QUESTION ONE (Start a new writing booklet)

(a) Simplify $\sqrt{18} + \sqrt{8}$.

(b) Solve $4x - 64 = 0$.

(c) Factorise $x^2 + 3x - 28$.

(d) Write down the derivative of x^6.

(e) Write down the gradient of the line $y = 7 - 3x$.

(f) Sketch the graph of $y = x^2 + 1$.

(g) Find the exact value of $\tan 30°$.

(h) Evaluate $\dfrac{a+b}{a^2 - b^2}$ when $a = -3$ and $b = 1$.

(i) Rationalise the denominator of $\dfrac{2}{\sqrt{6}}$.

(j) Write down the midpoint of the interval joining $(-a, b)$ and (a, b).

(k) Solve $2 - x \geq 4$.

QUESTION TWO (Start a new writing booklet)

(a) Factorise $3x^2 - x - 24$.

(b) Sketch the graph of $x^2 + y^2 = 4$.

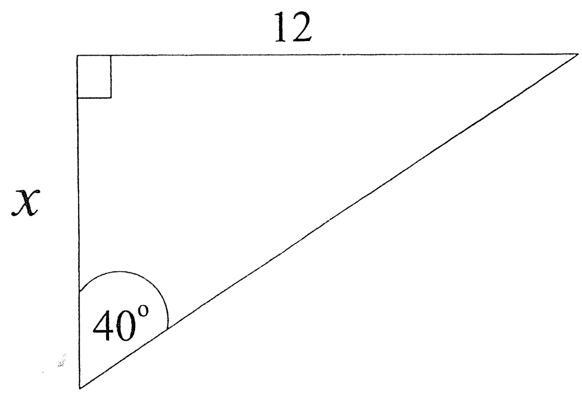
(c) Write down the equation of the line that passes through $(2,3)$ and is parallel to the x-axis.

(d) Differentiate $3x^4 + 7x + 2$ with respect to x.

(e) Express $\dfrac{3}{3 - \sqrt{3}}$ with a rational denominator.

(f) Find the exact value of $\sin 240°$.

(g)



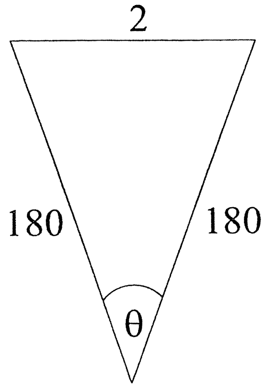
In the diagram above, find the value of x, correct to three decimal places.

(h) Solve $x^2 + 4x - 12 = 0$.

(i) Solve $\sin \theta = \frac{1}{2}$, for $0° \leq \theta \leq 360°$.

QUESTION THREE (Start a new writing booklet)

(a)



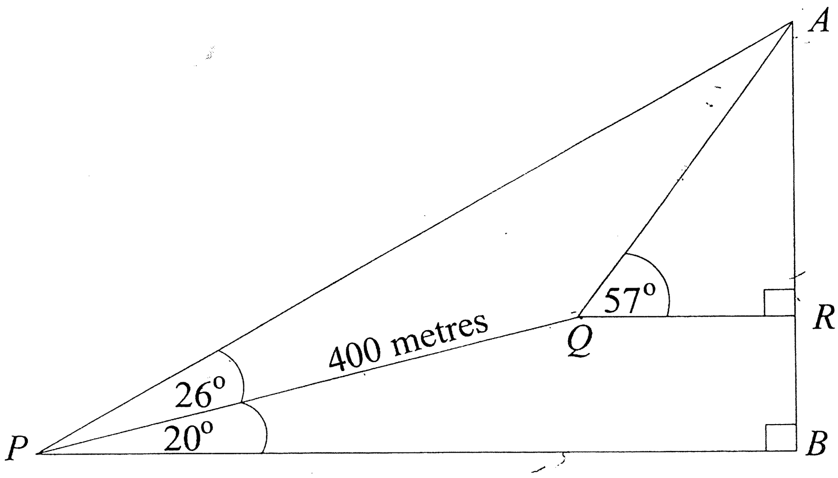
In the diagram above, use the cosine rule to find the value of θ, correct to the nearest minute.

