
CANDIDATE NUMBER

SYDNEY GRAMMAR SCHOOL

2019 Annual Examination

FORM V

MATHEMATICS ADVANCED

Wednesday 4th September 2019

General Instructions

- Writing time — 2 hours
- Write using black pen.
- NESA-approved calculators may be used.

Total — 80 Marks

- All questions may be attempted.

Section I — 8 Marks

- Questions 1–8 are of equal value.
- Record your answers to the multiple choice on the sheet provided.

Section II — 72 Marks

- Questions 9–14 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your candidate number on each answer booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Write your candidate number on this question paper and hand it in with your answers.
- Place everything inside the answer booklet for Question Nine.

Checklist

- SGS booklets — 6 per boy
- Multiple choice answer sheet
- Reference Sheet
- Candidature — 177 boys

Examiner

REJ

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

Which of the following expressions is equivalent to $\left(3\sqrt{5}-5\right)^2$?

(A) 20

(B) $70-30\sqrt{5}$

(C) $20-30\sqrt{5}$

(D) 70

QUESTION TWO

It is known that 14 apples and 4 oranges cost \$8, and 5 apples and 4 oranges cost \$4.40. If $\$x$ represents the cost of an apple and $\$y$ represents the cost of an orange, which of the following pairs of simultaneous equations could be used to find the cost of each apple and orange?

(A) $4x+14y=8$ and $5x+4y=4{\cdot}4$

(B) $14x+4y=8$ and $5x+4y=4{\cdot}4$

(C) $14x+4y=8$ and $4x+5y=4{\cdot}4$

(D) $4x+14y=8$ and $4x+5y=4{\cdot}4$

QUESTION THREE

Which expression is equivalent to $\log_a 2m-\log_a m$?

(A) $\log_a m$

(B) $\dfrac{\log_a 2m}{\log_a m}$

(C) $\log_a 2m$

(D) $\log_a 2$

QUESTION FOUR

Which of the following is the derivative of e^{2x}?

(A) $2e^{2x-1}$

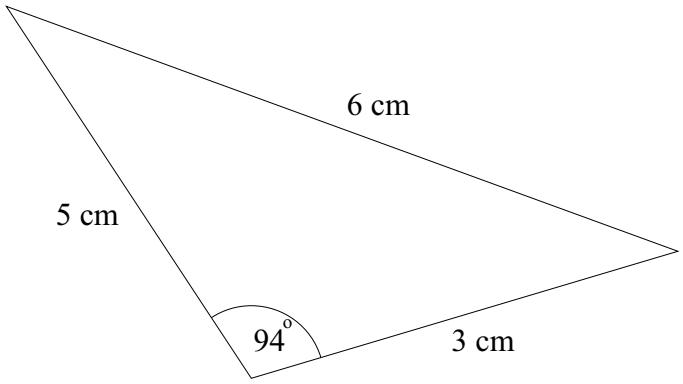
(B) $2e^{2x}$

(C) $2xe^{2x-1}$

(D) e^{2x}

Examination continues next page ...

QUESTION FIVE



Which expression gives the correct area of the triangle above?

(A) $\frac{1}{2} \times 5 \times 3 \times \cos 94^\circ$

(B) $\frac{1}{2} \times 5 \times 6 \times \cos 94^\circ$

(C) $\frac{1}{2} \times 5 \times 3 \times \sin 94^\circ$

(D) $\frac{1}{2} \times 5 \times 6 \times \sin 94^\circ$

QUESTION SIX

What is the natural domain of the function $f(x) = \dfrac{1}{\sqrt{4 - x^2}}$?

(A) $-2 \leq x \leq 2$

(B) $x \leq -2$ or $x \geq 2$

(C) $-2 < x < 2$

(D) $x < -2$ or $x > 2$

QUESTION SEVEN

Each of the following experiments involves two events A and B. In which case are the events A and B independent?

(A) $P(A|B) = 0{\cdot}5$ and $P(A) = 0{\cdot}4$ and $P(B) = 0{\cdot}5$

(B) $P(A) = 0{\cdot}3$ and $P(B) = 0{\cdot}7$ and $P(A \cap B) = 0{\cdot}21$

(C) $P(A|B) = \frac{3}{4}$ and $P(A) = \frac{2}{5}$ and $P(B) = \frac{3}{10}$

(D) $P(A \cap B) = \frac{1}{5}$ and $P(A) = \frac{1}{5}$ and $P(B) = \frac{2}{3}$

Examination continues overleaf ...

QUESTION EIGHT

Let $f(x) = x^2 + 3x - 4$ and $g(x) = |x|$. What is the correct expression for $f\big(g(x)\big)$?

(A) $f\left(g(x)\right) = \begin{cases} x^2 + 3x - 4, & \text{for } x \geq 0, \\ x^2 - 3x - 4, & \text{for } x < 0 \end{cases}$

(B) $f\left(g(x)\right) = \begin{cases} x^2 - 3x - 4, & \text{for } x \geq 0, \\ x^2 + 3x - 4, & \text{for } x < 0 \end{cases}$

(C) $f\left(g(x)\right) = \begin{cases} x^2 + 3x - 4, & \text{for } x \leq -4 \text{ or } x \geq 1, \\ -x^2 - 3x + 4, & \text{for } -4 < x < 1 \end{cases}$

(D) $f\left(g(x)\right) = \begin{cases} -x^2 - 3x + 4, & \text{for } x \leq -4 \text{ or } x \geq 1, \\ x^2 + 3x - 4, & \text{for } -4 < x < 1 \end{cases}$

End of Section I

Examination continues next page ...

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION NINE (12 marks) Use a separate writing booklet. **Marks**

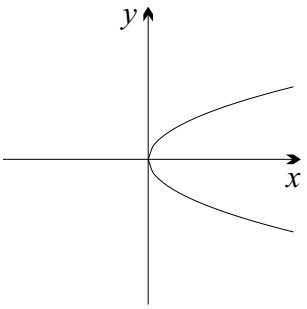
(a) Consider the function $f(x) = x^2 - 2x + 3$.

(i) Find the value of $f(2)$. **1**

(ii) Find a simplified expression for $f(a+2)$. **1**

(iii) Find a simplified expression for $f(a) + 2$. **1**

(b) **1**

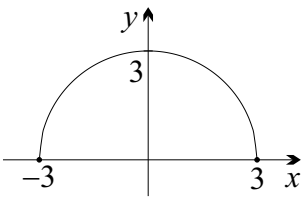


Classify the graph shown above as one-to-one, many-to-one, one-to-many or many-to-many.

(c) Rationalise the denominator of $\dfrac{2}{\sqrt{5}-2}$. **1**

(d) Solve $|2x - 1| = 11$. **2**

(e) **2**



The diagram above shows the graph of the semicircle $y = \sqrt{9 - x^2}$. State the domain and range of this function.

Examination continues overleaf ...

