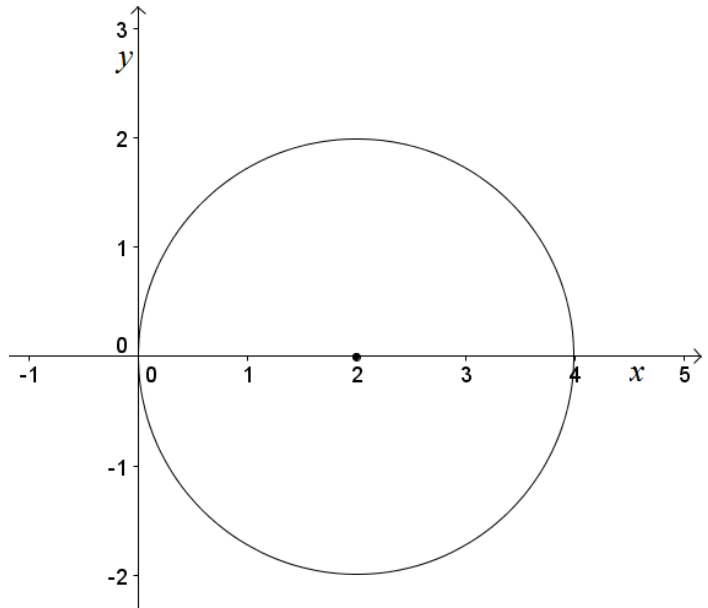
(B)  $\frac{1}{2}$

(C) -1

(D) 5

(8) The equation of the circle is:

- (A)  $x^2 + (y - 2)^2 = 4$
- (B)  $(x - 2)^2 + y^2 = 16$
- (C)  $(x + 2)^2 + y^2 = 16$
- (D)  $(x - 2)^2 + y^2 = 4$

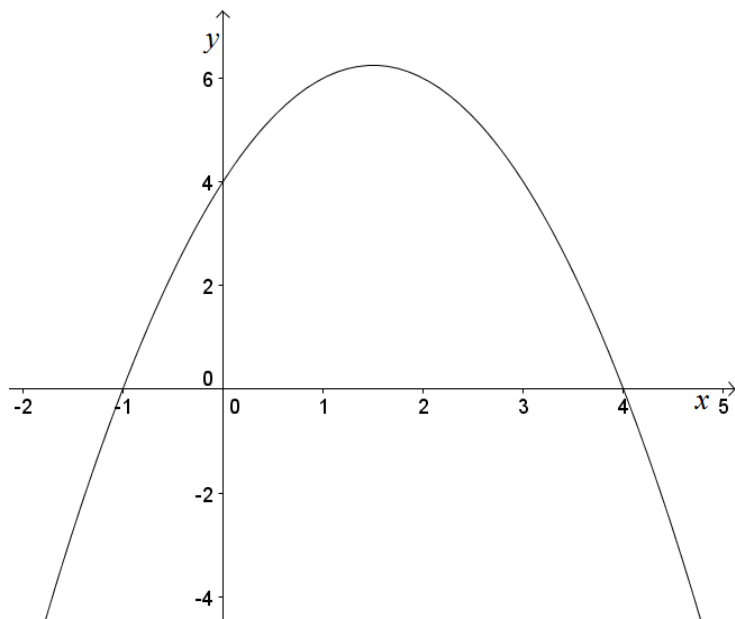


(9) The domain for the function  $f(x) = \sqrt{2 - x}$  is:

- (A)  $x \geq 0$
- (B)  $x \geq 2$
- (C)  $x \leq 2$
- (D) All reals

(10) The graph best represents

- (A)  $y = (x + 1)(x - 4)$
- (B)  $y = (x - 1)(x + 4)$
- (C)  $y = (1 - x)(x - 4)$
- (D)  $y = (x + 1)(4 - x)$



## Section II (60 marks)

Answer this section in the Writing Booklets

Question 11 [11 marks]

(a) Sketch the following on separate number plane graphs [3]