(b) Solve $\cos\alpha = \dfrac{1}{\sqrt{2}}$, where $0^\circ \le \alpha \le 360^\circ$.

(c) Given that $\tan\theta = -\sqrt{3}$ and θ is obtuse, find the exact value of $\sin\theta$.

(d) Show that $\tan^2 x\cos^2 x + \cot^2 x\sin^2 x = 1$. (Begin your solution with LHS $= \ldots$).

(e)



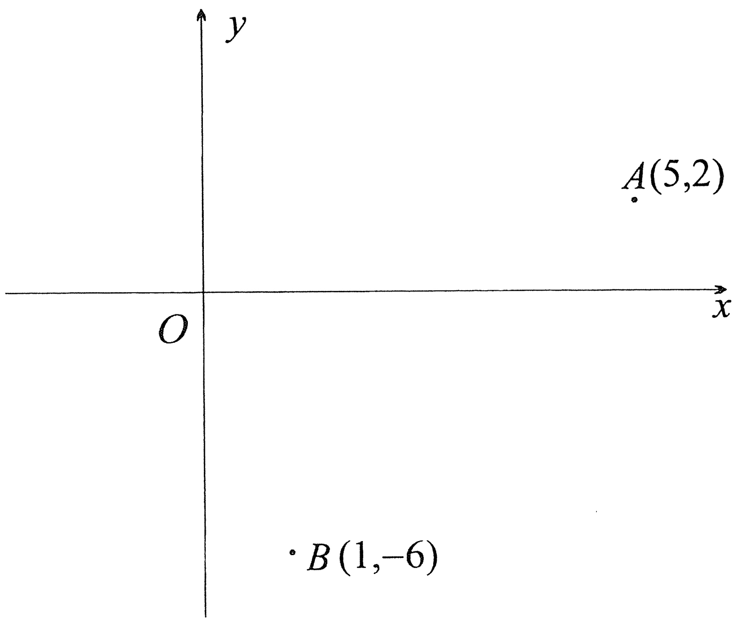
In the diagram above, $\triangle ABP$ and $\triangle ARQ$ are right-angled triangles. It is also known that $\angle APQ = 26^\circ$, $\angle QPB = 20^\circ$, $\angle AQR = 57^\circ$ and $PQ = 400$ metres.

(i) Show that $\angle AQP = 143^\circ$.

(ii) Find the length AB, correct to the nearest metre.

QUESTION FOUR (Start a new writing booklet)

(a)



In the diagram above, the point A has coordinates (5,2) and the point B has coordinates $(1,-6)$. Copy the diagram into your booklet.

(i) Find the exact length of the interval AB.

(ii) Find the gradient of the interval AB.

(iii) Find the coordinates of the point C such that B is the midpoint of AC.

(iv) Find the acute angle at which the line AB meets the x-axis. (Give your answer correct to the nearest degree.)

(v) Show that the equation of the line AB is $2x - y - 8 = 0$.

(vi) Find the perpendicular distance from the origin $(0, 0)$ to the line AB.

(vii) Hence or otherwise find the area of $\triangle AOB$.

(b) Find the points of intersection of the line $y = 12 - 5x$ and the parabola $y = x^2 - x - 20$.

QUESTION FIVE (Start a new writing booklet)

(a) Differentiate the following with respect to x:

(i) $y = \dfrac{1}{x^2}$

(ii) $y = (1 - 4x)^5$

(iii) $y = \sqrt{x + 3}$

(iv) $y = 3x^2(x + 4)^3$

(v) $y = \dfrac{1 - 5x}{x + 3}$

(b) A function is given by $f(x) = 10x - x^3$.

(i) Find $f'(x)$ and $f'(2)$.

(ii) Find the equations of the tangent and normal to the curve $y = f(x)$ at the point $A(2, 12)$.

(c) (i) Differentiate $y = (x - n)^3$, where n is a constant.

(ii) Find the value of n for which $\dfrac{dy}{dx} = 0$ when $x = 3$.