QUESTION NINE (Continued)

(f) A bag contains 2 red discs and 3 white discs. Two discs are drawn from the bag without replacement. What is the probability that both discs are red? **1**

(g) Find the exact value of:

(i) $\sin \frac{2\pi}{3}$ **1**

(ii) $\cos \frac{5\pi}{4}$ **1**

Examination continues next page ...

QUESTION TEN (12 marks) Use a separate writing booklet. **Marks**

(a) Find the derivative of each of the following functions.

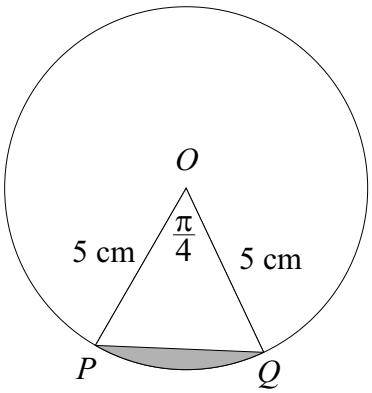
(i) $y = 2x^3$ **1**

(ii) $y = \dfrac{1}{x^2}$ **1**

(iii) $y = (3x + 2)^4$ **1**

(iv) $y = \dfrac{x^3}{3x + 2}$ **2**

(b)



The diagram above shows a circle with centre O, radius 5 cm and $\angle POQ = \frac{\pi}{4}$. The points P and Q lie on the circumference of the circle.

(i) Find the exact area of the sector OPQ. **1**

(ii) Find the exact area of $\triangle OPQ$. **1**

(iii) Hence find the exact area of the shaded minor segment. **1**

(c) Find the equation of the tangent to the curve $y = e^{3x+1}$ at the point $(0, e)$. **2**

(d) Solve $\cos x = -\dfrac{\sqrt{3}}{2}$, for $0 \leq x \leq 2\pi$. **2**

Examination continues overleaf ...

QUESTION ELEVEN (12 marks) Use a separate writing booklet. **Marks**

(a) Sketch graphs of the following functions on separate number planes, showing all intercepts with the axes and other important features.

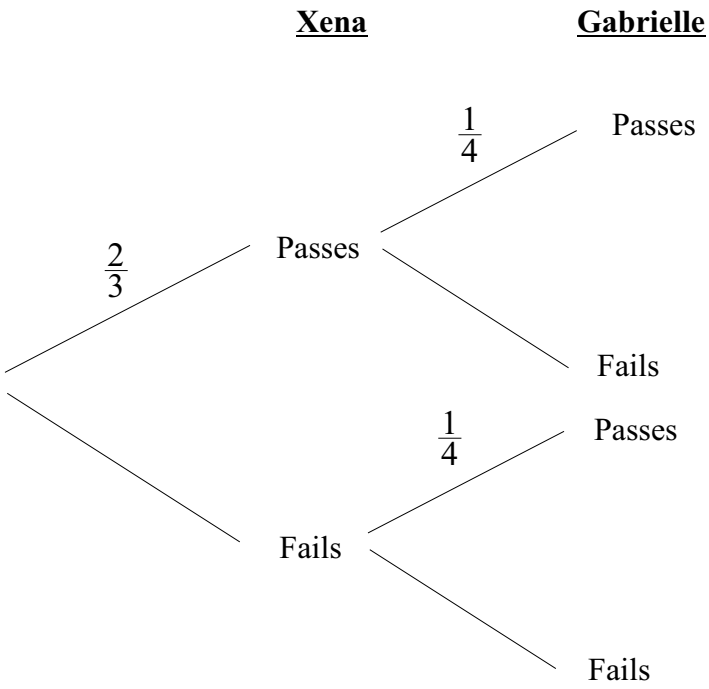
(i) $y = 2^x - 4$ **2**

(ii) $y = \dfrac{1}{x-3}$ **2**

(iii) $y = \log_3 x$ **2**

(iv) $y = x(x-1)(x+1)(x+3)$ **2**

(b) Xena and Gabrielle and are going to sit their driving test. The probability that Xena passes her driving test is $\frac{2}{3}$. The probability that Gabrielle passes her driving test is $\frac{1}{4}$. Part of the probability tree diagram for this scenario is shown below.



(i) Copy and complete the tree diagram showing the probabilities on each branch. **1**

(ii) Write a list of possible outcomes. **1**

(iii) What is the probability that only one of the girls passes their driving test? **2**

Examination continues next page ...

QUESTION TWELVE (12 marks) Use a separate writing booklet. **Marks**

(a) If $\sin\theta = -\dfrac{2}{5}$ and $\cos\theta > 0$, find the exact value of $\tan\theta$. **2**

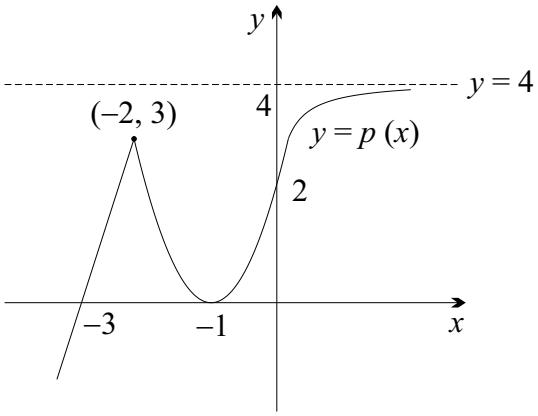
(b) Consider the function $f(x) = x^2 + 7x - 10$.

(i) Write down an expression for $f(x+h)$. **1**

(ii) Differentiate $f(x)$ from first principles to show that $f'(x) = 2x + 7$. **2**

(iii) What is the gradient of the normal to $y = f(x)$ at the point $(1, -2)$? **1**

(c) **2**



The diagram above shows an unknown function $y = p(x)$.

The function $y = p(x)$ is transformed to give $y = -p(-x)$. Sketch $y = -p(-x)$ showing any intercepts with the axes and other important features.

(d) If $x = \log_a 3$, $y = \log_a 5$ and $z = \log_a 7$, write the expression $\log_a\left(\frac{75}{49a}\right)$ in terms of x, y and z only. **2**

(e) Determine whether the function $f(x) = \dfrac{x^3 + 3x}{x^4 + x^2 - 1}$ is even, odd or neither. You must show all working. **2**

Examination continues overleaf ...

