SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2013

Year 11 Mathematics Yearly

Mathematics

General Instruction

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Start each **NEW** question in a separate answer booklet.
- Marks may **NOT** be awarded for messy or badly arranged work.
- All necessary working should be shown in every question.
- Answer in simplest exact form unless otherwise instructed.

Total Marks – 70

- **Attempt Questions 1 – 6**

Examiner: *R. Elliott*

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x,\ x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax,$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a},\ \ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$

START A NEW ANSWER BOOKLET

QUESTION 1 (10 marks)

(a) Simplify $4x^3y^{-2} \div 2xy^{-3}$. **1**

(b) What is the gradient of $2x + 3y = 5$? **1**

(c) If $(2, a)$ is on the line $2x + 3y = 5$ find the value of a. **1**

(d)

(i) Find where the curve $y = x^2 - x - 6$ cuts the x-axis. **2**

(ii) What is the minimum value of the same function? **2**

(e) Simplify $3^x \times 5^x$. **1**

(f) Solve $\frac{x}{2} - \frac{x-1}{3} = 5$. **2**

End of Question 1

START A NEW ANSWER BOOKLET

QUESTION 2 (11 marks)

(a) On separate graphs sketch the following. **4**

(i) $y = 2 - x^2$

(ii) $xy - 3 = 0$

(iii) $y = |x + 1|$

(iv) $y = 3^{-x}$

(b) For the parabola $x^2 = 4y - 8$ write down the coordinates of **4**

(i) the focus

(ii) the vertex

(iii) the equation of the directrix

(iv) the equation of the line through the focus parallel to the directrix.

(c) Solve the following quadratic equations **3**

(i) $2x^2 - 5x - 3 = 0$

(ii) $2x^2 - 4x - 1 = 0$ (leave answer as a surd)

End of Question 2

START A NEW ANSWER BOOKLET

QUESTION 3 (12 marks)

(a) Given α and β are the roots of $2x^2 + 3x - 4 = 0$ find the values of **4**

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\alpha^{-1} + \beta^{-1}$

(iv) $\alpha^3 + \beta^3$

(b) A triangle has sides of length $7, 5$ and 4 cm.

(i) Find the smallest angle. **2**

(ii) Find its area. **2**

(c) Solve each of the following equation and graph the solutions on a number line.

(i) $|x + 2| = 7$ **2**

(ii) $3x + 15 \leq 0$ **2**

End of Question 3

START A NEW ANSWER BOOKLET

QUESTION 4 (13 marks)

(a) Find derivatives of the following **5**

(i) $x^2 - \frac{2}{x} + 5$

(ii) $x(x^2 - 3)^3$

(iii) $\frac{x}{\sqrt{x}-1}$

(b) Find the equation of the tangent drawn to the curve $y = \sqrt{x} + 1$ at the point where $x = 4$. **2**

(c) Determine whether any of the following are odd, even or neither functions, justifying your answer. **6**

(i) $f(x) = x\sqrt{x^2 - 1}$

(ii) $f(x) = \frac{4}{x^2 - x}$

(iii) $f(x)(x + 2) = 1$

End of Question 4

START A NEW ANSWER BOOKLET

QUESTION 5 (11 marks)

(a) Graph on the number plane the region that is solved simultaneously by **3**

$$y - 1 > |x| \qquad \text{and} \qquad y \leq x^2 + 2$$

(b) If $\log a = 1.47$ and $\log b = 0.86$ find correct to 2 decimal places **2**

(i) $\log\left(\frac{a}{b}\right)$

(ii) $\log\left(\sqrt{ab}\right)$

(c) Solve for x **3**

$$\log_3\left(\frac{x^2}{x-2}\right) = 2$$

(d) Factorise fully **3**

(i) $2x^2 - 4x - 30$

(ii) $x^3 - y^3 - x + y$

End of Question 5

START A NEW ANSWER BOOKLET

QUESTION 6 (13 marks)

(a)

(i) Find the 37^{th} term of the series **2**

$$19 + 15 + 11 + \cdots$$

(ii) Find the sum of these 37 terms. **2**

(b) Find the 19^{th} term of the series $96 - 48 + 24 - \cdots$ (leave answer in simplified index form) **2**