QUESTION SIX (Start a new writing booklet)

(a) If the roots of the quadratic equation $2x^2 + x + 7 = 0$ are α and β, find:

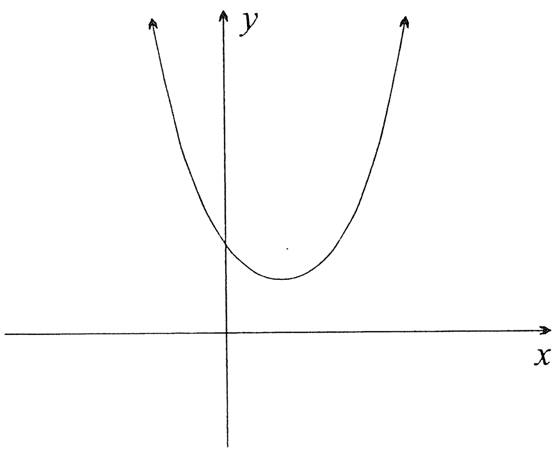
(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$

(iv) $\alpha^2 + \beta^2$

(b)



In the diagram above, the graph of $y = ax^2 + bx + c$ is drawn. From the following five possibilities, choose the letter which corresponds to a true statement and write it in your answer booklet.

A: $a < 0$ and $\triangle > 0$

B: $a > 0$ and $\triangle > 0$

C: $a < 0$ and $\triangle < 0$

D: $a > 0$ and $\triangle < 0$

E: None of the above

(c) Write down the coordinates of the vertex of the parabola with equation $y = (x-2)^2+3$.

(d) What is the effect, on the graph of $y = (x - p)^2$, of varying p?

(e) (i) Find the discriminant of the equation $2x^2 - 5x + 2 = 0$.

(ii) Hence describe the roots of the equation $2x^2 - 5x + 2 = 0$.

(f) Find the values of k for which $4x^2 - kx + 49 = 0$ has exactly one root.

(g) (i) Write down the equation of the axis of symmetry of $y = 1 + 3x - 3x^2$.

(ii) Find the maximum value of $1 + 3x - 3x^2$.

QUESTION SEVEN (Start a new writing booklet)

(a) Consider the sequence $3, \frac{10}{3}, \frac{11}{3}, \ldots$.

(i) Explain why the sequence is arithmetic.

(ii) Find the one-hundredth term of the sequence.

(iii) Find which term of the sequence is 40.

(iv) Find the sum of the first one hundred terms.

(b) The sequence $4, 8, 16, \ldots$ is a geometric sequence.

(i) Write down the values of the first term a and the common ratio r.

(ii) Find T_{10}.

(iii) Find S_{10}.

(c) The sixth term of an AP is 17 and the thirteenth term is 31.

(i) Show that $a + 5d = 17$.

(ii) Form another similar equation and solve the pair simultaneously to find the first term and the common difference of the AP.

QUESTION EIGHT (Start a new writing booklet)

(a) A point $P(x, y)$ moves so that it is always 3 units from the origin. Sketch the locus of P and then write down its equation.

(b) The points A and B have coordinates $(2, 1)$ and $(-1, -3)$ respectively. The point $P(x, y)$ moves so that it is equidistant from A and B, that is, $PA = PB$.

(i) Use the distance formula to find expressions for PA and PB.

(ii) Equate the expressions to find the equation of the locus of P.

(c) Write down the missing words in the sentences below:

The locus of a point which moves so that its distance from a fixed point is (i) .... to its distance from a fixed straight line is called a (ii) .... The fixed point is called the (iii) .... and the fixed straight line is called the (iv) ....

(d) (i) Write down the equation of the parabola with vertex at the origin $(0, 0)$, focus at the point $(0, 3)$, and the axis of symmetry vertical.

(ii) Write down the equation of the parabola with vertex at the origin $(0, 0)$, directrix $y = 5$, and the axis of symmetry vertical.

(e) For the parabola $x^2 = 2y$, write down:

(i) the focal length,

(ii) the coordinates of the focus,

(iii) the equation of the directrix.

