NAME _____ MASTER _____

SYDNEY GRAMMAR SCHOOL

2015 Annual Examination

FORM V

MATHEMATICS 2 UNIT

Wednesday 26th August 2015

General Instructions

- Writing time — 2 hours
- Write using black or blue pen.
- Board-approved calculators and templates may be used.

Total — 100 Marks

- All questions may be attempted.

Section I – 9 Marks

- Questions 1–9 are of equal value.
- Record your answers to the multiple choice on the sheet provided.

Section II – 91 Marks

- Questions 10–16 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your name, class and Master on each answer booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Write your name, class and Master on this question paper and hand it in with your answers.
- Place everything inside the answer booklet for Question Ten.

5A: DS	5B: RCF	5C: SO	5D: DNW
5E: DWH	5F: REJ	5G: SJE	5H: KWM
5P: NL	5Q: TCW	5R: LRP	

Checklist

- SGS booklets — 7 per boy
- Multiple choice answer sheet
- Candidature — 185 boys

Examiner

NL

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

What is the length of the interval joining the points $A(-2,-4)$ and $B(3,8)$?

(A) 5

(B) $\sqrt{119}$

(C) 12

(D) 13

QUESTION TWO

What are the solutions to the quadratic equation $2(x-1)(x+3)=0$?

(A) $x=-1$ or $x=3$

(B) $x=-1$ or $x=-3$

(C) $x=1$ or $x=-3$

(D) $x=1$ or $x=3$

QUESTION THREE

Which of the following is an equivalent expression to $\dfrac{4\sqrt{x}}{x^{\frac{3}{2}}}$?

(A) $\dfrac{\sqrt{x}}{4}$

(B) $\dfrac{4}{x}$

(C) $\dfrac{1}{4x}$

(D) $\dfrac{4}{\sqrt[3]{x}}$

QUESTION FOUR

Which of the following is the correct statement of the product rule used to differentiate $f(x)=uv$?

(A) $f'(x)=vu'-uv'$

(B) $f'(x)=uv'-vu'$

(C) $f'(x)=vu'+uv'$

(D) $f'(x)=\dfrac{vu'+uv'}{v^2}$

Exam continues next page ...

QUESTION FIVE

Which of the following statements is always true?

(A) The derivative of a linear function is zero.

(B) The derivative of a quadratic function is a parabola.

(C) The derivative of a cubic function is a linear function.

(D) The derivative of a linear function is a constant.

QUESTION SIX

What is the perpendicular distance from the point $(2, -3)$ to the line $x + 2y - 5 = 0$?

(A) $\dfrac{9}{\sqrt{5}}$

(B) $\dfrac{13}{\sqrt{5}}$

(C) $\dfrac{13}{5}$

(D) $\dfrac{9}{5}$

QUESTION SEVEN

The circle with radius 6 and centre $(-3, 4)$ is shifted 2 units to the right and 3 units up. The equation of the shifted circle is:

(A) $(x-1)^2 + (y-7)^2 = 36$

(B) $(x+1)^2 + (y-1)^2 = 36$

(C) $(x+5)^2 + (y+1)^2 = 36$

(D) $(x+1)^2 + (y-7)^2 = 36$

QUESTION EIGHT

Which of the following is the derivative of $4x^5 - \dfrac{3}{2}x^2 + 7x - 10$?

(A) $4x^4 - \dfrac{3}{2}x + 7$

(B) $20x^4 - 3x + 7$

(C) $20x^4 - 6x^2 + 7$

(D) $\dfrac{2}{3}x^6 - \dfrac{1}{2}x^3 + \dfrac{7}{2}x^2 - 10x$

Exam continues overleaf ...

QUESTION NINE

Which one of the following expressions is equivalent to $\dfrac{\sec\theta}{\operatorname{cosec}\theta}$?

(A) $\tan\theta$

(B) $\cos\theta$

(C) $\sin\theta$

(D) $\cot^2\theta$

End of Section I

Exam continues next page ...

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION TEN (13 marks) Use a separate writing booklet. **Marks**

(a) State the equation of the axis of symmetry of the parabola with equation $y = x^2 + 4x + 1$. **1**

(b) Evaluate $\log_2 32$. **1**

(c) Consider the function $f(x) = 2x^3$. Find:

(i) $f(2)$. **1**

(ii) $f'(2)$. **2**

(d) What is the angle of inclination of the line $y = -2x + 1$? Express your answer correct to the nearest degree. **2**

(e) Rationalise the denominator of $\dfrac{2}{3+\sqrt{2}}$. Express your answer in simplest form. **2**

(f) Consider the arithmetic sequence $4, 7, 10, 13, \ldots$

(i) Find the common difference d. **1**

(ii) Find the two hundredth term, T_{200}. **1**

(iii) Find the sum of the first two hundred terms, S_{200}. **2**

Exam continues overleaf ...