(c) Express $0.\dot{1}\dot{7}$ as a geometric series and hence write it as a fraction. **3**

(d) A boy starts work in a Hamburger shop and his father tells him to save \$10 out of his first week's salary. He then tells him to save an additional 50 cents each week from then on. In other words he saves \$10.50 in the second and \$11 in the third and so on. How much does he save in a year. **4**

End of Question 6
End of Exam

Y11 Mathematics Yearly - Solutions (2013)

Question 1 (10 marks)

(a) $\frac{4x^3}{y^2} \div \frac{2x}{y^3} = \frac{4x^3}{y^2} \times \frac{y^3}{2x}$

$\boxed{= 2x^2y}$ (1)

(b) $2x + 3y = 5$

$\Rightarrow (y = mx + b)$

$3y = 5 - 2x$

$y = \frac{5}{3} - \frac{2x}{3}$

$\boxed{\therefore m = -\frac{2}{3}}$ (1)

(c) $2x + 3y = 5$, $(2, a)$

$2(2) + 3a = 5$

$4 + 3a = 5$

$3a = 1$

$\boxed{a = \frac{1}{3}}$ (1)

(d)

(i) x-intercepts $(y = 0)$

$\therefore x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x - 3 = 0$ or $x + 2 = 0$

$\therefore \boxed{x = 3}$ $\boxed{(3, 0)}$ and $\boxed{x = -2}$ $\boxed{(-2, 0)}$ (1) (1)

(ii) $a > 0$ $\therefore$ minimum value at vertex.

$x = \frac{-b}{2a} \Rightarrow x = \frac{-(-1)}{2(1)}$

$\boxed{x = \frac{1}{2}}$ (1)

$y = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 6$

$= -6\frac{1}{4}$

$\boxed{\therefore \left(\frac{1}{2}, -6\frac{1}{4}\right)}$ (1)

(e) $3^x \times 5^x = \boxed{15^x}$ (1)

(f) $\frac{x}{2} - \frac{x-1}{3} = 5$

$3x - 2(x - 1) = 30$ (1)

$3x - 2x + 2 = 30$

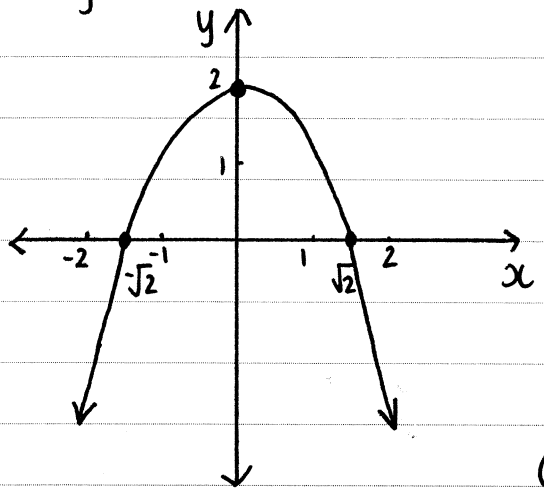
$x + 2 = 30$

$\boxed{x = 28}$ (1)

Question 2 (11 marks)

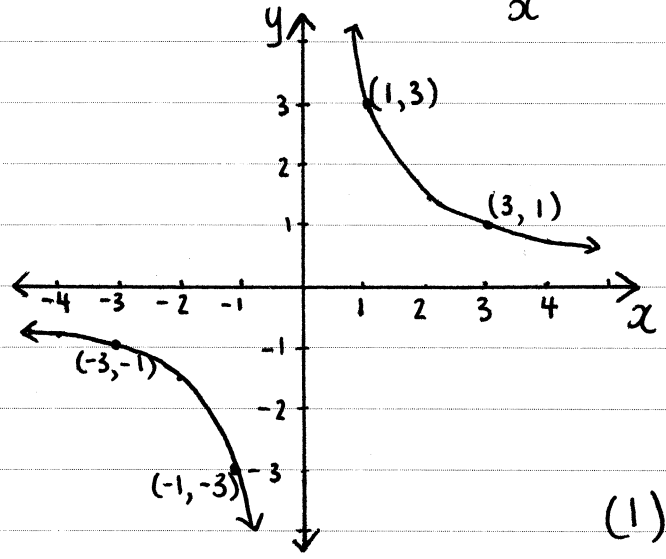
(a)