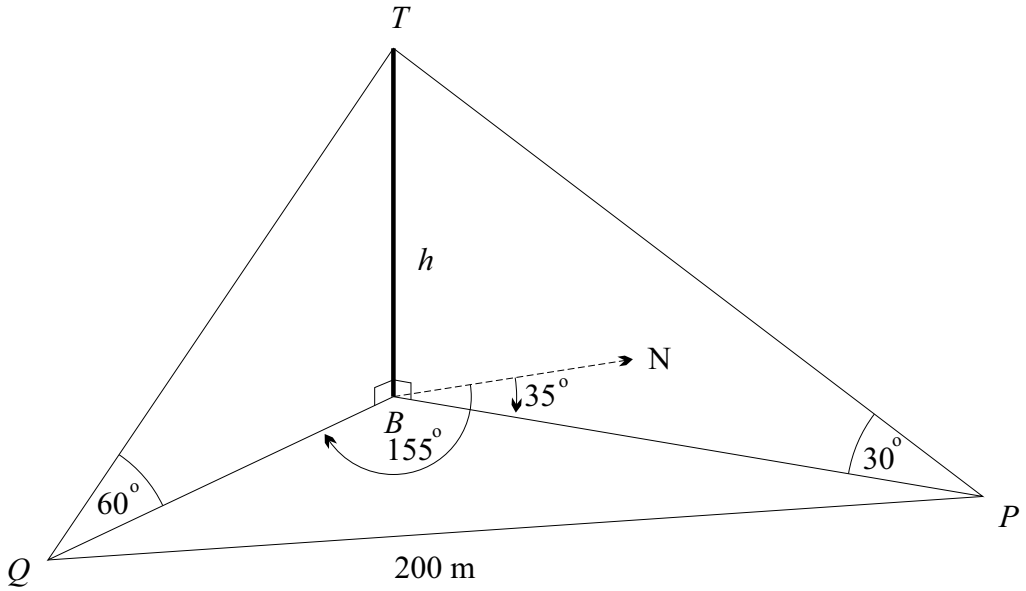
QUESTION THIRTEEN (12 marks) Use a separate writing booklet. **Marks**

(a) On any given day in Sydney, the probability that it is raining is $\frac{1}{3}$, the probability that there is heavy traffic is $\frac{7}{12}$ and the probability that there is heavy traffic or it is raining is $\frac{2}{3}$.

(i) Find the probability that it is raining and there is heavy traffic. **1**

(ii) Find the probability that there is heavy traffic given that it is raining. **2**

(b)



A vertical tower is observed from two landmarks P and Q on level ground. From P, the angle of elevation to the top of the tower T is 30°. From Q, the angle of elevation to the top of the tower T is 60°. From the base of the tower B, P is on a bearing of 035°T and Q is on a bearing of 155°T. It is known that P and Q are 200 m apart. Let h represent the height of the tower BT.

(i) Explain why $\angle PBQ = 120°$. **1**

(ii) Show that $BP = \sqrt{3}h$ and write a similar expression for BQ. **2**

(iii) Show that $\frac{13h^2}{3} = 200^2$ and hence find the height of the tower h. Give your answer correct to the nearest metre. **2**

Examination continues next page ...

QUESTION THIRTEEN (Continued)

(c) In January 2018, a farmer accidentally spread a dangerous chemical on a paddock. The concentration of the chemical in the soil was initially measured to be 7 kg/ha. One year later, the concentration was found to be 3·2 kg/ha.

It is known that the concentration C of the chemical in kg/ha is given by the formula $C = C_0 e^{-kt}$ where t is the time in years after the chemical was spread and C_0 is a constant.

(i) Find the values of C_0 and k, giving your answers in exact form. **2**

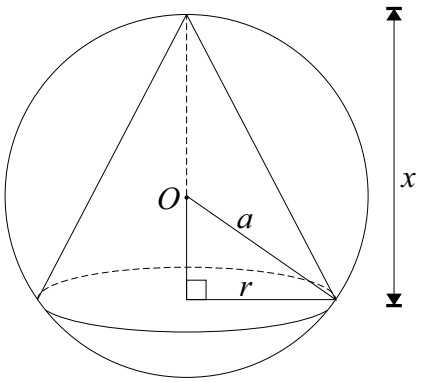
(ii) It is safe to use the paddock when the concentration falls below 0·2 kg/ha. During which year will it first become safe for the farmer to use the paddock again? **2**

Examination continues overleaf ...

QUESTION FOURTEEN (12 marks) Use a separate writing booklet. **Marks**

(a) Prove that $\dfrac{(1-\sin\theta)(1+\sin\theta)}{\cos^2\theta} = 1$. **2**

(b)



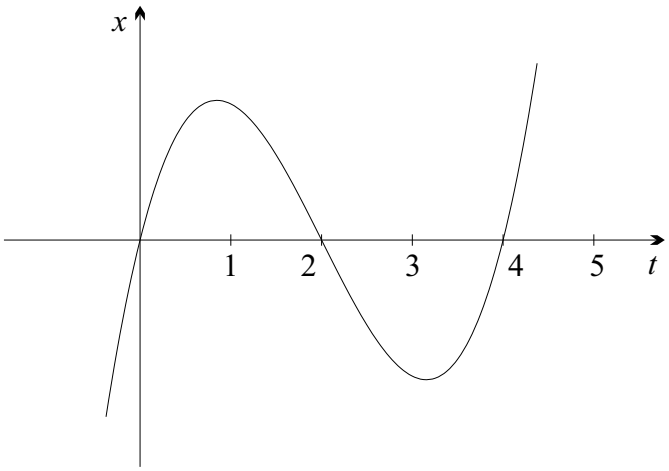
The diagram above shows a cone inscribed in a sphere. The sphere has radius a and centre O. The cone has a base of radius r and height x.

(i) Show that $r^2 = 2ax - x^2$. **1**

(ii) The volume of a cone is given by the formula $V = \dfrac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height of the cone. Show that the volume of this cone is given by $V = \dfrac{1}{3}\pi\left(2ax^2 - x^3\right)$. **1**

(iii) Find the value of x so that $\dfrac{dV}{dx} = 0$, given $0 < x < 2a$. **2**

(c) **2**



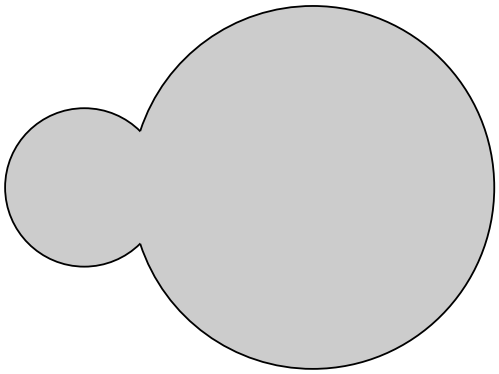
The graph above shows the function $x = f(t)$.

Sketch a possible graph of $\dfrac{dx}{dt}$ as a function of t.

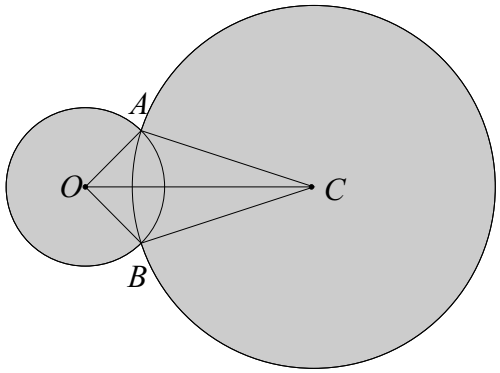
Examination continues next page ...