QUESTION ELEVEN (13 marks) Use a separate writing booklet. **Marks**

(a) Evaluate the expression $|x-2|$ when $x=-10$. **1**

(b) State the domain of $y=\sqrt{x-3}$. **1**

(c) Simplify $5^{4n}\times 25^{2n}\times 125^{3n}$. **2**

(d) Using knowledge of the discriminant, show that $x^2-3x-1=0$ has two distinct real solutions. **2**

(e) Solve $3^{2x-1}=\dfrac{1}{27}$. **2**

(f) Differentiate:

(i) $y=\dfrac{3}{4}x^4$ **1**

(ii) $y=\dfrac{5}{x^3}$ **1**

(g) Let α and β be the roots of the equation $x^2+5x-12=0$. Without solving the equation find:

(i) $\alpha+\beta$ **1**

(ii) $\alpha^2\beta+\alpha\beta^2$ **2**

QUESTION TWELVE (13 marks) Use a separate writing booklet. **Marks**

(a) Consider the parabola with equation $y=-x^2-4x+6$.

(i) State the y-intercept. **1**

(ii) By completing the square, express the equation in the form $y=a(x-h)^2+k$. **2**

(iii) State the coordinates of the vertex. **1**

(iv) Find the x-intercepts, expressing each in exact form. **2**

(v) Sketch the graph of $y=-x^2-4x+6$, clearly marking the intercepts and vertex. **2**

(vi) Hence solve the inequation $-x^2-4x+6\geq 0$. **1**

(b) (i) Solve $\log_3 x=\dfrac{3}{2}$, leaving your answer as a simplified surd. **2**

(ii) Solve $\sin\theta=0.4$, for $0^\circ\leq\theta\leq 360^\circ$. Express your solutions correct to the nearest minute. **2**

Exam continues next page ...

QUESTION THIRTEEN (13 marks) Use a separate writing booklet. **Marks**

(a) Solve $x^2 + 6x - 4 = 0$. Express your answer in exact form. **2**

(b) Evaluate $\sum_{b=2}^{5}(-2)^b$. **2**

(c) Sketch the graph of $\log_2 x$, showing clearly any asymptotes or intercepts. **2**

(d) (i) Given that $y = 3^{-x} - 2$, show that $x = -\log_3 2$ when $y = 0$. **1**

(ii) Sketch the graph of $y = 3^{-x} - 2$, showing clearly any asymptotes or intercepts. **2**

(e) Given $f(x) = -3x^3 + x$, determine whether $f(x)$ is odd, even or neither. **2**

(f) Differentiate $y = (2x^3 + 7)^4$. **2**

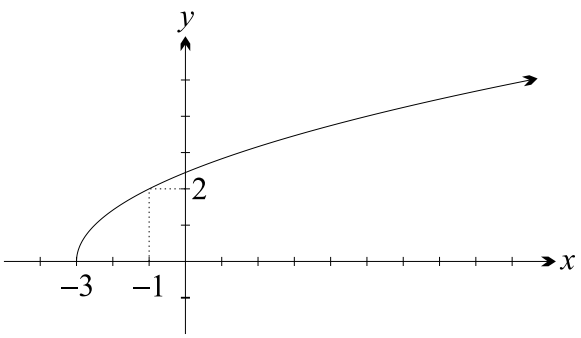
QUESTION FOURTEEN (13 marks) Use a separate writing booklet. **Marks**

(a) It is known that in triangle ABC, $\angle B = 27^\circ$, $b = 4\,\text{cm}$ and $c = 5\,\text{cm}$, where sides a, b and c are opposite angles A, B and C respectively. **2**

Find the possible sizes of angle C. Express your answers correct to one decimal place.

(b) Express $2\log_{10} x - log_{10}7$ as a single logarithm. **2**

(c)



The diagram shows the graph of $y = \sqrt{2x+6}$.