QUESTION NINE (Start a new writing booklet)

(a) Find an expression for the nth term of the series in which the sum of the first n terms is given by $S_n = n(4+n)$.

(b) Find the value(s) of k for which the equation $x^2 - 3kx + (k+3) = 0$ has one root which is the reciprocal of the other.
Begin your solution by letting the roots be α and $\dfrac{1}{\alpha}$.

(c) Determine the value of the constant A if the tangent to the curve $y = \dfrac{A}{x+2}$ at the point where $x = 2$ has gradient $\dfrac{1}{4}$.

(d) Consider the parabola $(x-1)^2 = 8(y+3)$.

(i) Write down the focal length.

(ii) Write down the coordinates of the vertex.

(iii) Write down the coordinates of the focus.

(iv) Write down the equation of the directrix.

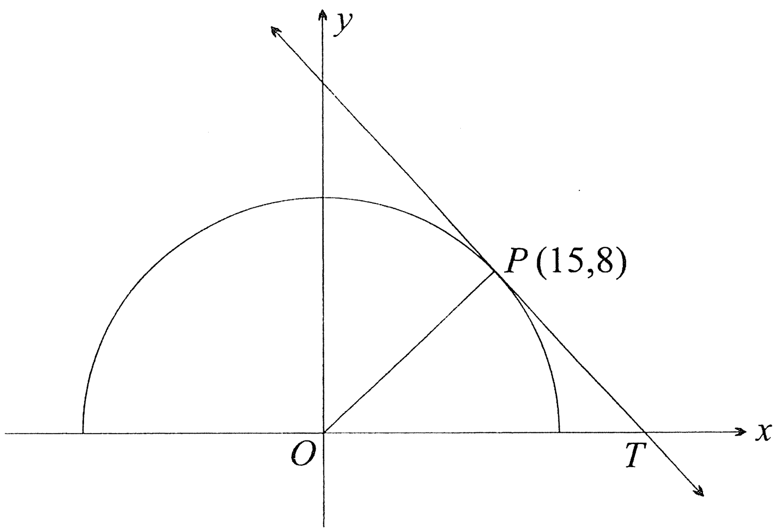
(v) Sketch the parabola showing all the features found in parts (i)–(iv).

QUESTION TEN (Start a new writing booklet)

(a) (i) For what values of x will the infinite geometric series $1 - 2x + 4x^2 - 8x^3 + \cdots$ have a limiting sum?

(ii) If the limiting sum is $\frac{3}{5}$, find the value of x.

(b)



The diagram above shows the semi-circle $y = \sqrt{289 - x^2}$ and the tangent at $P(15, 8)$ on it. The tangent at P meets the x-axis at T.

(i) Show that the tangent at P has gradient $-\frac{15}{8}$.

(ii) Find the equation of the tangent at P.

(iii) Show that the tangent at P is perpendicular to the radius.

(iv) Find the exact area of the triangle OPT.

(v) Find the size of $\angle PTO$, correct to the nearest degree.

JNC

FY II UNIT 2003 YEARLY

Question 1

(a) $3\sqrt{2}+2\sqrt{2} = 5\sqrt{2}$ ✓

(b) $4x = 64$

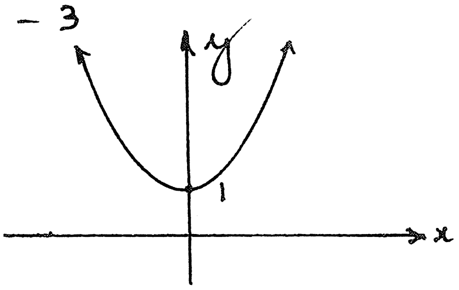
$\therefore x = 16$ ✓

(c) $x^2+3x-28 = (x+7)(x-4)$ ✓

(d) $6x^5$ ✓

(e) $m = -3$ ✓

(f)



✓

(g) $\frac{1}{\sqrt{3}}$ ✓

(h) $\frac{-2}{8} = -\frac{1}{4}$ ✓ ✓ { (1 for correct numerator or denominator) (1 for simplification) }