QUESTION FOURTEEN (Continued)

(d)



An advertising logo is formed from two circles which intersect as shown in the diagram above.



The circles intersect at A and B and have centres at O and C. The diagram above shows the radii OA, OB, CA and CB of each circle. The radius of the circle centred at O is $0{\cdot}5$ m and the radius of the circle centred at C is $\dfrac{\sqrt{3}}{2}$ m. The length of OC is 1 m.

(i) Show that $\angle OAC = \dfrac{\pi}{2}$. **1**

(ii) Find the area of the quadrilateral $AOBC$. **1**

(iii) Find the total area of the logo. Give your answer in exact form. **2**

End of Section II

END OF EXAMINATION

B L A N K P A G E

B L A N K P A G E

B L A N K P A G E

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- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

Question One

A ◯ B ◯ C ◯ D ◯

Question Two

A ◯ B ◯ C ◯ D ◯

Question Three

A ◯ B ◯ C ◯ D ◯

Question Four

A ◯ B ◯ C ◯ D ◯

Question Five

A ◯ B ◯ C ◯ D ◯

Question Six

A ◯ B ◯ C ◯ D ◯

Question Seven

A ◯ B ◯ C ◯ D ◯

Question Eight

A ◯ B ◯ C ◯ D ◯

Mathematics Advanced Form V Solutions

Question 1

$(3\sqrt{5} - 5)^2$

$= 45 - 30\sqrt{5} + 25$

$= 70 - 30\sqrt{5}$ (B)

Question 2

$14x + 4y = 8$

$5x + 4y = 4 \cdot 4$ (B)

Question 3

$\log_a 2m - \log_a m$

$= \log_a \frac{2m}{m}$

$= \log_a 2$ (D)

Question 4

$\frac{d}{dx} e^{2x} = 2e^{2x}$ (B)

Question 5

$A = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 3 \times 5 \sin 94$ (C)

Question 6

$4 - x^2 > 0$

$(2 - x)(2 + x) > 0$

$-2 < x < 2$ (C)

Question 7

either $P(A|B) = P(A)$

or $P(A \cap B) = P(A) \times P(B)$

A X

B → $P(A \cap B) = P(A) \times P(B)$

C → X

D X

Answer (B)

Question 8

$f(g(x)) = |x|^2 + 3|x| - 4$

$= x^2 + 3|x| - 4$

$= \begin{cases} x^2 + 3x - 4 & x \geq 0 \\ x^2 - 3x - 4 & x < 0 \end{cases}$

(A)

Question 9

a) $f(x) = x^2 - 2x + 3$

i) $f(2) = 2^2 - 2 \times 2 + 3$

$= 3$ ✓

ii) $f(a+2) = (a+2)^2 - 2(a+2) + 3$

$= a^2 + 4a + 4 - 2a - 4 + 3$

$= a^2 + 2a + 3$ ✓

ii) $f(a) + 2 = a^2 - 2a + 3 + 2$

$= a^2 - 2a + 5$ ✓

b) one-to-many ✓

c) $\frac{2(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)}$

$= \frac{2(\sqrt{5}+2)}{5-4}$

$= 2\sqrt{5} + 4$ or $2(\sqrt{5}+2)$ ✓

d) $|2x - 1| = 11$

$2x - 1 = 11$

$2x = 12$

$x = 6$

$2x - 1 = -11$

$2x = -10$

$x = -5$

$x = 6$ or $x = -5$

✓ ✓

e) Domain: $-3 \leq x \leq 3$ ✓

Range: $0 \leq y \leq 3$ ✓

f) $P(RR) = \frac{2}{5} \times \frac{1}{4}$

$= \frac{1}{10}$ ✓

g) i) $\sin \frac{2\pi}{3}$

$= \sin \pi/3$

$= \frac{\sqrt{3}}{2}$ ✓

ii) $\cos \frac{5\pi}{4}$

$= -\cos \pi/4$

$= -\frac{1}{\sqrt{2}}$ ✓

Question 10

a) i) $y = 2x^3$

$\frac{dy}{dx} = 6x^2$ ✓

ii) $y = x^{-2}$

$\frac{dy}{dx} = -2x^{-3}$ ✓

$= \frac{-2}{x^3}$

iii) $y = (3x+2)^4$

$\frac{dy}{dx} = 4(3x+2)^3 \times 3$ ✓

$= 12(3x+2)^3$

iv) $u = x^3$ $\quad v = 3x+2$

$u' = 3x^2$ $\quad v' = 3$

$v^2 = (3x+2)^2$

$\frac{dy}{dx} = \frac{3x^2(3x+2) - 3x^3}{(3x+2)^2}$ ✓

$= \frac{9x^3 + 6x^2 - 3x^3}{(3x+2)^2}$

$= \frac{6x^3 + 6x^2}{(3x+2)^2}$ ✓

b) i) $A = \frac{r^2\theta}{2}$

$= \frac{5^2 \times \pi/4}{2}$

$= \frac{25\pi}{8}$ ✓

ii) $A = \frac{r^2 \sin\theta}{2}$

$= \frac{5^2}{2} \times \sin \pi/4$

$= \frac{25}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{25\sqrt{2}}{4}$ ✓

