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Sydney Girls High School

Y11 2u Yearly

Syd Girls 2000 Y11 2u CR 85

2000 Yearly Exam

MATHEMATICS 2U/3U

Year 11

Time allowed - 90 minutes

Instructions

NAME _____

- Attempt all eight questions.
- Questions are NOT of equal value.
- All necessary working should be shown in every question.
  Marks may be deducted for careless or badly arranged work.
- Each question attempted should be started on a new sheet.
  Write on one side of the paper only.

Question One (10 marks)

a) Evaluate $\dfrac{1}{\sqrt{5.3^2 - 1.7}}$ to 2 decimal places

b) Evaluate $|-5| - |7|$

c) Evaluate $8.3 \times 10^{15} - 7.1 \times 10^{13}$ and give your answer in scientific notation correct to 2 significant figures.

d) Simplify $2 - 2(2x - 1)$

e) Write $2\sqrt{44}$ in simplest surd form.

f) Remove the grouping symbols: $(\sqrt{3} + 4)(\sqrt{2} - \sqrt{5})$

g) Solve for x and plot your solution on a number line: $-5 < 2x + 7 \leq 11$

Question Two (10 marks)

a) Write $\sqrt[5]{(3+x)^2}$ in index form

b) Simplify $2ab - b^2 + 5ab - 2b^2$

c) Factorise $4y^2 - 1$

d) Factorise $x^2 - 5x + 2xy - 10y$

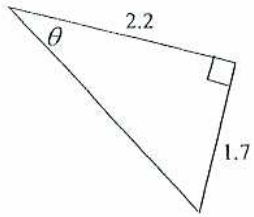
e) Factorise $25 - (x - b)^2$

f) If $\tan\theta = \dfrac{6}{11}$ and θ acute, find the exact ratios of $\sin\theta$ and $\cos\theta$.

Question Three (12 marks)

a) Solve $2x^2 + 3x - 7 = 0$ (leave your answer in exact form)

b) Find θ in the triangle below;



c) A plane leaves Sydney and flies on a bearing of 235 for 750 km.

i) Draw a diagram and mark this information on it.

ii) Calculate how far west of Sydney the plane is.

d) i) Sketch $x^2 + y^2 = 9$

ii) Is this graph a function ? (give a reason for your answer)

e) Solve $|2x+1| = 5$

Question Four (13 marks)

a) Simplify $\dfrac{a-3}{a^3-27}$

b) Sketch $y = x^2 - 4$ on a number plane, and state its domain and range.

c) Sketch the graph of $y = \cos x$ for $0 \le \theta \le 360$.

d) A straight line has x-intercept 1 and y-intercept -2. Find its equation.

e) Simplify $\sqrt{9 - 9\cos^2 \alpha}$

Question Five (12 marks)

a) Solve $\dfrac{x}{2} - \dfrac{x+1}{5} = 1$

b) If $A = \dfrac{1}{2}h(a+b)$, find the value of a if $A = 100, h = 10$ and $b = 8$.

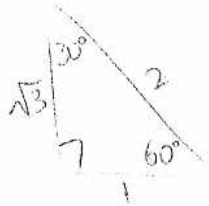
c) For the function $F(x) = x^4 + 1$;

i) Find $F(1)$

ii) Find x if $F(x) = 17$

iii) Show that $F(x)$ is an even function

d) Solve $2\sin\theta = 1$ for $0 \le \theta \le 360$.



Question Six (16 marks)

Draw sketch graphs of each function showing relevant features. (do not use calculus)

a) $y = 2$

b) $x + y = 2$

c) $y = -2^x$

d) $y = \dfrac{1}{x+2}$

f) $y = |2 + x|$

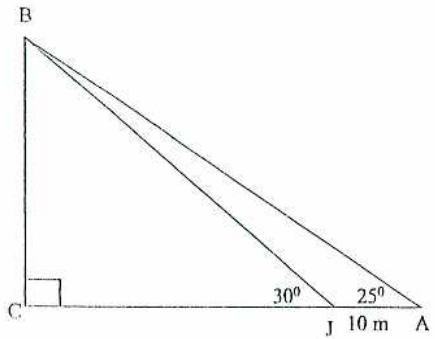
g) $y = |x^2 - 2|$

h) $y = \dfrac{-2}{x^2}$

i) $f(x) = \begin{cases} 2x - 1 & \text{if } x > 2 \\ x^2 & \text{if } x \le 2 \end{cases}$

