

SYDNEY GRAMMAR SCHOOL



2014 Annual Examination

FORM V

MATHEMATICS 2 UNIT

Wednesday 27th August 2014

General Instructions

- Writing time — 2 hours
- Write using black or blue pen.
- Board-approved calculators and templates may be used.

Total — 100 Marks

- All questions may be attempted.

Section I – 9 Marks

- Questions 1–9 are of equal value.
- Record your solutions to the multiple choice on the sheet provided.

Section II – 91 Marks

- Questions 10–16 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your name, class and master on each booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Place your multiple choice answer sheet inside the answer booklet for Question Ten.
- Write your name and master on this question paper and submit it with your answers.

5A: BDD

5B: MLS

5C: LYL

5D: LRP

5E: PKH

5F: BR

5G: SG

5P: REJ

5Q: NL

5R: TCW

Checklist

- SGS booklets — 7 per boy
- Multiple choice answer sheet
- Candidature — 185 boys

Examiner

PKH

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

The gradient of the line $3x - 2y = 6$ is

- (A) $\frac{2}{3}$ (B) $-\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $-\frac{3}{2}$

QUESTION TWO

The exact value of $\cos 210^\circ$ is

- (A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $-\frac{\sqrt{3}}{2}$

QUESTION THREE

The expression $\sqrt{16x^{16}}$ simplifies to

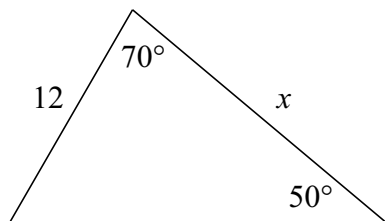
- (A) $8x^4$ (B) $4x^8$ (C) $4x^4$ (D) $8x^8$

QUESTION FOUR

Which of the following is rational?

- (A) $\sqrt[3]{18}$ (B) $\sqrt{8}$ (C) $0.\dot{4}9$ (D) π

QUESTION FIVE



In the diagram above which of the following is correct?

- (A) $x = \frac{12 \sin 50^\circ}{\sin 70^\circ}$ (B) $x = \frac{6\sqrt{3}}{\sin 50^\circ}$
 (C) $x = \frac{12 \sin 70^\circ}{\sin 50^\circ}$ (D) $x = \frac{6}{\sin 50^\circ}$

QUESTION SIX

The quadratic equation $2x^2 - 4x + 6 = 0$ has roots α and β . Which of the following is true?

- (A) $\alpha + \beta = 4$
- (B) $\alpha + \beta = 2$
- (C) $\alpha\beta = 6$
- (D) $\alpha\beta = -6$

QUESTION SEVEN

What is the domain of $y = \sqrt{x^2 - 1}$?

- (A) $x \geq 1$
- (B) $x \geq 1$ or $x \leq -1$
- (C) $x > 1$
- (D) $-1 \leq x \leq 1$

QUESTION EIGHT

For the geometric sequence $2, -4, 8, -16, \dots$ which of the statements below is true?

- (A) The limiting sum is $\frac{2}{3}$.
- (B) The sequence does not have a limiting sum.
- (C) The n th term is never greater than 100 000.
- (D) The sum of the first n terms is never less than $-100\,000$.

QUESTION NINE

Let $y = \frac{x}{1+x}$. A correct expression for the derivative $\frac{dy}{dx}$ is:

- (A) $-\frac{1}{(1+x)^2}$
- (B) $\frac{2x+1}{(1+x)^2}$
- (C) $\frac{1}{(1+x)^2}$
- (D) $\frac{2x-1}{(1+x)^2}$

————— End of Section I —————

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION TEN	(13 marks) Use a separate writing booklet.	Marks
(a) Solve $\frac{x}{2} + 2x = 5$.		2
(b) Differentiate $y = x^3 - 5x$.		1
(c) Sketch the line $y = 2x - 6$, marking the intercepts with the axes.		2
(d) Expand and simplify $(2x - 3)^2$.		1
(e) Simplify:		
(i) $\log_2 30 - \log_2 6$		1
(ii) $\log_3 81$		1
(f) Differentiate $y = x^{\frac{1}{2}} + x^{-1}$.		2
(g) Given $f(x) = x^{\frac{3}{2}} - x$, evaluate $f(9)$.		2
(h) Write down the range of $y = \sqrt{4 - x}$.		1

QUESTION ELEVEN (13 marks) Use a separate writing booklet. **Marks**

- (a) Solve the equation $\tan \theta = -0.64$, for $0^\circ \leq \theta \leq 360^\circ$. Leave your answers correct to the nearest minute. 3