iii) $A = \frac{25\pi}{8} - \frac{25\sqrt{2}}{4}$ ✓

$= \frac{25\pi - 50\sqrt{2}}{8}$

c) $y = e^{3x+1}$ $(0, e)$

$\frac{dy}{dx} = 3e^{3x+1}$

$m_T = 3e^{0+1}$

$= 3e$ ✓

$y - e = 3e(x - 0)$ ✓

$y = 3ex + e$

d) $\cos x = -\frac{\sqrt{3}}{2}$

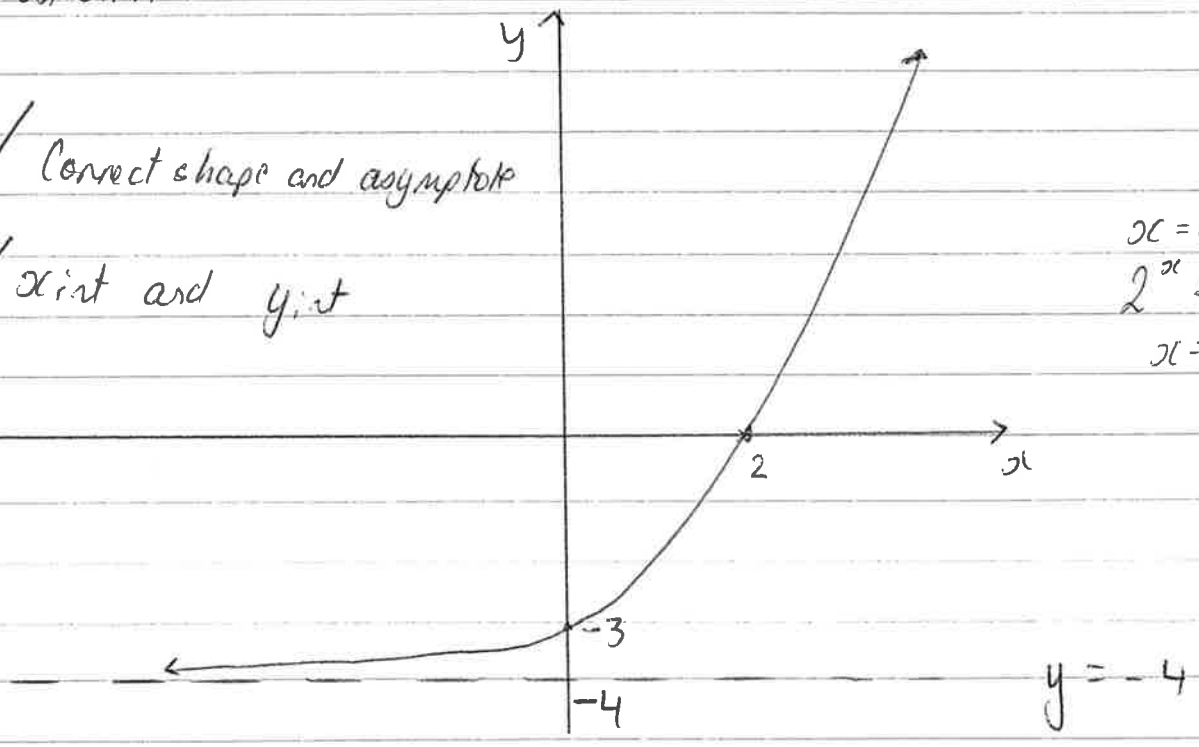
$\cos \alpha = \frac{\sqrt{3}}{2}$ α is acute

$\alpha = \frac{\pi}{6}$ ✓ @ 2 + 3

$x = \pi - \frac{\pi}{6}$ or $\pi + \frac{\pi}{6}$

$= \frac{5\pi}{6}$ or $\frac{7\pi}{6}$ ✓

Question 11



✓ Correct shape and asymptote

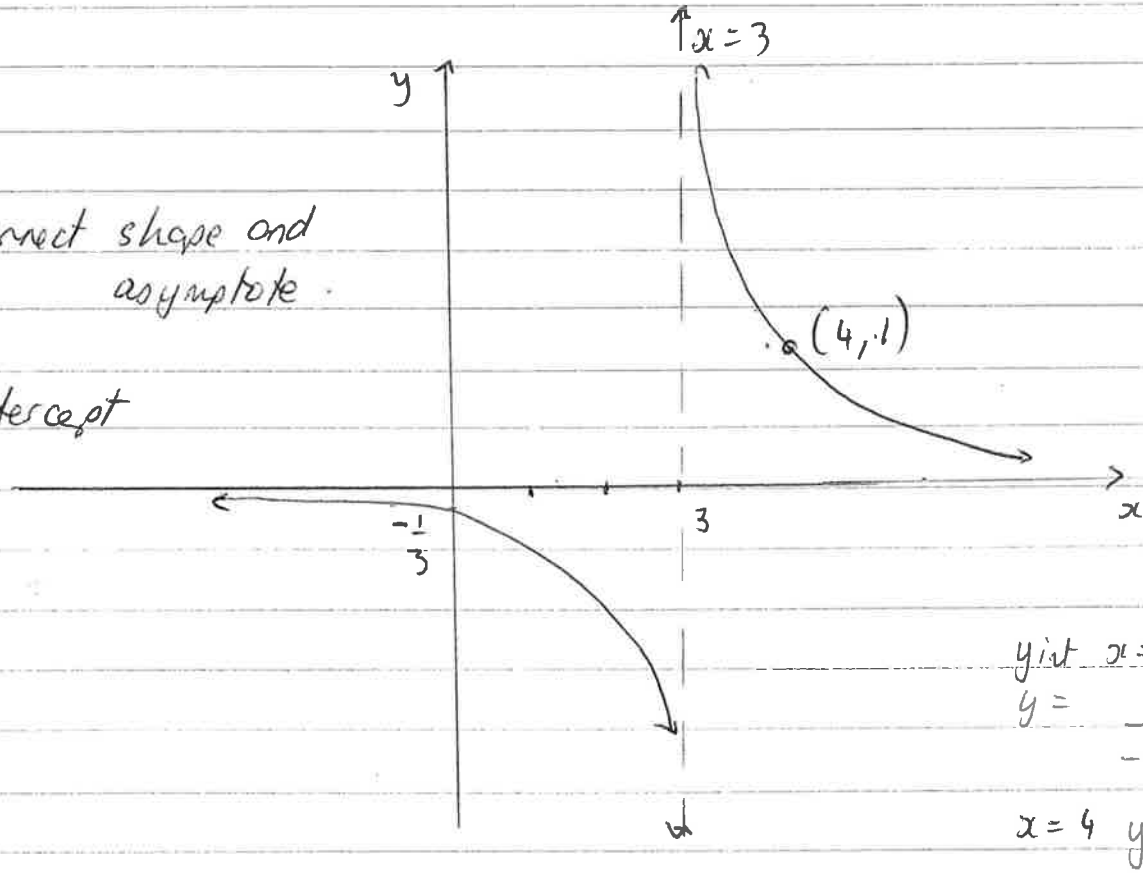
✓ x int and y int

$x = 0$

$2^x = 4$

$x = 2$

ii)



✓ correct shape and asymptote.

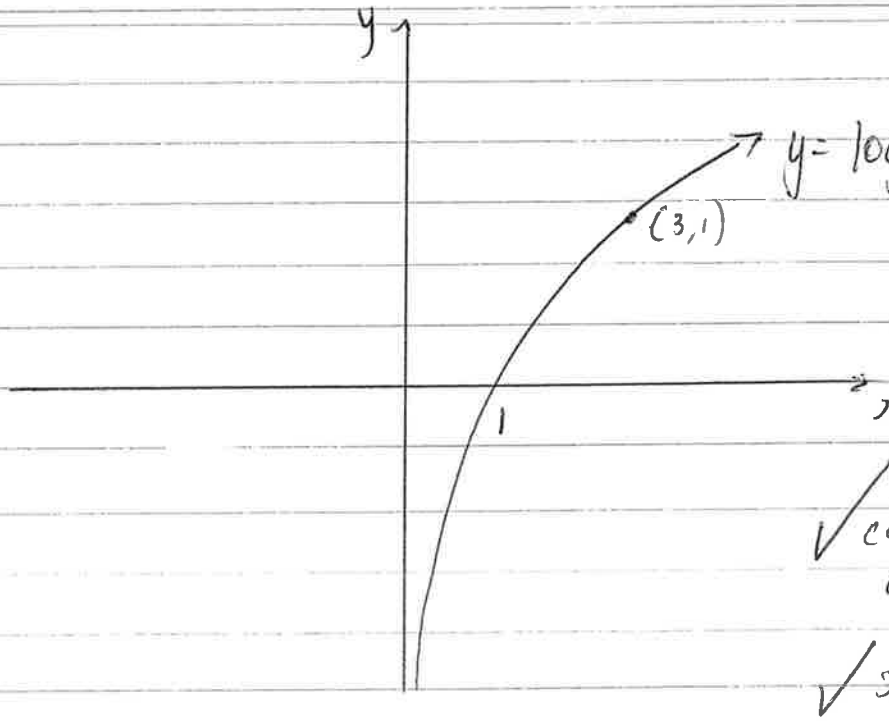
✓ y intercept

y int $x = 0$

$y = \frac{1}{-3}$

$x = 4 \quad y = 1$

iii)