(i)  $x^2 + y^2 = 25$

(ii)  $y = 4^x$

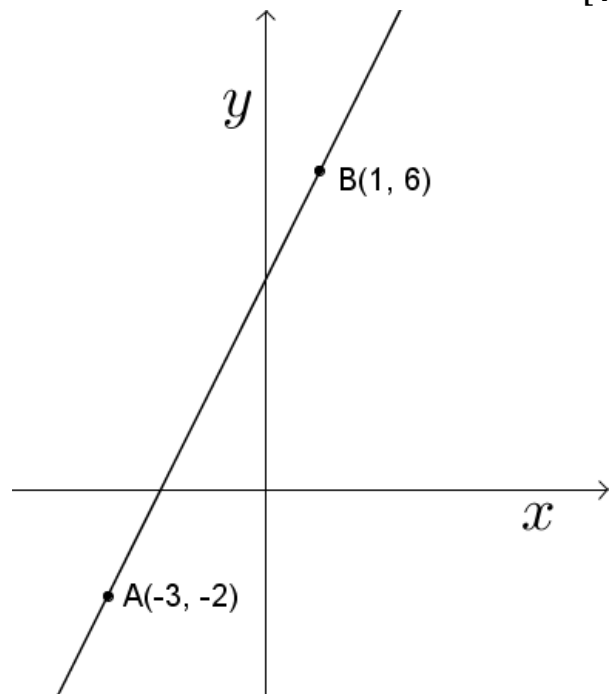
(iii)  $xy \leq 3$

(b) Find [4]

(i) Mid-point of AB.

(ii) The gradient of AB.

(iii) The equation of AB.



(c) Find the gradient of the line  $2x + 3y - 11 = 0$  [1]

(d) The point  $(-3, c)$  lies on  $x + 4y - 9 = 0$ , find the value of  $c$ . [1]

(e) Find the perpendicular distance of the line  $3x - 4y - 10 = 0$  from the origin. [2]

Question 12 [12 marks]

(a) Find the value of  $x$  in the following:

[5]

(i)  $\log_x 36 = 2$

(ii)  $x = \log_3 \frac{1}{9}$

(iii)  $\log_4 x = 3$

(iv)  $2^{2x} = 16^{x-8}$

(b) Sketch  $y = \log_2 x$  for  $0 < x \leq 8$ .

[2]

(c) If  $f(x) = x^2 - x$ , find the values of  $a$  such that  $f(a) = f(2)$ .

[2]

(d) State whether the following functions are even (E), odd (O) or neither (N).

[3]

(i)  $f(x) = 2 - x^2$

(ii)  $f(x) = \frac{x}{1+x}$

(iii)  $f(x) = x\sqrt{x^2 + 1}$

Question 13 [12 marks]

(a) Find the solutions to the following quadratic equations (leave in simplest exact form).

(i)  $x^2 + 6x - 4 = 0$

(ii)  $6x^2 - 17x = 14$

[4]

(b) Find the centre and radius of the circle  $x^2 - 6x + y^2 + 8y = 0$ .

[2]

(c) Find

(i) Focus

(ii) Directrix

(iii) Vertex

[6]

for the following

( $\alpha$ )  $x^2 = 12y$

( $\beta$ )  $x^2 - 4x + 8y = 0$

Question 14 [11 marks]

(a) Find A, B and C such that

[3]

$$A(x + 1)^2 + B(x + 1) + C \equiv 3x^2 + 4x + 5$$

(b) Solve the following simultaneous equations

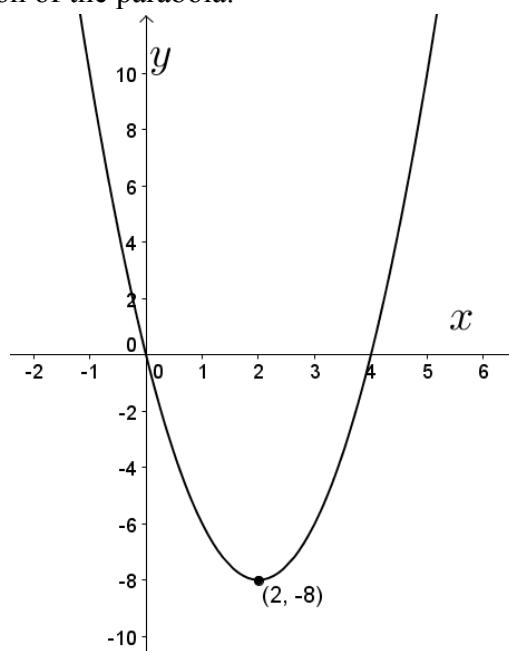
[3]

$$5^{x+y} = \frac{1}{5}$$

$$5^{3x+2y} = 1$$

(c) Find the equation of the parabola.

[2]



(d) Find the equation of the line perpendicular to  $2x + y - 5 = 0$  and passing through  $(5, 1)$ .

[3]



Question 15 [14 marks]

(a) If the roots of  $x^2 - 2x - 4 = 0$  are  $\alpha$  and  $\beta$  find the value of: [5]

(i)  $\alpha + \beta$

(ii)  $\alpha\beta$

(iii)  $\frac{1}{\alpha} + \frac{1}{\beta}$

(iv)  $\alpha^2 + \beta^2$

(b) For what values of  $k$  does  $x^2 - (k + 5)x + 9 = 0$  have [3]

(i) Equal roots

(ii) No real roots

(c) Form a quadratic equation with roots  $2 + \sqrt{3}$  and  $2 - \sqrt{3}$ . [2]

(d) The cost of producing  $y$  items per day is  $\$ \left( \frac{1}{3}y^2 + 45y + 27 \right)$ . The price at which each item may be sold is  $\$ \left( 60 - \frac{1}{2}y \right)$ . [4]

(i) Find an expression for the daily profit.