(i) Differentiate $y = \sqrt{2x+6}$ with respect to x. **1**

(ii) Find the equation of the tangent to $y = \sqrt{2x+6}$ at the point $A(-1, 2)$. **2**

(iii) Find the equation of the normal to $y = \sqrt{2x+6}$ at the point $B(5, 4)$. **2**

(iv) Find the acute angle formed at the intersection of the lines found in parts (ii) and (iii). Express your answer to the nearest minute. **2**

(d) If $\cos\theta = \dfrac{2}{5}$ and θ is a reflex angle, find $\tan\theta$. Leave your answer in exact form. **2**

Exam continues overleaf ...

QUESTION FIFTEEN (13 marks) Use a separate writing booklet. **Marks**

(a) Fred, an elderly gardener, has 250 kg of soil to shovel. He shovels 64 kg during the first hour, 48 kg the next hour and 36 kg in the third hour. Fred continues in this pattern and is determined to shovel all 250 kg of soil before taking a break.

(i) How much soil will Fred shovel during the 4th hour? **1**

(ii) How much soil will Fred have shovelled in total by the end of the seventh hour? **2**

(iii) Will Fred ever shovel all 250 kg of soil? Give a reason for your answer. **2**

(b) Differentiate $f(x) = \dfrac{x}{x^2+1}$ expressing your answer as a simplified fraction. **2**

(c) Differentiate $f(x) = (3x+1)^{\frac{3}{2}}(2x+4)$ expressing your answer in fully factorised form. **3**

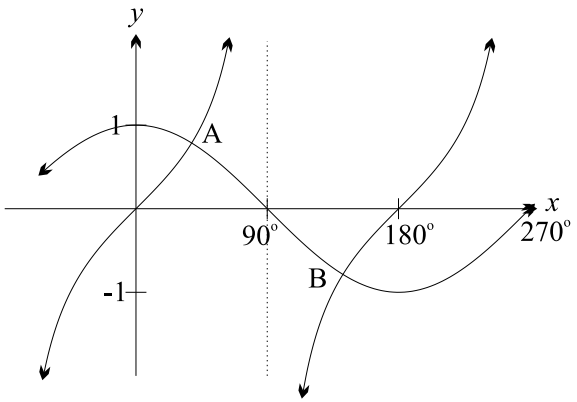
(d) Find the value of c such that $y = -10x+c$ is a tangent to the parabola $y = 3x^2+2x-4$. **3**

QUESTION SIXTEEN (13 marks) Use a separate writing booklet. **Marks**

(a) Differentiate $f(x) = \dfrac{1}{x}$ from first principles. **3**

(b) The quadratic equation $2x^2 - 9x + k = 0$ has two roots, α and β. One root is five times the other. Find the value of k. **3**

(c)

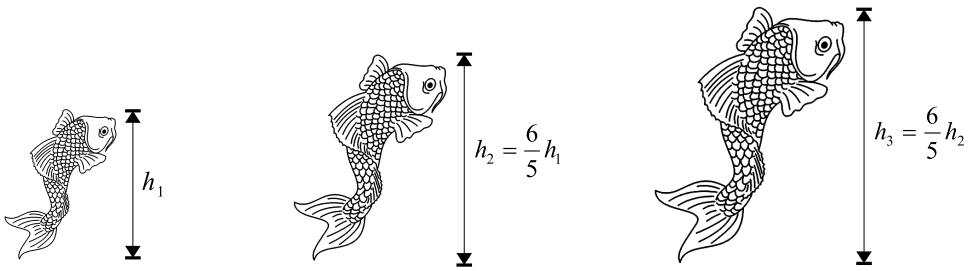


The diagram above shows the graphs of $y = \cos x$ and $y = \tan x$ for $0^\circ \leq x \leq 180^\circ$.

Find the coordinates of the intersection points A and B. Express your answers correct to 1 decimal place. **4**

Exam continues next page ...

(d) An artist is making a fountain out of solid bronze sculptures modelled on fish. Each fish-sculpture will be the same shape but of a different size.



As shown in the diagram above, the height of each sculpture is six-fifths the height of the preceeding sculpture. Only one sculpture of each size is made and the smallest one requires a volume of $20\,\text{cm}^3$ of molten bronze.
How many fish can be made with six litres of molten bronze? **3**

End of Section II

END OF EXAMINATION

B L A N K P A G E

NAME: .....

CLASS: .....MASTER: .....