Question Seven (14 marks)

a) Angela measures the angle of elevation of the top of a tower as 25^0 and Jessica measures the angle of elevation as 30^0. Jessica is standing 10 metres closer to the tower than Angela



i) Copy the diagram onto your test paper

ii) Show that $BJ = \dfrac{10\sin 25^0}{\sin 5^0}$

iii) Hence find the height of the tower.

b) Plot points A (2,1) and B (-2,-3) on a number plane.

i) Find the midpoint, C, of AB.

ii) Show that the line through C perpendicular to AB has equation $x + y + 1 = 0$

iii) Show that this line passes through D(-3,2).

iv) Find the area of triangle ABD

Question Eight(13 marks)

a) Find the exact value of $\cos 315^0$.

b) Find the equation of the straight line passing through (-3,-2) and passing through the intersection of $2x - 5y - 3 = 0$ and $3x - 4y - 8 = 0$.

c) Solve $2^{3x-1} = \dfrac{1}{4}$

d) Given $x = \sqrt{5} + 2$, find b if $x + \dfrac{1}{x} = 2\sqrt{b}$

e) Prove $\left(\sec^2\theta\right)\left(\cos ec^2\theta\right) = \sec^2\theta + \cos ec^2\theta$

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Soln's 2U Yrly Year 11

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Q1.

a) 0.19

b) $5 - 7 = -2$

c) 8.2×10^{15}

d) $2 - 4x + 2$
$= 4 - 4x$

e) $2\sqrt{44} = 2 \times 2\sqrt{11}$
$= 4\sqrt{11}$

f) $(\sqrt{3} + 4)(\sqrt{2} - \sqrt{5})$
$= \sqrt{6} - \sqrt{15} + 4\sqrt{2} - 4\sqrt{5}$

g) $-5 < 2x + 7 \leq 11$
$-12 < 2x \leq 4$
$-6 < x \leq 2$

Q2

a) $\sqrt[3]{(3+x)^2} = (3+x)^{\frac{2}{3}}$

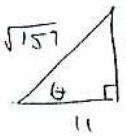
b) $7ab - 3b^2$

c) $4y^2 - 1 = (2y+1)(2y-1)$

d) $x^2 - 5x + 2xy - 10y$
$x(x-5) + 2y(x-5)$
$(x+2y)(x-5)$

e) $25 - (x-b)^2$
$= [5 - (x-b)][5 + (x-b)]$
$= (5 - x + b)(5 + x - b)$

f)



$\cos\theta < 0$ is θ in Q_2, Q_3

$\sin\theta = \frac{6}{\sqrt{157}}$, $\tan\theta = \frac{6}{11}$

$\cos\theta = \frac{11}{\sqrt{157}}$

Q3

a) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

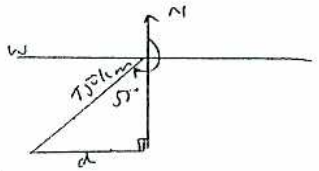
$= \dfrac{-3 \pm \sqrt{9 - 4(2)(-7)}}{4}$

$= \dfrac{-3 \pm \sqrt{65}}{4}$

b) $\tan\theta = \frac{1.7}{2.1}$

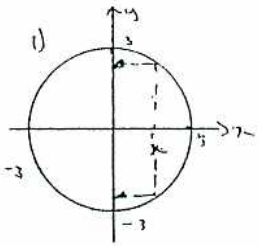
$\theta = 55°$

c) i)



d
$750 = \sin 55°$
$d = 750 \sin 55°$
$= 614$ km

a) i)



ii) No as for some given x there are 2 y's (see diagram)