(ii) Find the maximum value of the profit.

**End of Exam**



Student Number: ANSWERS

## Mathematics

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:  $2 + 4 =$  (A) 2 (B) 6 (C) 8 (D) 9  
A  B  C  D

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A  B  C  D

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

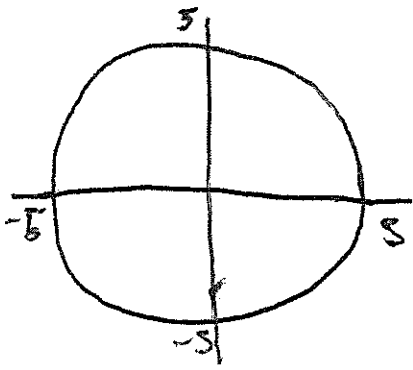
A  B  C  D   
*correct* (arrow pointing to B)

### Section I: Multiple choice answer sheet.

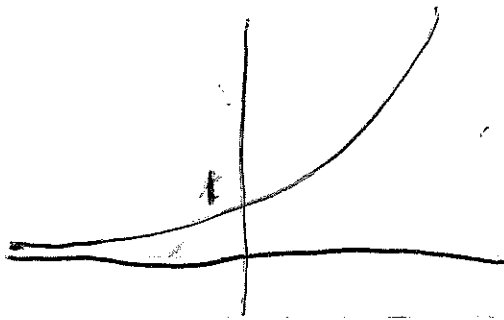
Completely colour the cell representing your answer. Use black pen.

1. A  B  C  D
2. A  B  C  D
3. A  B  C  D
4. A  B  C  D
5. A  B  C  D
6. A  B  C  D
7. A  B  C  D
8. A  B  C  D
9. A  B  C  D
10. A  B  C  D

Q11 (a)  
(i)

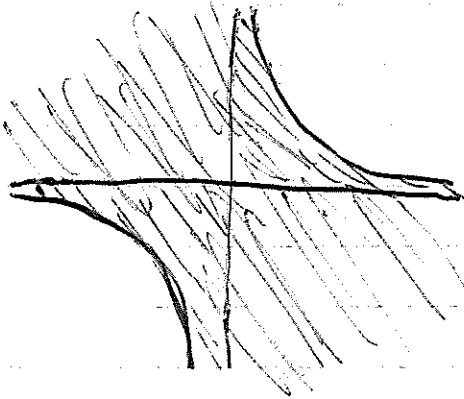


(ii)



3

(iii)



(b) (i) Mid Point  $AB = \left( \frac{1+3}{2}, \frac{6+2}{2} \right)$   
 $= (-1, 2)$

(ii)  $m_{AB} = \frac{6+2}{1+3}$

4

$= 2.$

(iii)  $y+2 = 2(x+3)$

$y = 2x + 4.$

(c)  $2x + 3y - 11 = 0$

$$y = -\frac{2}{3}x + \frac{11}{3}$$

$$m = -\frac{2}{3}$$

(d)  $-3 + 4c - 9 = 0$

$$4c = 12$$

$$c = 3$$

(e)

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|10|}{\sqrt{9+16}}$$

$$= 2 \text{ units}$$

Q12 (a) (i)  $x = 6$

(ii)  $x = -2$

(iii)  $x \cdot 4^3 = 64$   
 $4x = 32$

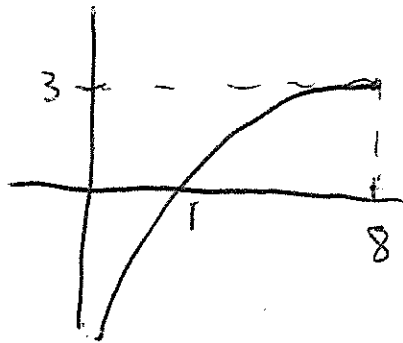
(iv)  $2^{2x} = 2$

$$2x = 4x - 32$$

$$2x = 32$$

$$x = 16$$

(b)



2

(c)  $a^2 - a = 4 - 2$

$$a^2 - a - 2 = 0.$$

~~$a = \frac{1 \pm \sqrt{1+8}}{2}$~~

2

$$(a-2)(a+1) = 0$$

$$a = 2, -1.$$

- (d) (i) Even  
(ii) Neither  
(iii) Odd.