SYDNEY GRAMMAR SCHOOL

2015
Annual Examination
FORM V
MATHEMATICS 2 UNIT
Wednesday 26th August 2015

- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

Question One

A ○ B ○ C ○ D ○

Question Two

A ○ B ○ C ○ D ○

Question Three

A ○ B ○ C ○ D ○

Question Four

A ○ B ○ C ○ D ○

Question Five

A ○ B ○ C ○ D ○

Question Six

A ○ B ○ C ○ D ○

Question Seven

A ○ B ○ C ○ D ○

Question Eight

A ○ B ○ C ○ D ○

Question Nine

A ○ B ○ C ○ D ○

M.C.

1.) $A(-2,-4)$ $B(3,8)$.

$l = \sqrt{(3+2)^2 + (8+4)^2}$

$l = 13$

(D)

2.) $x = 1$ and $x = -3$

~~(D)~~ (C)

3.) $\frac{4x^{1/2}}{x^{3/2}} = \frac{4}{x}$

(B)

4.) (C)

5.) (D)

6.) $d = \frac{|2-6-5|}{\sqrt{4+1}}$

$d = \frac{9}{\sqrt{5}}$

$a=1 \quad b=2 \quad c=-5$

$x_1 = 2 \quad y_1 = -3$

(A)

~~5~~ 7.) $(-3,4) \longrightarrow (-1,7)$ $(x+1)^2 + (y-7)^2 = 36$

(D)

8.) $20x^4 - 3x + 7$ (B)

9.) $\frac{1}{\cos\theta} \div \frac{1}{\sin\theta} = \tan\theta$ (A).

10) a) $x = \frac{-b}{2a}$

$= \frac{-4}{2}$

$x = -2$

b) 5.

c) $f(x) = 2x^3$. i) $f(2) = 16$.

ii) $f'(x) = 6x^2$ ✓

$f'(2) = 24$ ✓

d) $m = -2$, $\tan\alpha = 2$, $\alpha = 63.43$ } Any ✓

$\therefore \theta = 180 - 63.43$

$\theta = 116.6^\circ$ ✓

e) $\frac{2(\sqrt{3} - \sqrt{2})}{(\sqrt{3} + \sqrt{2})(3 - \sqrt{2})}$ ✓

$= \frac{2(3 - \sqrt{2})}{9 - 2}$

$= \frac{2(3 - \sqrt{2})}{7}$ ✓

f) i) $d = 3$

ii) $T_n = a + (n-1)d$

$T_{200} = 199 \times 3 + 4$

$= 601$ ✓

iii) $S_n = \frac{1}{2}n(a + l)$

$= \frac{1}{2} \times 200 \times (4 + 601)$ ✓

$= 60,500$ ✓

11.) a) $|-10-2|-4 = 8$

b) $x \geq 3$

c) $5^{4n} \times 5^{4n} \times 5^{9n} = 5^{17n}$ ✓ ✓

d) $a=1 \quad b=-3 \quad c=-1$

$b^2-4ac > 0$ ✓

either

$(-3)^2 - 4\times1\times-1 > 0$

$13 > 0$ ✓

e) $3^{2x-1} = 3^{-3}$ ✓

$2x-1 = -3$

$2x = -2$

$\underline{x = -1}$ ✓

f) i) $y' = \frac{3}{2} \times 5x^{1/2}$

$= \frac{15}{2}x^{1/2}$ ✓

ii) $y = 5x^{-3}$

$y' = -15x^{-4}$

$= \frac{-15}{x^4}$

g) $\alpha+\beta = -5 \qquad \alpha\beta = -12.$

(i) $\alpha+\beta = -5$

(ii) $-5\times-12 = 60$ ✓

(1 mark for $\alpha\beta(\alpha+\beta) = \alpha^2\beta + \alpha\beta^2$ in any form). ✓

12.) a) i) 6

ii) $y = -((x+2)^2 - 10)$

$y = -(x+2)^2 + 10$

iii) Vertex $(-2, 10)$

iv) $-(x+2)^2 + 10 = 0$

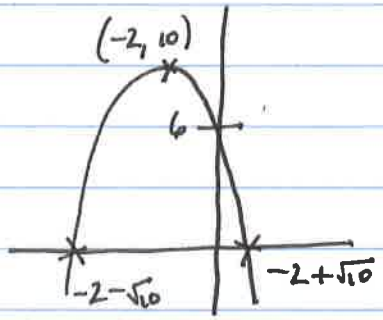
$(x+2)^2 = 10$

$x+2 = \pm\sqrt{10}$

$x = -2 \pm \sqrt{10}$

$x = -2 + \sqrt{10}$ and $x = -2 - \sqrt{10}$

v)



vi) $-2 - \sqrt{10} \leq x \leq -2 + \sqrt{10}$

or

$-5.16 \leq x \leq 1.16$

} either. Don't penalise rounding

b) i) $x = 3^{3/2}$ ✓

$x = \sqrt{3^2 \times 3}$

$x = 3\sqrt{3}$ ✓

ii) $\sin\theta = 0.4$

$\theta = 23.6^\circ, 156.4^\circ$ ✓✓ (one each)