(i) $y = 2 - x^2$



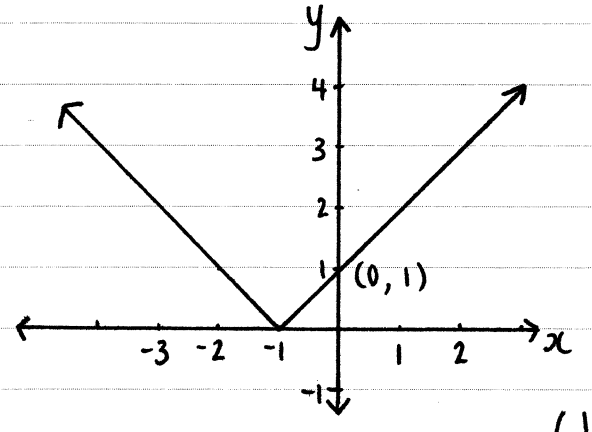
(1)

(ii) $xy - 3 = 0 \Rightarrow y = \frac{3}{x} \quad x \neq 0$



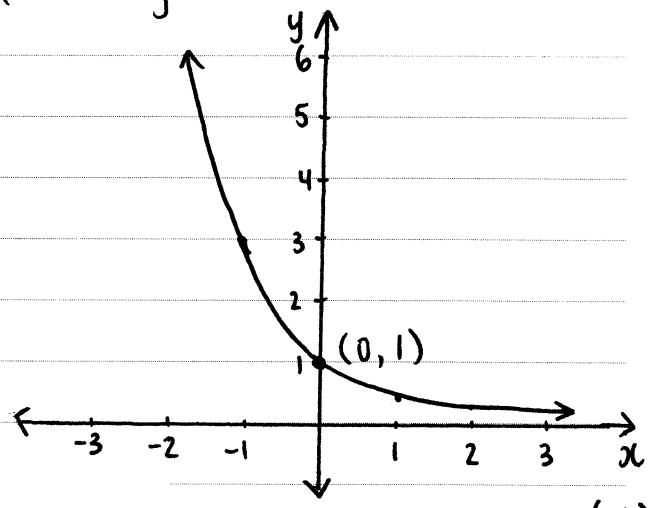
(1)

(iii) $y = |x + 1|$



(1)

(iv) $y = 3^{-x}$



(1)

(b) $x^2 = 4y - 8$

$x^2 = 4(y - 2)$

$x^2 = 4(1)(y - 2)$

(i) focus $\Rightarrow$

focal length $a = 1$

$\therefore$ Focus at $(0, 2+1)$

Focus $(0, 3)$

(1)

(ii) vertex $(0, 2)$

(1)

(iii) directrix with equation

$y = 2 - 1$

$\therefore y = 1$

(1)

(iv) $y = 3$

(1)

(c)

(i) $2x^2 - 5x - 3 = 0$

P: −6 } −6, +1
S: −5 }

$$\frac{(2x-6)(2x+1)}{2} = 0$$

$$\frac{\cancel{2}(x-3)(2x+1)}{\cancel{2}} = 0$$

$x - 3 = 0$ or $2x + 1 = 0$

$\boxed{x = 3}$ $\quad 2x = -1$

$\boxed{x = -\frac{1}{2}}$

(1)

(ii) $2x^2 - 4x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{4 \pm \sqrt{24}}{4}$$

$$= \frac{4 \pm 2\sqrt{6}}{4}$$ (1)

$$= \frac{\cancel{2}(2 \pm \sqrt{6})}{\cancel{4}\,2}$$

$\therefore \boxed{x = \frac{2+\sqrt{6}}{2}}$ or $\boxed{x = \frac{2-\sqrt{6}}{2}}$

(1)

Question 3 (12 marks)

(a) $2x^2 + 3x - 4 = 0$

(i) $\alpha + \beta = -\frac{b}{a}$

$= -\frac{3}{2}$ (1)