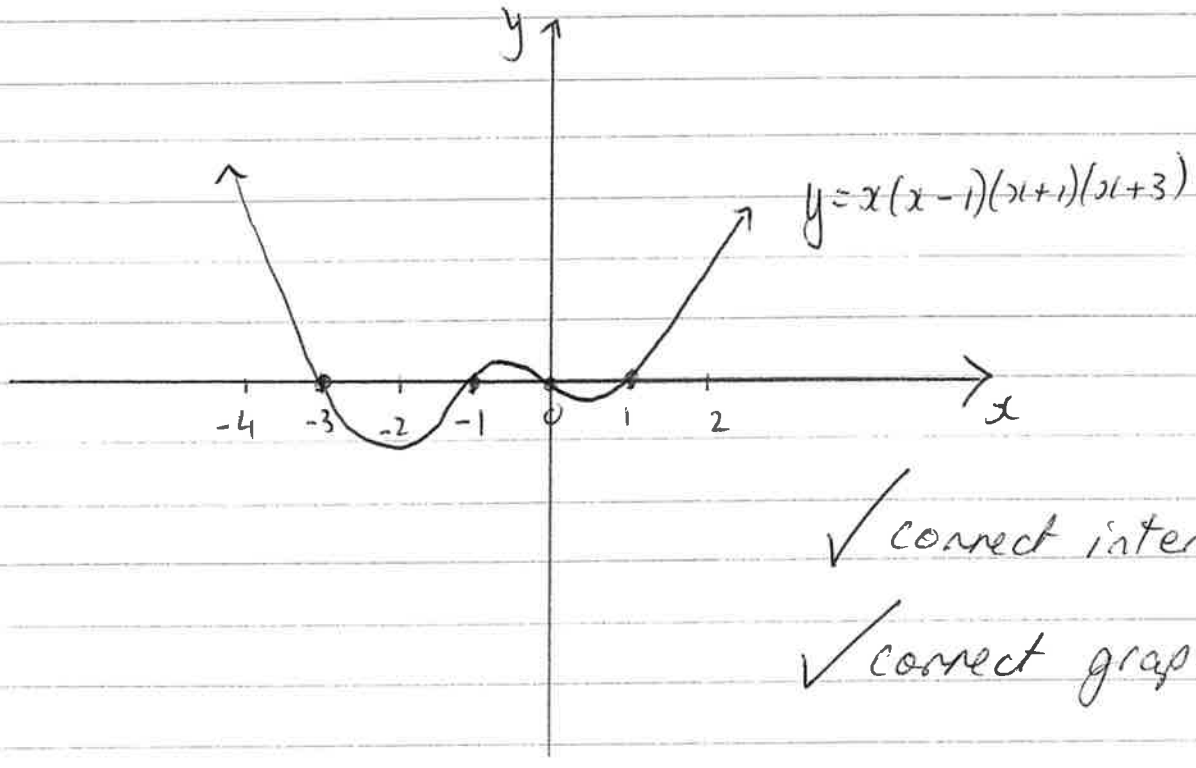
✓ correct shape and approaching y-axis

✓ x-intercept

iv)

x	-4	-3	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
y	$+$	0	$-$	0	$+$	0	$-$	0	$+$

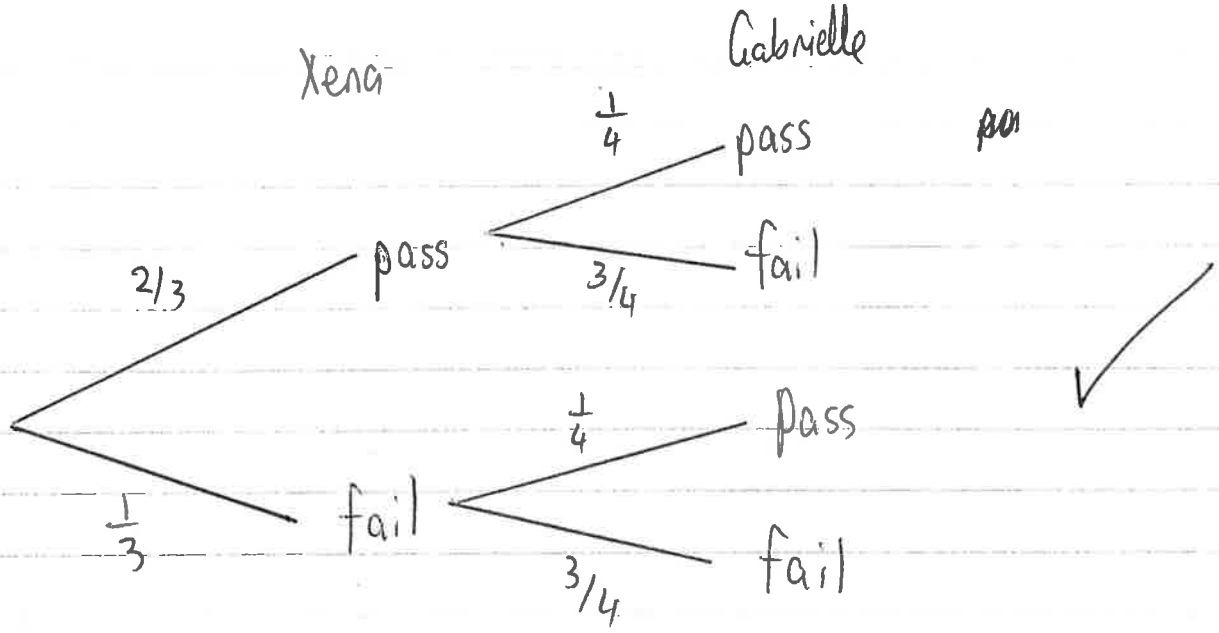
x int $x = 0, 1, -1, -3$ table of signs



✓ correct intercepts

✓ correct graph

b) i)



✓

ii)

Xena	Gabrielle
P	P
P	F
F	P
F	F

✓

iii) $P(\text{only one fails}) = P(PF) + P(FP)$

$= \frac{2}{3} \times \frac{3}{4} + \frac{1}{3} \times \frac{1}{4}$ ✓

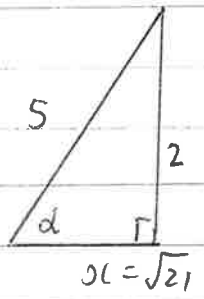
$= \frac{7}{12}$ ✓

Question 12

a) $\sin\theta = -\frac{2}{5}$ $\cos\theta > 0$

Quadrant 4

let $\sin\alpha = \frac{2}{5}$ α acute



$x^2 = 5^2 - 2^2$ (pythagoras' theorem)