- (b) Solve $x^2 + 2x - 24 = 0$. 2

- (c) Find the gradient of the line passing through $A(2, -4)$ and $B(-6, 2)$. 2

- (d) Solve $|x - 5| = 3$. 2

- (e) Differentiate $y = (2x + 3)^5$. 1

- (f) Rationalise the denominator of $\frac{6}{3 - \sqrt{6}}$, giving your answer in simplest form. 3

QUESTION TWELVE (13 marks) Use a separate writing booklet. **Marks**

- (a) Find the gradient of the tangent to the curve $y = x^2 - 4x$ when $x = -4$. 2

- (b) Simplify $\sqrt{50} - 2\sqrt{8}$. 2

- (c) Solve:
 - (i) $8^x = 2^{2x+1}$ 2
 - (ii) $\log_2 x = 3$ 1
 - (iii) $2 - x \leq 5$ 1

- (d) For the arithmetic sequence $60, 56, 52 \dots$ find:
 - (i) the twenty-first term, 2
 - (ii) the sum of the first twenty one terms. 2

- (e) The numbers $3, x$ and 12 are consecutive terms of a geometric sequence. What are the possible values for x ? 1

QUESTION THIRTEEN (13 marks) Use a separate writing booklet. **Marks**

(a) Sketch the following functions on separate axes, showing any x -intercepts, y -intercepts and asymptotes:

(i) $y = \frac{4}{x - 2}$ 2

(ii) $y = -\sqrt{9 - x^2}$ 2

(b) For what values of k does the quadratic equation $2x^2 - 4x + (k + 2) = 0$ have no real roots? 2

(c) Differentiate $y = \frac{1}{(2x - 5)^3}$. 2

(d) Use the product rule to differentiate $y = x^3(2 + x)^5$. Leave your answer in fully factored form. 3

(e) Find the equation of the tangent to $y = 4x - x^3$ at the point when $x = -2$. 2

QUESTION FOURTEEN (13 marks) Use a separate writing booklet. **Marks**

(a) Simplify $2^x \times 3^x$. 1

(b) The first term of a geometric sequence is 5 and the eighth term is 640.

(i) Find the 12th term. 2

(ii) Find the sum of the first 8 terms. 2

(c) Solve the equation $2 \cos^2 \theta = 1$, for $0^\circ \leq \theta \leq 360^\circ$. 3

(d) Find the points of intersection of the straight line $y = 2x + 5$ and the hyperbola $y = \frac{3}{x}$. 3

(e) Use the quotient rule to differentiate $y = \frac{x}{x^2 + 1}$. 2

QUESTION FIFTEEN (13 marks) Use a separate writing booklet. **Marks**

- (a) Using the perpendicular distance formula from a point to a line, find the values of a for which the line $4x + 3y - a = 0$ is 4 units from the point $(2, -1)$. **3**
- (b) In triangle ABC , $\angle BAC = 24^\circ$, $AB = 2.6$ and $BC = 1.1$. Find the possible values of $\angle ACB$ to the nearest degree. **3**
- (c) Prove that $(1 - \cos A)(1 + \sec A) = \sin A \tan A$. **3**
- (d) Two men are at an intersection of two straight roads which cross at an angle of 58° . They set off at the same time with one man walking at 6 km/h along one road and the other walking along the other road at 5 km/h. **4**

How long before they are 20 km apart, as the crow flies, to the nearest minute?

QUESTION SIXTEEN (13 marks) Use a separate writing booklet. **Marks**

- (a) Find the value of $7 + 14 + 21 + \dots + 1008$. **3**
- (b) Using first principles differentiation, find the derivative of $f(x) = 4x - x^2$. **3**
- (c) Simplify $\log_a b^2 \times \log_b a^3$. **1**
- (d) (i) Find the equation of the normal to the hyperbola $y = \frac{1}{x}$ at the point A where $x = 2$. **2**
(ii) The normal at A meets the hyperbola again at B . Find the co-ordinates of M the mid-point of interval AB . **3**
(iii) How many times does the circle with diameter AB meet this hyperbola? You must justify your answer. **1**

————— End of Section II —————

END OF EXAMINATION

B L A N K P A G E



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- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

NAME:

CLASS: MASTER:

Question One

A B C D

Question Two

A B C D

Question Three

A B C D

Question Four

A B C D

Question Five

A B C D

Question Six

A B C D

Question Seven

A B C D

Question Eight

A B C D

Question Nine

A B C D