(i) $\frac{2}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{2\sqrt{6}}{6}$ ✓

$= \frac{\sqrt{6}}{3}$ ✓

(j) $(0, b)$ ✓ ✓ (1 for 0, 1 for b)

(k) $2-x \geq 4$

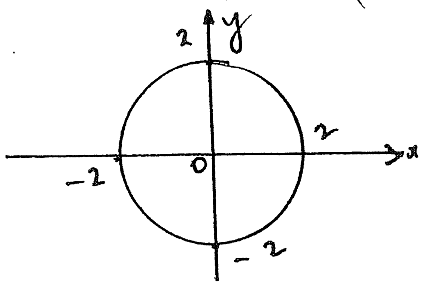
$-x \geq 2$ ✓

$x \leq -2$ ✓

Question 2

(a) $3x^2 - x - 24 = (3x+8)(x-3)$ ✓

(b)



✓

(c) $y = 3$ ✓

(d) $12x^3 + 7$ ✓✓ {(1 for each part) or (−1 per error)}

(e) $\dfrac{3}{3-\sqrt{3}} \times \dfrac{3+\sqrt{3}}{3+\sqrt{3}} = \dfrac{3(3+\sqrt{3})}{6}$ ✓

$= \dfrac{3+\sqrt{3}}{2}$ ✓

f) $\sin 240^\circ = -\sin 60$

$= -\dfrac{\sqrt{3}}{2}$ ✓✓ (1 for sign, 1 for ratio)

g) $\tan 40^\circ = \cancel{\dfrac{x}{12}} \quad \dfrac{12}{x}$ ✓

$\therefore \cancel{x = 12\tan 40}$

$\cancel{= 10.069}$ ✓ $x = \dfrac{12}{\tan 40}$

$= 14.301$

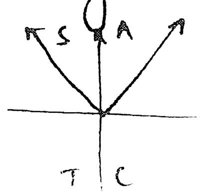
h) $x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$ ✓

$\therefore x = -6$ or 2 ✓

i) $\sin\theta = \frac{1}{2}$

Related angle $= 30^\circ$ ✓



$\therefore \theta = 30^\circ$ or 150° ✓

Question 3

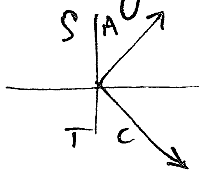
(a) $\cos\theta = \dfrac{180^2 + 180^2 - 2^2}{2 \times 180 \times 180}$ ✓

$= 0.9999\ldots$ ✓

$\therefore \theta = 0°38'$ ✓

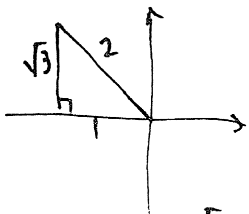
(b) $\cos\alpha = \dfrac{1}{\sqrt{2}}$

Related angle $= 45°$ ✓



$\therefore \alpha = 45°$ or $315°$ ✓.

c)



$\sin\theta = \dfrac{\sqrt{3}}{2}$ ✓✓

d) LHS $= \tan^2 x \cos^2 x + \cot^2 x \sin^2 x$

$= \dfrac{\sin^2 x}{\cancel{\cos^2 x}} \cdot \cancel{\cos^2 x} + \dfrac{\cos^2 x}{\cancel{\sin^2 x}} \cdot \cancel{\sin^2 x}$ ✓

$= \sin^2 x + \cos^2 x$ ✓

$= 1$

e) (i) $\angle PQR = 360 - 200$ (angle sum PQRB)

$= 160°$ ✓

$\therefore \angle AQP = 360 - (160 + 57)$ (angles at point)