$x^2 = 21$

$x = \sqrt{21}$ ✓

$\tan\theta = -\tan\alpha$

$= -\frac{2}{\sqrt{21}}$ ✓

$= -\frac{2\sqrt{21}}{21}$

b) i) $f(x) = x^2 + 7x - 10$ ✓

$f(x+h) = (x+h)^2 + 7(x+h) - 10$

$= x^2 + 2xh + h^2 + 7x + 7h - 10$

ii) $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 7x + 7h - 10 - x^2 - 7x + 10}{h}$ ✓

$= \lim_{h\to 0} \frac{2xh + h^2 + 7h}{h}$

$= \lim_{h \to 0} \quad 2x + h + 7$ ✓ or showing $\frac{h(2x+h+7)}{h}$ factorised

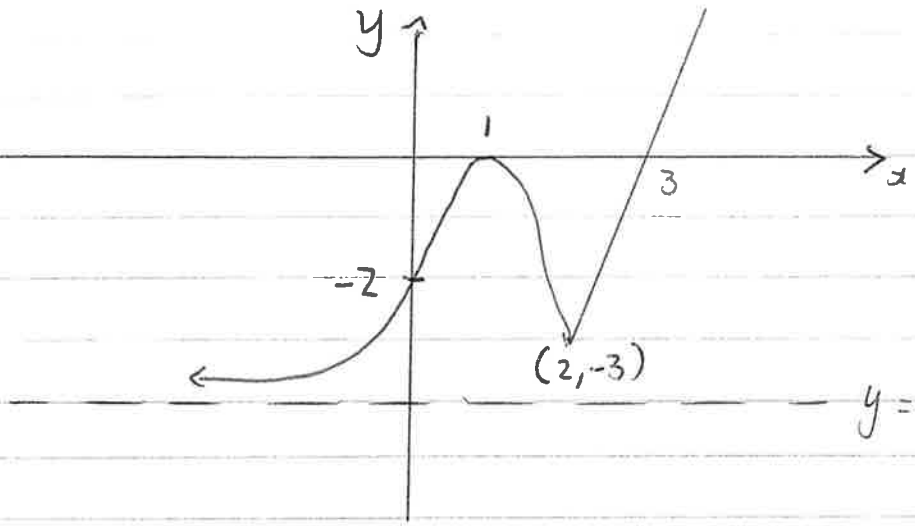
$= 2x + 7$

iii) $m_T = 2(1) + 7$

$= 9$

$m_N = -\frac{1}{9}$ ✓

c)



- x intercepts
- y intercepts
- sharp point
- correct orientation
- asymptote clearly shown.

✓ 2 of 5 features

✓ all 5 features

d) $\log_a \left(\frac{75}{49a}\right)$

$= \log_a 75 - \log_a 49a$

$= \log_a 3 + 2\log_a 5 - (\log_a 7^2 + \log_a a)$ ✓

$= \log_a 3 + 2\log_a 5 - 2\log_a 7 - 1$

$= x + 2y - 2z - 1$ ✓

e) $f(x) = \dfrac{x^3 + 3x}{x^4 + x^2 - 1}$

$f(-x) = \dfrac{(-x)^3 + 3(-x)}{(-x)^4 + (-x)^2 - 1}$

$= \dfrac{-x^3 - 3x}{x^4 + x^2 - 1}$ ✓ showing not even

$\neq f(x)$ So the function is not even

$-f(-x) = -\left(\dfrac{-x^3 - 3x}{x^4 + x^2 - 1}\right)$

Must explicitly show either multiplying $f(-x)$ by -1 or multiplying $f(x)$ by -1.

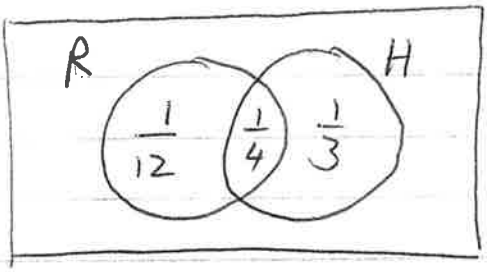
$= \dfrac{x^3 + 3x}{x^4 + x^2 - 1}$ ✓ Showing odd

$= f(x)$

Since $f(x) = -f(-x)$ the function is odd.

Question 13

a) i)



$$P(R \cap H) = \frac{1}{3} + \frac{7}{12} - \frac{2}{3}$$

$$= \frac{1}{4} \quad \checkmark$$

ii) $P(H/R) = \dfrac{P(H \cap R)}{P(R)}$ ✓

$$= \frac{\frac{1}{4}}{\frac{1}{3}}$$

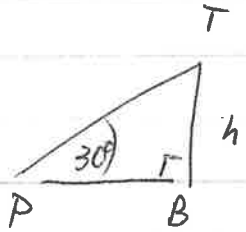
$$= \frac{3}{4} \quad \checkmark$$

✓

b) i) $\angle PBQ = 155° - 35°$ (adjacent angles)

$\angle PBQ = 120°$

ii) $\tan 30° = \dfrac{h}{BP}$ ✓



$$BP \times \frac{1}{\sqrt{3}} = h$$

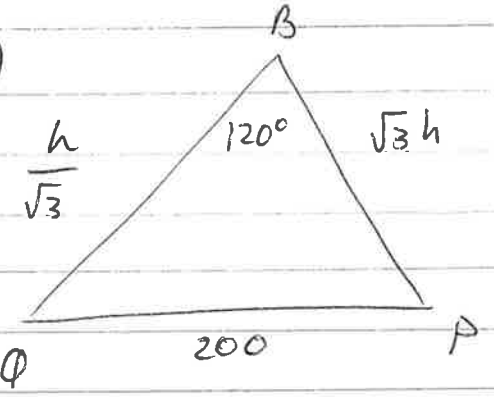
$BP = \sqrt{3}h$ as required

$$\tan 60^\circ = \frac{h}{BQ}$$

$$\sqrt{3} \times BQ = h$$

$$BQ = \frac{h}{\sqrt{3}}$$ ✓

iii)



$$200^2 = (\sqrt{3}h)^2 + \left(\frac{h}{\sqrt{3}}\right)^2 - 2 \times \sqrt{3}h \times \frac{h}{\sqrt{3}} \cos 120^\circ$$ ✓