13.

a) $x = \dfrac{-6 \pm \sqrt{36 - 4 \times 1 \times -4}}{2}$

$x = \dfrac{-6 \pm \sqrt{52}}{2}$

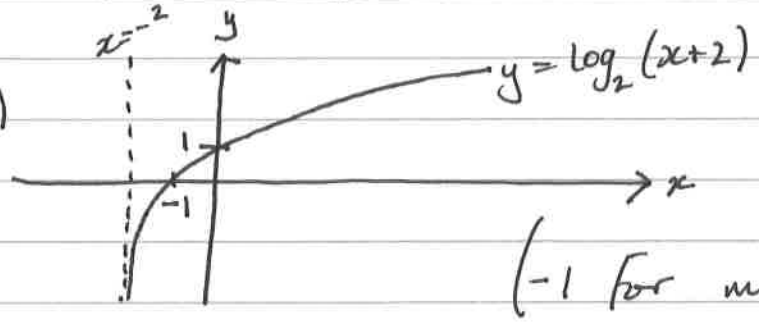
$x = \dfrac{-6 \pm 2\sqrt{13}}{2}$

$x = -3 \pm \sqrt{13}$

b) $\sum_{2}^{5} (-2)^b = (-2)^2 + (-2)^3 + (-2)^4 + (-2)^5$ ✓

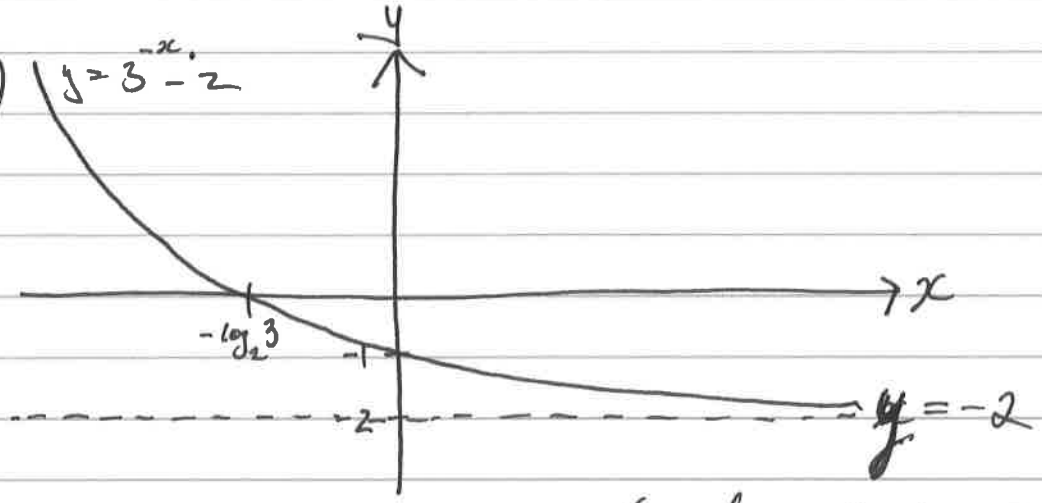
$= -20$ ✓

c) i)



(-1 for missing intercept/asymptote)

ii)



(-1 for missing intercept/asymptote).

iii) Range: $y > -2$

d) $f(x) = -3x^3 + x$

$-f(x) = 3x^3 - x$

$f(-x) = 3x^3 - x$

$f(-x) = -f(x)$ $\therefore$ odd.