(ii) $\alpha\beta = \frac{c}{a}$

$= -\frac{4}{2}$

$= -2$ (1)

(iii) $\alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta}$

$= \frac{\beta + \alpha}{\alpha\beta}$

$= \frac{-3}{\frac{2}{-2}}$

$= \frac{3}{4}$ (1)

(iv) $\alpha^3 + \beta^3$

$= (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$\downarrow$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\therefore = (\alpha + \beta)[(\alpha + \beta)^2 - 2\alpha\beta - \alpha\beta]$

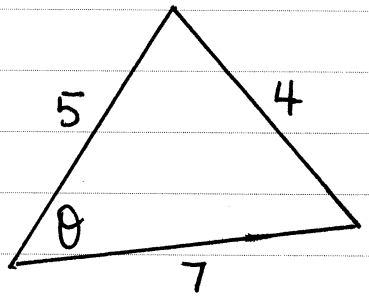
$= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$

$= \left(\frac{-3}{2}\right)\left[\left(\frac{-3}{2}\right)^2 - 3(-2)\right]$

$= \left(\frac{-3}{2}\right)\left(\frac{33}{4}\right)$

$= -\frac{99}{8}$ or -12.375 (1)

(b)



(i) smallest angle θ

$\cos\theta = \frac{7^2 + 5^2 - 4^2}{2(7)(5)}$ (1)

$\theta = \cos^{-1}\left(\frac{7^2 + 5^2 - 4^2}{2(7)(5)}\right)$

$= 34^\circ 2' 52''$

$\theta = 34^\circ$ (1)

(ii) Area:

$A = \frac{1}{2} \times 5 \times 7 \times \sin 34^\circ$ (1)

$= 9.78586 \text{ cm}^2$

$A = 9.8 \text{ cm}^2$ (1)

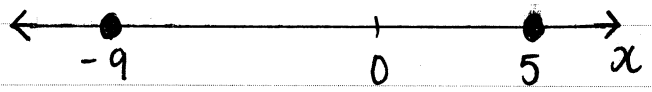
(c)

(i) $|x + 2| = 7$

$x + 2 = 7$ $\quad$ $x + 2 = -7$

$x = 5$ $\quad$ $x = -9$

(1)



(1)

(ii) $3x + 15 \leq 0$

$3x \leq -15$

$x \leq -5$ (1)

(1)

Solutions to 4R11 2U Maths Yearly (Sept) 2013

(4) (a) (i) $\frac{d}{dx}(x^2 - 2x^{-1} + 5)$

$= 2x + 2x^{-2} = 2x + \frac{2}{x^2}$ (1) //

(ii) $\frac{d}{dx}\left(\underset{u}{x}\underset{v}{(x^2-3)^3}\right)$

$= x \times 3(x^2-3)^2 \times 2x + (x^2-3)^3 \times 1$

$= 6x^2(x^2-3)^2 + (x^2-3)^3$

$= (x^2-3)^2\left[6x^2 + (x^2-3)^1\right]$

$= (x^2-3)^2[7x^2-3]$ // (2)

(iii) $\frac{d}{dx}\left(\frac{x}{x^{\frac{1}{2}}-1}\right)$ $\frac{u}{v}$

$= \dfrac{(\sqrt{x}-1) \times 1 - x \times \frac{1}{2}x^{-\frac{1}{2}}}{(\sqrt{x}-1)^2}$

$= \dfrac{\frac{\sqrt{x}}{1} - \frac{1}{1} - \frac{x}{2\sqrt{x}}}{(\sqrt{x}-1)^2} = \dfrac{\frac{2x - 2\sqrt{x} - x}{2\sqrt{x}}}{(\sqrt{x}-1)^2}$

$= \dfrac{(x - 2\sqrt{x})}{2\sqrt{x}(\sqrt{x}-1)^2} \times \dfrac{\sqrt{x}}{\sqrt{x}}$

$= \dfrac{x\sqrt{x} - 2x}{2x(\sqrt{x}-1)^2}$ ←

$\dfrac{\sqrt{x}-2}{2(\sqrt{x}-1)^2}$ // (2)