$$200^2 = 3h^2 + \frac{h^2}{3} - 2h^2(-\cos 60)$$

$$200^2 = 3h^2 + \frac{h^2}{3} + h^2$$

$$200^2 = \frac{13h^2}{3}$$ as required

$$13h^2 = 120\,000$$

$$h^2 = \frac{120\,000}{13}$$

$$h \simeq 96.07689\ldots$$

$$h \simeq 96\text{m} \text{ (nearest metre)}$$ ✓

Question 13

c)i) $C = C_0 e^{-kt}$

$t = 0 \quad C = 7 \quad$ so $C_0 = 7$ ✓

$C = 7e^{-kt}$

$t = 1 \quad C = 3 \cdot 2$

$3 \cdot 2 = 7e^{-k}$

$\frac{16}{35} = e^{-k}$

$-k = \log_e \frac{16}{35}$

$k = -\log_e \frac{16}{35}$ or $k = \log_e \frac{35}{16}$ ✓

ii) $C = 7e^{\log_e \frac{16}{35} \times t}$

We want $C = 0 \cdot 2$

$0 \cdot 2 = 7e^{\log_e \frac{16}{35} \times t}$

$\frac{1}{35} = e^{\log_e \frac{16}{35} \times t}$ ✓ substitute & make progress

$\log_e \frac{1}{35} = \log_e \frac{16}{35} \times t$

$t = \frac{\log_e 1/35}{\log_e 16/35}$

$t \simeq 4 \cdot 54207\ldots$ Answer 2018+4

During the year 2022 ✓

Question 14

a) LHS = $\frac{(1-\sin\theta)(1+\sin\theta)}{\cos^2\theta}$

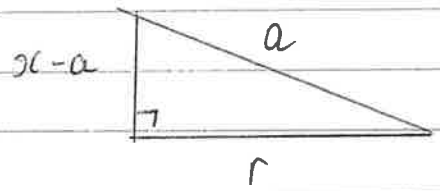
$= \frac{1-\sin^2\theta}{\cos^2\theta}$ ✓

$= \frac{\cos^2\theta}{\cos^2\theta}$ ✓

$= 1$

= RHS as required

b) i)



$r^2 + (x-a)^2 = a^2$ ✓ (pythagoras' theorem)

$r^2 + x^2 - 2ax + a^2 = a^2$

$r^2 = 2ax - x^2$ as required

ii) $A = \pi r^2$

$= \pi(2ax - x^2)$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(2ax - x^2)\,x$ ✓

$= \frac{1}{3}\pi(2ax^2 - x^3)$ as required ✓

iii) $\frac{dV}{dx} = \frac{1}{3}\pi(4ax - 3x^2)$ ✓

Solve $\frac{dV}{dx} = 0$

$\frac{1}{3}\pi(4ax - 3x^2) = 0$

$4ax - 3x^2 = 0$

$x(4a - 3x) = 0$

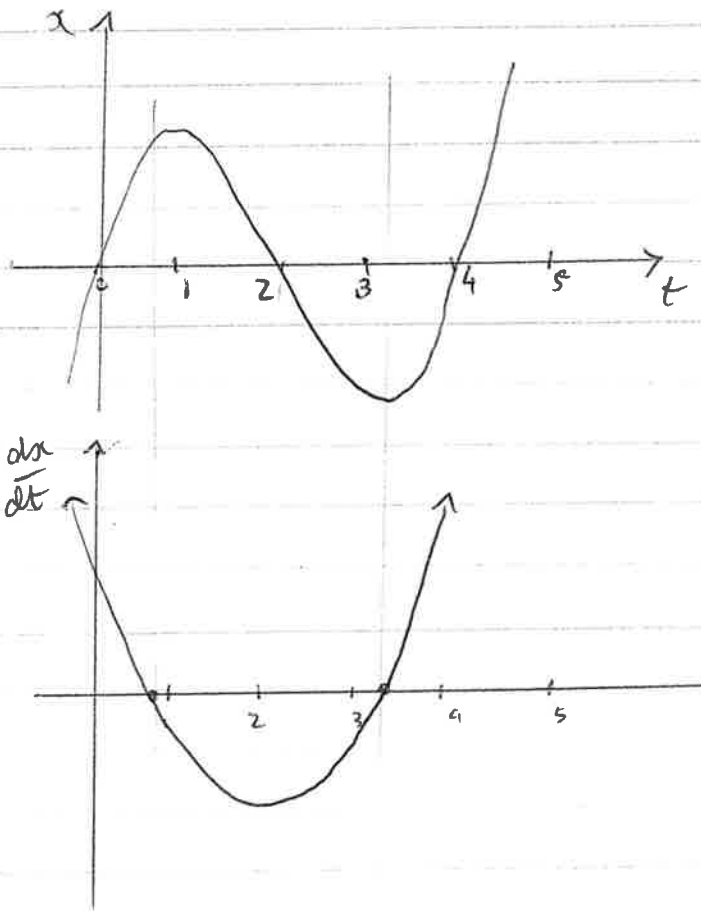
$x = 0$ or $3x = 4a$

$x = \frac{4a}{3}$ ✓

but $0 < x \leq 2a$

So $x = \frac{4a}{3}$ only.

c)



✓ shape

✓ intercepts.

(note x ints are slightly off $x=1$ & $x=3$)

d) i) $OA^2 + AC^2$

$= 0.5^2 + \left(\frac{\sqrt{3}}{2}\right)^2$ ✓

$= \frac{1}{4} + \frac{3}{4}$

$= 1$

$= OC^2$

Since pythagoras theorem holds, $\angle OAC$ is $\pi/2$

ii) Area $= 2 \times \frac{1}{2} \times OA \times AC$

$= OA \times AC$

$= \frac{1}{2} \times \frac{\sqrt{3}}{2}$ ✓

$= \frac{\sqrt{3}}{4}\ m^2$

iii) let $\angle OAC = \alpha$ and $\angle ACO = \theta$

$\sin \alpha = \frac{\frac{\sqrt{3}}{2}}{1}$

$\alpha = \frac{\pi}{3}$ So $\theta = \frac{\pi}{6}$ (angle sum $\triangle AOC$)

So $\angle AOB = 2\alpha = \frac{2\pi}{3}$ and $\angle ACB = \frac{\pi}{3}$

Reflex $\angle AOB = \frac{4\pi}{3}$ and reflex $\angle ACB = \frac{5\pi}{3}$

Area Sector $AOB = \frac{r^2\theta}{2}$

$= \left(\frac{1}{2}\right)^2 \times \frac{4\pi}{3} \times \frac{1}{2}$

$= \frac{1}{4} \times \frac{4\pi}{3} \times \frac{1}{2}$

$= \frac{\pi}{6}$ ✓ finding at least one reflex angle and correct area of sector

Area sector $ACB = \left(\frac{\sqrt{3}}{2}\right)^2 \times \frac{5\pi}{3} \times \frac{1}{2}$

$= \frac{3}{4} \times \frac{5\pi}{3} \times \frac{1}{2}$

$= \frac{5\pi}{8}$

Total Area $= \frac{\pi}{6} + \frac{5\pi}{8} + \frac{\sqrt{3}}{4}$

$= \frac{4\pi + 15\pi + 6\sqrt{3}}{24}$

$= \frac{19\pi + 6\sqrt{3}}{24}$ ✓