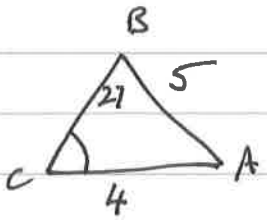
e) $y = (2x^3 + 7)^4$

$y' = 4(2x^3 + 7)^3 \times 6x^2$

one mark for $24x^2$ or $(2x^3+7)^3$

$y' = 24x^2(2x^3 + 7)^3$

(14) a)



$\frac{\sin C}{5} = \frac{\sin 27}{4}$ ✓

$C = \sin^{-1}\left(\frac{5\sin 27}{4}\right)$

$C = 34.575°\ldots$

$c = 34.6°$

or $c = 180 - 34.575°$

$= 145.4°$

} Both answers needed for 2nd mark.

b) $\log_2 12^2 + \log_2 5^3 - \log_2 15 - \log_2 150$

$= \log_2\left(\frac{12^2 \times 5^3}{15 \times 150}\right)$ ✓

$= \log_2 8$

$= 3$ ✓

c) i) $y = (2x+6)^{1/2}$

$y^{-1/2}$

$y' = \frac{1}{2}(2x+6)^{-1/2} \times 2$

$y' = \frac{1}{\sqrt{2x+6}}$ (✓) A(−1, 2) $\quad m = \frac{1}{\sqrt{-2+6}} = \frac{1}{2}$ (✓)

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{2}(x+1)$

$2y - 4 = x + 1$

$x - 2y + 5 = 0$ or $y = \frac{1}{2}x + \frac{5}{2}$ (✓)

ii) $y' = \frac{1}{\sqrt{2x+6}}$ $\quad B(5, 4)$

$m_2 = \frac{1}{\sqrt{16}}$

$m_2 = \frac{1}{4}$ $\quad$ Normal: $m_3 = -4$

$y - y_1 = -4(x - x_1)$

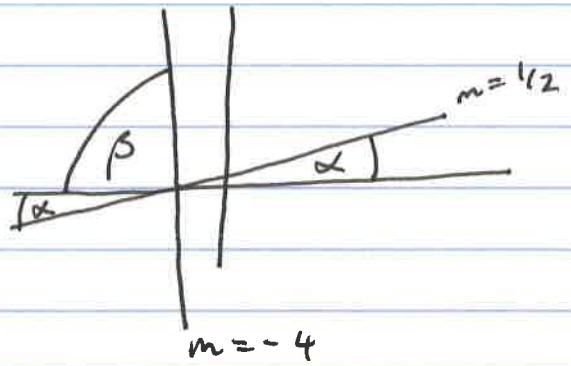
$y - 4 = -4(x - 5)$

$y - 4 = -4x + 20$

$y = -4x + 24$ $\quad$ or $\quad 4x + y - 24 = 0$

etc...

iii) $m = \frac{1}{2}$ $\quad m_3 = -4$

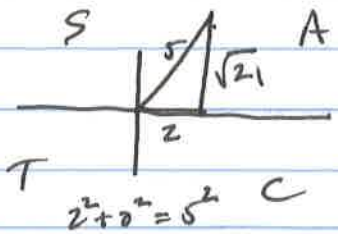


$\tan\alpha = \frac{1}{2}$ $\quad \tan\beta = 4$

$\alpha = 26.565°$ $\quad \beta = 75.9637°$

$180 - (\alpha + \beta) = 77.47°$ (to 2dp)

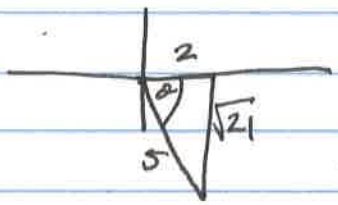
iv) $\cos\theta = \frac{2}{5} = \frac{A}{H}$



$2^2 + o^2 = 5^2$ C

$o = \sqrt{21}$ ✓

θ reflex $\therefore$ 4th quadrant



$\tan\theta = -\frac{\sqrt{21}}{2}$ ✓

(15). 250 kg.

i) 64, 48, 36, 27, ...

27 kg.

ii) Geometric, $r = \frac{3}{4}$ $a = 64$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_7 = \frac{64(1-(3/4)^7)}{1/4}$$

$$S_7 = 221.828125$$
$$= 221.83 \text{ kg}.$$

iii) $S_\infty = \frac{a}{1-r} = \frac{64}{1/4} = 256$.