④ (b) $y = \sqrt{x} + 1$

When $x = 4$, $y = \sqrt{4} + 1 = 3$ $(4, 3)$

$y = x^{\frac{1}{2}} + 1$

$y' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

when $x = 4$, $m = \frac{1}{2 \times \sqrt{4}} = \frac{1}{4}$

using $(y - y_1) = m(x - x_1)$

$(y - 3) = \frac{1}{4}(x - 4)$

$y - 3 = \frac{x}{4} - 1$

$y = \frac{x}{4} + 2$ ② or $4y = x + 8$

$x - 4y + 8 = 0$

(c) (i) $f(x) = x.\sqrt{x^2 - 1}$

$f(-x) = (-x)\sqrt{(-x)^2 - 1} = -x\sqrt{x^2 - 1}$

not even

now for odd, $f(x) = -f(-x)$

$= -\left[-x\sqrt{x^2 - 1}\right]$

$= x.\sqrt{x^2 - 1}$

is odd ②

④ (c) (ii) $f(x) = \frac{4}{x^2 - x}$

now $f(-x) = \frac{4}{(-x)^2 - (-x)} = \frac{4}{x^2 + x}$

not even

now for odd, $f(x) = -f(-x)$

$= \frac{-4}{x^2 + x}$ not odd

(2)

So neither //

(iii) $f(x) \cdot (x+2) = 1$

$f(x) = \frac{1}{x+2}$

now $f(-x) = \frac{1}{(-x)+2} = \frac{1}{-x+2}$ not even

now for odd. $f(x) = -f(-x)$

$= \frac{-1}{-x+2}$

$= \frac{-1}{-(x-2)} = \frac{1}{x-2}$

not odd.

So neither // (2)

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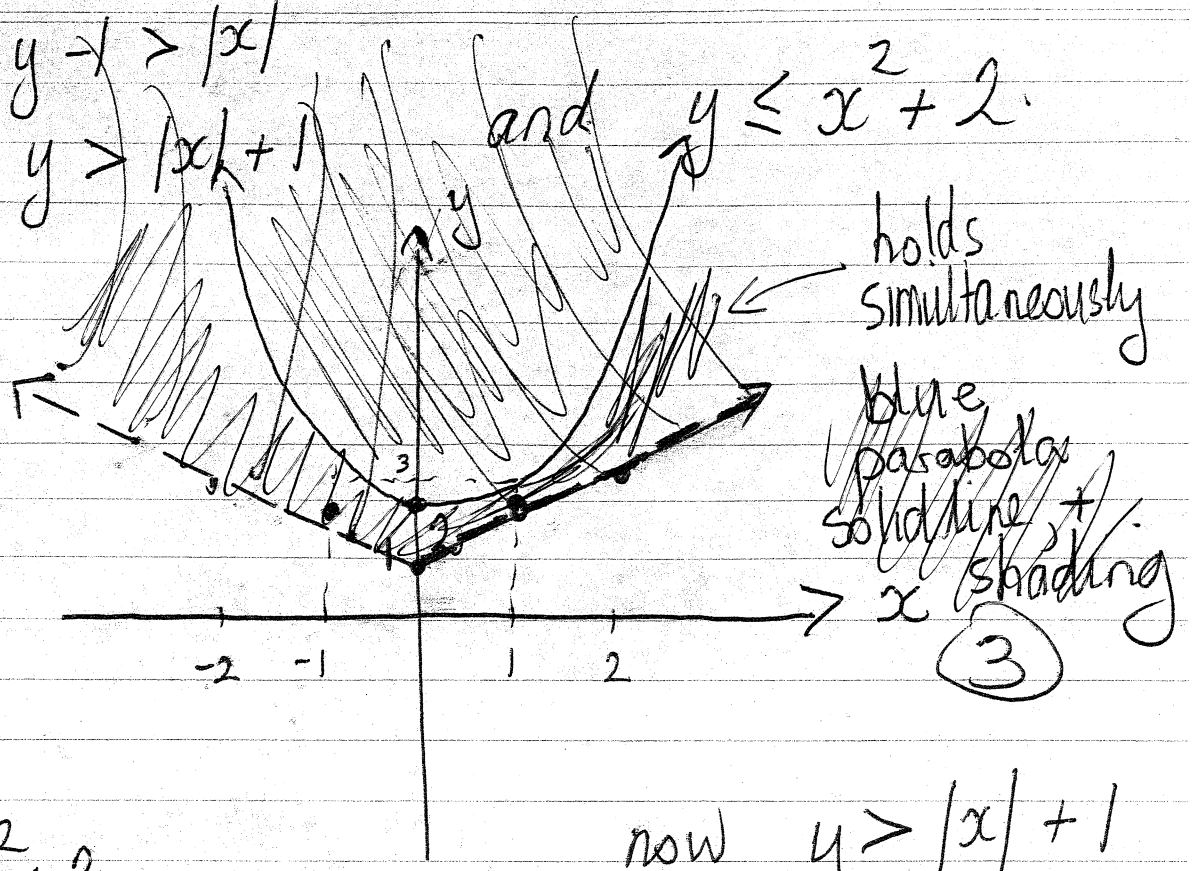
(5) (a) $y > |x|$ and $y \leq x^2 + 2$.