$= 143°$ ✓

(ii) $\dfrac{AP}{\sin 143} = \dfrac{400}{\sin 11}$ ✓

$AP = \dfrac{400 \sin 143}{\sin 11}$

$= 1261.607\ldots$

$\sin 46 = \dfrac{AB}{AP}$ ✓

$\therefore AB = \dfrac{400 \sin 143}{\sin 11} \cdot \sin 46$

$= 907.52\ldots$ ✓ $= 908$ m

Question 4

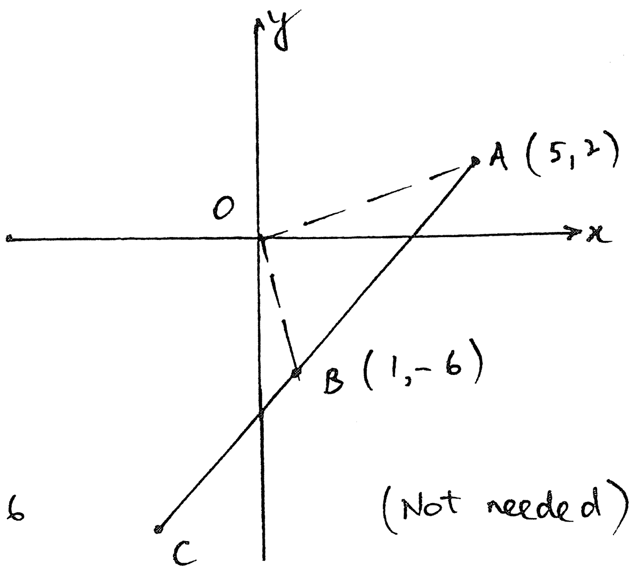
(a)

(i) $d = \sqrt{4^2 + 8^2}$

$= \sqrt{80}$ ✓

(ii) $M = \dfrac{-8}{-4}$

$= 2$ ✓



(Not needed)

(iii) $\dfrac{5+x}{2} = 1$ and $\dfrac{2+y}{2} = -6$

$x = -3$ $\quad y = -14$ ✓✓

$C = (-3, -14)$

(iv) $\tan\alpha = 2$ ✓

$\alpha = 63^\circ$ ✓

(v) $\left.\begin{array}{l} y - 2 = 2(x-5) \\ y - 2 = 2x - 10 \end{array}\right\}$ ✓ (either)

$2x - y - 8 = 0$ ✓

(vi) $d = \dfrac{|0 + 0 - 8|}{\sqrt{2^2 + 1^2}}$

$= \dfrac{8}{\sqrt{5}}$ ✓✓

(vii) Area AOB $= \dfrac{1}{2} \times \sqrt{80} \times \dfrac{8}{\sqrt{5}}$ ✓

$= 16$ ✓

(b)(i) $x^2 - x - 20 = 12 - 5x$

$\left.\begin{array}{l} x^2 + 4x - 32 = 0 \\ (x+8)(x-4) = 0 \end{array}\right\}$ ✓ (either)

$\therefore x = 4$ or -8 ✓

$\left.\begin{array}{l} \text{and } y = -8 \text{ or } 52 \\ \text{Pts of Int. are } (4, -8) \text{ and } (-8, 52) \end{array}\right\}$ ✓ (Must have y coordinc somehow)

Question 5

a) (i) $\dfrac{dy}{dx} = -2x^{-3}$

(ii) $\dfrac{dy}{dx} = 5(1-4x)^4 \times -4$

$= -20(1-4x)^4$

(either 1 for index
1 for -20)

(iii) $y = (x+3)^{\frac{1}{2}}$

$\dfrac{dy}{dx} = \dfrac{1}{2}(x+3)^{-\frac{1}{2}}$

(iv) $y = 3x^2(x+4)^3$

$\dfrac{dy}{dx} = 6x(x+4)^3 + 3x^2 \times 3(x+4)^2$

$= 6x(x+4)^3 + 9x^2(x+4)^2$

$= 3x(x+4)^2(5x+8)$

either

(v) $\dfrac{dy}{dx} = \dfrac{(x+3)\times(-5) - (1-5x)}{(x+3)^2}$

$= \dfrac{-16}{(x+3)^2}$

(b)(i) $f'(x) = 10 - 3x^2$

$f'(2) = 10 - 3\times 2^2$

$= -2$

(ii) Tangent: $y - 12 = -2(x-2)$

$y = -2x + 16$

Normal: $y - 12 = \frac{1}{2}(x-2)$

$2y - 24 = x - 2$

$2y = x + 22$

c) $y = (x-n)^3$

$\dfrac{dy}{dx} = 3(x-n)^2$

$0 = 3(3-n)^2$

$\therefore n = 3.$

Question 6

a) (i) $\alpha+\beta = -\frac{1}{2}$ ✓

(ii) $\alpha\beta = \frac{7}{2}$ ✓

(iii) $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$ ✓ (either)