3

Q/3 (a) (i)  $x^2 + 6x - 4 = 0.$

$$x = \frac{-6 \pm \sqrt{36 + 16}}{2} = \frac{-6 \pm \sqrt{52}}{2}.$$

~~$= \frac{-6 \pm 2\sqrt{13}}{2} = -2$~~

~~$= -3 \pm \sqrt{13}.$~~

(ii)  $6x^2 - 17x - 14 = 0$ .

$$x = \frac{17 \pm \sqrt{17^2 - 4 \times 6 \times -14}}{12}$$

$$= \frac{17 \pm 25}{12}$$

$$= \frac{7}{2}, -\frac{2}{3} \quad 2$$

(b)  $(x-3)^2 - 9 + (y+4)^2 - 16 = 0$

$$(x-3)^2 + (y+4)^2 = 25$$

centre  $(3, -4)$  2  
radius 5.

(c) (a) (i)  $(0, 3)$

(ii)  $y = -3$  3

(iii)  $(0, 0)$

(b)  $(x-2)^2 = -8y + 4$

$$(x-2)^2 = -4 \times 2 (y - \frac{1}{2})$$

(i)  $(2, -\frac{1}{2})$

(ii)  $y = \frac{1}{2}$  3

(iii)  $(2, \frac{1}{2})$

Q14  
(a)

$$A(x+1)^2 + B(x+1) + C \equiv 3x^2 + 4x + 5.$$

when  $x = -1$ .

$$C = 3 - 4 + 5 \\ = 4$$

$x^2$   $A = 3.$

constant.

$$A + B + C = 5$$

$$3 + B + 4 = 5.$$

$$B = -2.$$

$$A = 3, B = -2, C = 4.$$

3

(b)  $x + y = -1$

$$3x + 2y = 0. \quad \textcircled{A}$$

$$3x + 3y = -3. \quad \textcircled{B}$$

$$\textcircled{B} - \textcircled{A}$$

$$y = -3.$$

sub into  
1st.

$$x = 2.$$

3

$$(c) \quad y = a(x-4)x. \quad (2, -8)$$

$$-8 = a(-2)2.$$

$$a = 2.$$

$$y = 2x(x-4)$$

$$y = 2x^2 - 8x. \quad 2$$

$$(d) \quad y = -2x + 5$$

$$m_1 = -2$$

$$m_1 \times m_2 = -1$$

$$m_2 = \frac{1}{2}. \quad (5, 1)$$

$$y - 1 = \frac{1}{2}(x - 5).$$

$$2y - 2 = x - 5$$

$$x - 2y - 3 = 0$$

$$Q15 (a) \quad \alpha + \beta = 2$$

$$(ii) \quad \alpha\beta = -4$$

$$(iii) \quad \frac{d\alpha\beta}{d\alpha\beta} = \frac{2}{-4} = -\frac{1}{2} \quad 3$$

$$(iv) \quad (\alpha + \beta)^2 - 2\alpha\beta = 4 + 8 \\ = 12 \quad 2$$



$$(b) \quad x^2 - (k+5)x + 9 = 0$$

$$(i) \quad \Delta = 0$$

$$\Delta = (k+5)^2 - 4 \times 9$$

$$= k^2 + 10k + 25 - 36$$

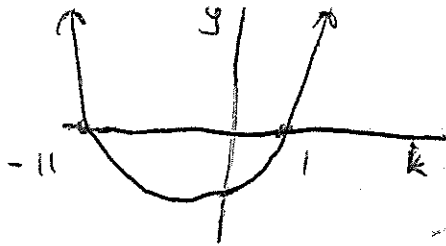
$$= k^2 + 10k - 11$$

$$k^2 + 10k - 11 = 0.$$

$$(k+11)(k-1) = 0.$$

$$k = -11, 1.$$

$$(ii) \quad \Delta < 0.$$



$$-11 < k < 1$$

$$(c) \quad (x - 2 - \sqrt{3})(x - 2 + \sqrt{3}) = 0.$$

$$(x - (2 + \sqrt{3}))(x - (2 - \sqrt{3})) = 0.$$

$$x^2 - 4x + (2 + \sqrt{3})(2 - \sqrt{3}) = 0$$

$$x^2 - 4x + 4 - 3 = 0$$

$$x^2 - 4x + 1 = 0.$$

$$(d) (i) P = (60 - \frac{1}{2}y) - (\frac{1}{3}y^2 + 45y + 27)$$

$$Profit = 33 - \frac{91}{2}y - \frac{1}{3}y^2.$$

$$(ii) y = \frac{\frac{91}{2}}{-\frac{2}{3}}$$

4

$$= -\frac{273}{4}.$$

$$\text{Max Profit} = \$1585.69.$$