$250 < 256$ ∴ yes, he will shovell all 250 kg.

b) i) $f(x) = \frac{x}{x^2+1}$ $u = x$ $v = x^2+1$

$u' = 1$ $v' = 2x$.

$$f'(x) = \frac{vu' - uv'}{v^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{-x^2-1}{(x^2+1)^2}$$

$$= \frac{-(x^2+1)}{(x^2+1)^2}$$

$$= -\frac{1}{x^2+1}$$

15b) ii) $f(x) = (3x+1)^{3/2}(2x+4)$

$u = (3x+1)^{3/2}$ $\quad v = 2x+4$

$u' = \frac{3}{2}(3x+1)^{1/2} \times 3$ $\quad v' = 2$

$u' = \frac{9}{2}(3x+1)^{1/2}$

$f'(x) = vu' + uv' = \frac{9}{2}(3x+1)^{1/2}(2x+4) + 2(3x+1)^{3/2}$

$= (3x+1)^{1/2}\left(\frac{9}{2}(2x+4) + 2(3x+1)\right)$

$= (3x+1)^{1/2}(9x+18+6x+2)$

$= (3x+1)^{1/2}(15x+20)$

$= 5(3x+1)^{1/2}(3x+4)$

c) $y = 3x^2 + 2x - 4$

$y' = 6x + 2.$

$-10 = 6x + 2$

$x = -2$ $\quad y = 3(-2)^2 + (2x-2) - 4$

$y = 12 - 4 - 4$

$y = 4$

$(-2, 4)$ in $y = -10x + c$

$4 = -10(-2) + c$

$c = -16$ $\quad y = -10x - 16$

16

a) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$

$= \lim_{h \to 0} \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h}$

$= \lim_{h \to 0} \frac{\frac{-h}{x(x+h)}}{h}$

$= \lim_{h \to 0} \frac{-1}{x(x+h)}$

$f'(x) = -\frac{1}{x^2}$

b) $2x^2 - 9x + k = 0$

$x^2 - \frac{9}{2}x + \frac{k}{2} = 0$ $\quad \alpha + \beta = \frac{9}{2}$

$\alpha\beta = \frac{k}{2}$

$5\alpha = \beta$ $\quad \therefore \quad 6\alpha = \frac{9}{2}$

$\alpha = 3/4$

$\beta = 15/4$

$\frac{k}{2} = \frac{3}{4} \times \frac{15}{4}$

$k = \frac{3}{2} \times \frac{15}{4}$

$k = 45/8$.

16) c) $\frac{h_2}{h_1} = \frac{6}{5}$ $\frac{V_2}{V_1} = \left(\frac{6}{5}\right)^3$ $r = \frac{216}{125}$ ✓

$a = 20\ cm^3$ Geometric sequence

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}$$

$6L = 6000\ cm^3$.

$$6000 = \frac{20\left(\left(\frac{216}{125}\right)^n - 1\right)}{\frac{91}{125}}$$ ✓

$$\frac{6000 \times 91}{20 \times 125} + 1 = \left(\frac{216}{125}\right)^n$$

$$219.4 = \left(\frac{216}{1.25}\right)^n$$

Trial and error: $\left(\frac{216}{125}\right)^4 = 8.9$

$\left(\frac{216}{125}\right)^7 = 46.005$

$\left(\frac{216}{125}\right)^9 = 137.37$

$\left(\frac{216}{125}\right)^{10} = 237.376$

$\therefore \quad n = 9$ ✓

16d) $y = \cos x$ and $y = \tan x$. $0° \leq x \leq 180°$

$\cos x = \tan x$.

$\cos x = \frac{\sin x}{\cos x}$

$\cos^2 x = \sin x$ $\quad$ $\sin^2 x + \cos^2 x = 1$

$1 - \sin^2 x = \sin x$ $\quad$ $\cos^2 x = 1 - \sin^2 x$ ✓

$0 = \sin^2 x + \sin x - 1$

$a = \sin x$

$0 = a^2 + a - 1$

$\sin x = \frac{-1 \pm \sqrt{1 - 4 \times 1 \times -1}}{2}$

$\sin x = \frac{-1 \pm \sqrt{5}}{2}$

$\sin x = \frac{-1 + \sqrt{5}}{2}$ $\quad$ $\sin x = \frac{-1 - \sqrt{5}}{2}$.

This solution is invalid.

$\sin \neq -1.6$.

$x_1 = 38.1727$ ✓

$x_2 = 90 + (90 - 38.1727)$

$= 141.8273$ ✓

~~$\sin(141.8273) =$~~

$\therefore$ ~~A(~~

$\cos(38.1727) = 0.78615146$.

$\cos(141.8273) = -0.78615146$.

$\therefore$ A(38.17, 0.79)

B(141.83, −0.79) to 2 d.p. ✓