$= \frac{-\frac{1}{2}}{\frac{7}{2}}$

$= -\frac{1}{7}$ ✓

(iv) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ✓

$= \left(-\frac{1}{2}\right)^2 - 2\times\frac{7}{2}$

$= \frac{1}{4} - 7$

$= -6\frac{3}{4}$ ✓

(b) D ✓

(c) (2, 3) ✓

(d) shifts horizontally p units ✓

(e)(i) $\Delta = 25 - 4\times2\times2$

$= 9$ ✓

(ii) Real, distinct and rational ✓

(f) $\Delta = k^2 - 4\times4\times49$

$= k^2 - 784$ ✓

Require : $\Delta = 0$

ie $k^2 - 784 = 0$

$\therefore k = \pm 28$ ✓

g)(i) $x = -\frac{3}{-6}$

$x = \frac{1}{2}$ ✓

(ii) $y = 1 + 3\times\frac{1}{2} - 3\times\left(\frac{1}{2}\right)^2$

$= \frac{7}{4}$ ✓

Question 7

a) (i) $d = \frac{10}{3} - 3 = \frac{1}{3}$ and $\frac{11}{3} - \frac{10}{3} = \frac{1}{3}$ ✓ (either or similar)

or $T_2 - T_1 = T_3 - T_2 = \frac{1}{3}$

(ii) $T_{100} = 3 + 99 \times \frac{1}{3}$

$= 36$ ✓

(iii) $a + (n-1)d = 40$ ✓

$3 + (n-1) \times \frac{1}{3} = 40$

$(n-1) \times \frac{1}{3} = 37$

$n = 1 + 3 \times 37$

$n = 112$ ✓

ie/ $T_{112} = 40$

(iv) $S_{100} = \frac{100}{2}(3 + 36)$ ✓

$= 1950$ ✓

b) (i) $a = 4$ $r = 2$ ✓ ✓

(ii) $T_{10} = 4 \times 2^9$

$= 2048$ ✓ either

(iii) $S_{10} = \frac{4(2^{10} - 1)}{(2-1)}$ ✓

$= 4092$ ✓

(c) (i) $T_6 = a + 5d = 17$ ✓ or $a + (n-1)d = 17$, $T_6 = a + 5d = 17$ or similar.

(ii) $a + 5d = 17$, $a + 12d = 31$ ✓

$7d = 14$

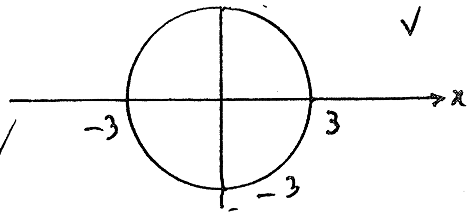
$\therefore d = 2$ ✓

and $a + 10 = 17$

$\therefore a = 7$ ✓

Question 8

(a) (i) $x^2 + y^2 = 9$ ✓



✓ (1 intercep needed)

(b) (i) $PA = \sqrt{(x-2)^2 + (y-1)^2}$ ✓

$PB = \sqrt{(x+1)^2 + (y+3)^2}$ ✓

(ii) $(x-2)^2 + (y-1)^2 = (x+1)^2 + (y+3)^2$

$x^2 - 4x + 4 + y^2 - 2y + 1 = x^2 + 2x + 1 + y^2 + 6y + 9$ ✓

$6x + 8y + 5 = 0$ ✓

(c) (i) equal ✓

(ii) parabola ✓

(iii) focus ✓

(iv) directrix ✓

d) (i) $x^2 = 4 \times 3\, y$

$x^2 = 12y$ ✓

(ii) $x^2 = -20y$ ✓

e) (i)