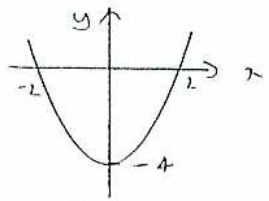
e) $2x + 1 = 5$ or $2x + 1 = -5$
$2x = 4$ $2x = -6$
$x = 2$ $x = -3$

Q4

a) $\frac{a-3}{a^2-27} = \frac{a-3}{(a-3)(a^2+3a+9)}$

$= \frac{1}{a^2+3a+9}$

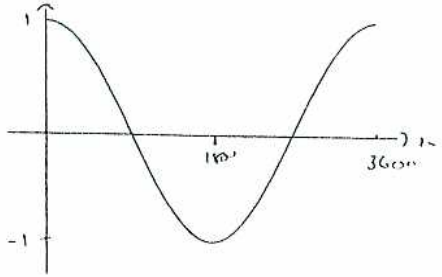
b) $y = x^2 - 4$



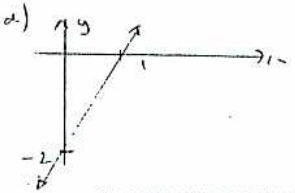
x : all real

$y \geq -4$

c)



d)



$y = 2x - 2$

e) $\sqrt{9 - 9\cos^2\theta} = \sqrt{9(1-\cos^2\theta)}$

$= \sqrt{9\sin^2\theta}$

$= 3\sin\theta$

Q5

a) $\frac{x}{2} - \frac{x+1}{5} = 1$

$5x - 2(x+1) = 10$

$5x - 2x - 2 = 10$

$3x = 12$

$x = 4$

b) $A = \frac{1}{2}h(a+b)$

$100 = 5(a+8)$

$20 = a+8$

$a = 12$

c) i) $F(1) = 1^4 + 1$

$= 2$

ii) $17 = x^4 + 1$

$16 = x^4$

$x = \pm 2$

iii) Even if $F(a) = F(-a)$

$F(a) = a^4 + 1$, $F(-a) = (-a)^4 + 1$

$= a^4 + 1$

$\therefore$ Even.

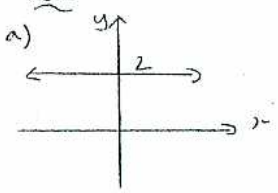
d) $2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

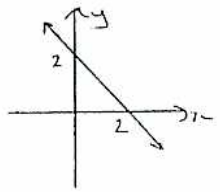
$\theta = 30°, 150°$

Q6

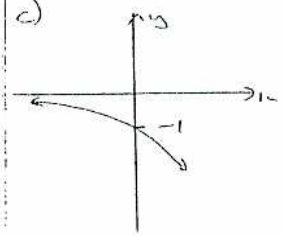
a)



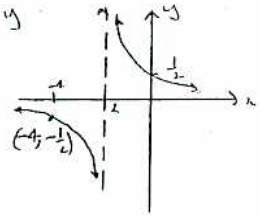
b)



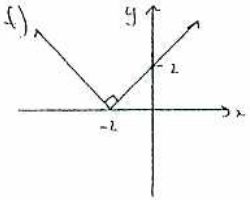
c)



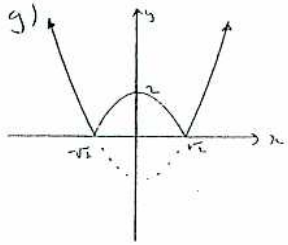
d)



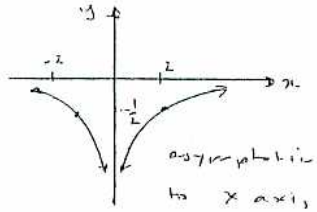
f)



g)

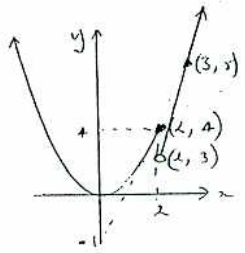


h)