$y > |x| + 1$



holds simultaneously

blue parabola solid line, + shading

(3)

For $y \leq x^2 + 2$

test (0,0) $0 \leq 0 + 2$ ✓

now $y > |x| + 1$

First graph $y = |x| + 1$

test (0,0) $0 > |0| + 1$ No

(b) $\log a = 1.47$, $\log b = 0.86$.

(i) $\log\left(\frac{a}{b}\right) = \log a - \log b = 1.47 - 0.86 = 0.61$ (1)

(ii) $\log(\sqrt{ab}) = \frac{1}{2}\log(ab) = \frac{1}{2}[\log a + \log b]$

$= \frac{1}{2}[1.47 + 0.86]$

$= 1.165$ (1)

(5) (c) $\log_3\left(\frac{x^2}{x-2}\right) = 2$

So $\frac{x^2}{x-2} = \frac{9}{1}$

$x^2 = 9x - 18$

$x^2 - 9x + 18 = 0$

$(x-3)(x-6) = 0$

$x = 3, \quad x = 6.$ (3)

(d) (i) $2x^2 - 4x - 30$

$= 2(x^2 - 2x - 15)$

$= 2(x-5)(x+3)$ (1½)

(ii) $x^3 - y^3 - x + y$

$x^3 - x - y^3 + y$

$x(x^2-1)_3 - y(y^2-1)$ NO!

Back to $x^3 - y - x + y$

$(x-y)(x^2+xy+y^2) - 1(x-y)$

$= (x-y)(x^2+xy+y^2-1)$ (1½)

13

6 @ (i) $19+15+11+\cdots$

arithmetic, $a=19$, $d=-4$ $n=37$

$u_n = a+(n-1)d$

$u_{37} = 19+36\times -4 = -125$ (2)

(ii) now $S_{37} = \frac{n}{2}(a+L)$ (2)

$= \frac{37}{2}(19+-125) = -1961$

(b) Find u_{19} $96-48+24-\cdots$

$a=96$, $r=-\frac{1}{2}$, $n=19$

$u_n = ar^{n-1}$

$= 96\times\left(\frac{-1}{2}\right)^{n-1}$

$= 96\times\left(\frac{1}{2}\right)^{n-1}\times(-1)^{n-1}$

now $u_{19} = 96\times\left(\frac{1}{2}\right)^{18}\times(-1)^{18}$

$= \frac{3\times 2^5}{2^{18}} = \frac{3}{2^{13}}$ (2)

(c) $0.\dot{1}\dot{7} = 0.17+0.0017+0.000017+\cdots$ (1)

geometric, $a=0.17$, $r=0.01$

$S_\infty = \frac{a}{1-r} = \frac{0.17}{0.99} = \frac{17}{99}$ (2)

(d) week 1 $10 saved
week 2 $10.50 "
week 3 $11 "
etc.

$a = 1000, \; n = 52, \; d = 50$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{52} = \frac{52}{2}(2 \times 1000 + 51 \times 50)$$

$$= \$1183$$ (4)