$4a = 2$

$\therefore a = \frac{1}{2}$ ✓

(ii) $S = (0, \frac{1}{2})$ ✓

(iii) $y = -\frac{1}{2}$ ✓

Question 9

(a) $x^4 - 10x^2 + 9 = 0$

Let $u = x^2$, then

$u^2 - 10u + 9 = 0$

$(u-9)(u-1) = 0$ ✓ either

$\therefore u = 1 \text{ or } 9$ ✓

and $x^2 = 1 \text{ or } 9$

$\therefore x = \pm 1 \text{ or } \pm 3.$ ✓

(b) Let the roots be α and $\frac{1}{\alpha}$,

$\alpha + \frac{1}{\alpha} = 3k$

$\alpha \cdot \frac{1}{\alpha} = (k+3)$ ✓

$k + 3 = 1$ ✓

$\therefore k = -2$ ✓

(c) $y = A(x+2)^{-1}$

$\frac{dy}{dx} = -A(x+2)^{-2}$ ✓

When $x = 2$, $\frac{dy}{dx} = \frac{1}{4}$

ie/ $\frac{1}{4} = \frac{-A}{(2+2)^2}$ ✓

$\frac{1}{4} = \frac{-A}{16}$

$\therefore A = -4$ ✓

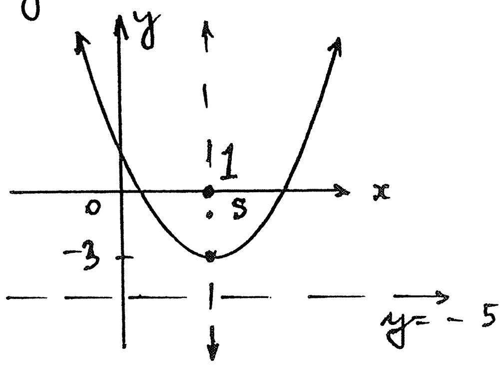
(d) (i) $a = 2$ ✓

(ii) Vertex $= (1, -3)$ ✓

(iii) $S = (1, -1)$ ✓

(iv) $y = -5$ ✓

(v)



✓✓

(−1 per error)

(a) $S_n = n(4+n)$

$T_n = S_n - S_{n-1}$ ✓

$= n(4+n) - (n-1)(3+n)$

$= 4n + n^2 - n^2 - 2n + 3$

$= 2n + 3.$ ✓

Question 10

a) (i) $r = -2x$

Limiting sum exists if $|r| < 1$ ✓ either

ie $-1 < -2x < 1$

$\frac{1}{2} > x > -\frac{1}{2}$

ie $-\frac{1}{2} < x < \frac{1}{2}$ ✓

(ii) $(S_\infty =) \frac{1}{1+2x} = \frac{3}{5}$ ✓✓ (1 for $\frac{1}{1+2x}$, 1 for equating to $\frac{3}{5}$)

$5 = 3(1+2x)$ ✓

$6x = 2$

$\therefore x = 3$ ✓

(b) (i) $y = (289 - x^2)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{-\cancel{2}x}{\cancel{2}}(289 - x^2)^{-\frac{1}{2}}$ ✓

At $(15, 8)$; $\frac{dy}{dx} = -15(289-225)^{-\frac{1}{2}}$

$= -\frac{15}{8}$

$= -\frac{15}{8}$ ✓

So eqtn of tangent is:

$y - 8 = -\frac{15}{8}(x - 15)$

$8y - 64 = -15x + 225$

$15x + 8y - 289 = 0$ ✓ or equivalent

(iii) Gradient $OP = \frac{8}{15}$

Gradient of tangent $= -\frac{15}{8}$ ✓

Since $\frac{8}{15} \times -\frac{15}{8} = 1$, $OP \perp$ tangent ✓

(iv) $OT = \frac{289}{15}$

Area $\Delta = \frac{1}{2} \times \frac{289}{15} \times 8$ ✓

$= \frac{1156}{15}$ units squar ✓

(v) $\angle PTO = 90 - \tan^{-1}\left(\frac{8}{15}\right)$ ✓

$= 62^\circ$ ✓