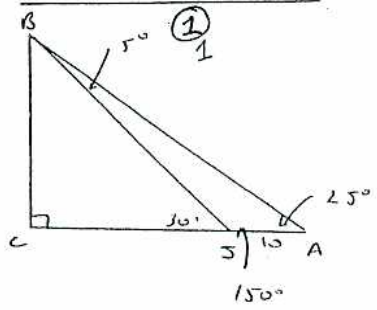
asymptotic to x axis,

i)



Question Seven

a)



(1)

ii) $\frac{BD}{\sin 25^\circ} = \frac{10}{\sin 5^\circ}$

$BD = \frac{10 \sin 25^\circ}{\sin 5^\circ}$ 1 (1)

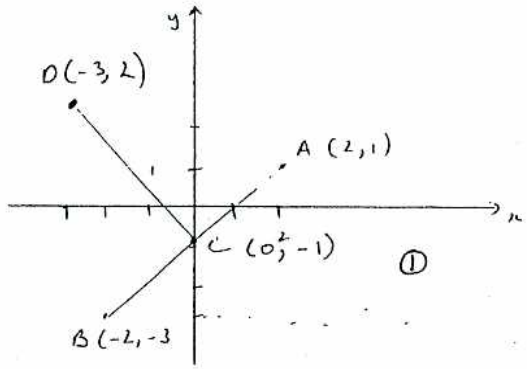
iii) $\frac{BC}{BD} = \sin 30^\circ$

$BC = BD \times \sin 30^\circ$

$= \frac{10 \sin 25^\circ \sin 30^\circ}{\sin 5^\circ}$

$BC = 24.2\text{ m}$ (2)

b)



(1)

i) co-ords C

$x = \frac{2 + -2}{2} = 0$ (2)

$y = \frac{1 - 3}{2} = -1$

ii) gradient AB

$m_1 = \frac{1 - -3}{2 - -2}$

$= 1$ (1)

$\therefore m_2 = -1$ ($m_1 m_2 = -1$) (1)

Eqn $y - y_1 = m(x - x_1)$

$y + 1 = -1(x - 0)$

$y + 1 = -x$

$x + y + 1 = 0$ (1)

iii) subst $x = -3$, $y = 2$ in above

$-3 + 2 + 1 = 0$ (1)

iv) $AB = \sqrt{4^2 + 4^2}$

$= 4\sqrt{2}$ (1)

$CD = \sqrt{3^2 + 3^2}$

$= 3\sqrt{2}$ (1)

$A = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 4\sqrt{2} \times 3\sqrt{2}$

$= 12\text{ units}^2$ (1)

Question 8

a) $\cos 315° = \cos 45°$

$= \frac{1}{\sqrt{2}}$

b) Eqn of form $2x - 5y - 3 + k(3x - 4y - 8) = 0$

$x = -3, y = -2$

$-6 + 10 - 3 + k(-9 + 8 - 8) = 0$

$1 + -9k = 0$

$k = \frac{1}{9}$

ie $2x - 5y - 3 + \frac{1}{9}(3x - 4y - 8) = 0$

$18x - 45y - 27 + 3x - 4y - 8 = 0$

$21x - 49y - 35 = 0$

c) $2^{3x-1} = \frac{1}{4}$

$3\ 2^{3x-1} = 2^{-2}$

$3x - 1 = -2$

$3x = -1$

$x = -\frac{1}{3}$

d) If $x + \frac{1}{x} = \sqrt{5} + 2$ then

$x + \frac{1}{x} = \sqrt{5} + 2 + \frac{1}{\sqrt{5}+2}$

$= \sqrt{5} + 2 + \sqrt{5} - 2$

$= 2\sqrt{5}$ ie $b = 5$

e) Prove $(\sec^2\theta)(\operatorname{cosec}^2\theta) = \sec^2\theta + \operatorname{cosec}^2\theta$

RHS $= \sec^2\theta + \operatorname{cosec}^2\theta$

$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta \cos^2\theta}$

$= \frac{1}{\sin^2\theta \cos^2\theta}$

$= \sec^2\theta \operatorname{cosec}^2\theta$