Name: .... Maths Class: ....

SYDNEY TECHNICAL HIGH SCHOOL

Year 11

Mathematics

Term 3 Examination

September 2002

TIME ALLOWED: 2 hours

Instructions:

- Write your name and class at the top of this page.
- At the end of the examination this examination paper must be attached to the front of your answers.
- All questions are of equal value and may be attempted.
- All necessary working must be shown. Marks will be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.

Q 1	Q 2	Q 3	Q 4	Q 5	Q 6	Q 7	Q 8	TOTAL
								/88

QUESTION 1: (11 MARKS)

(a) Fully factorise:

1 (i) $2x^2 + 5x - 3$

2 (ii) $a^2 - b^2 - a + b$

1 (b) Simplify $3\sqrt{8} - \sqrt{32}$

1 (c) Find the value, to 2 dec places, of $\cos^2 32°11'$

3 (d) If $f(x) = x^2 + 3$, find

(i) $f(-2)$ (ii) $\dfrac{f(x+h) - f(x)}{h}$

1 (e) Find the midpoint of the interval AB where A is (-2,3) and B is (6,-3)

2 (f) Rationalise the denominator of the following and simplify:

$$\frac{5\sqrt{3} - 1}{2 - \sqrt{3}}$$

QUESTION 2: (11 MARKS)

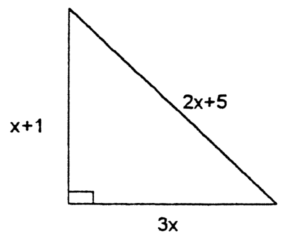
2 (a) Solve and plot the solution on a number line to

$$6 - x \geq -3$$

2 (b) Solve $(3x - 1)(2x + 5) = 0$

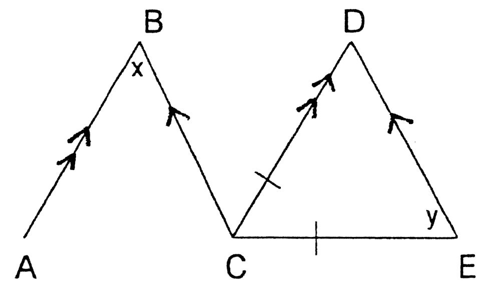
3 (c) Solve $2x^2 - 4x + 1 = 0$ giving your solutions correct to 2 decimal places.

4 (d) Form an equation to find the value of x in the following and then solve it to find x

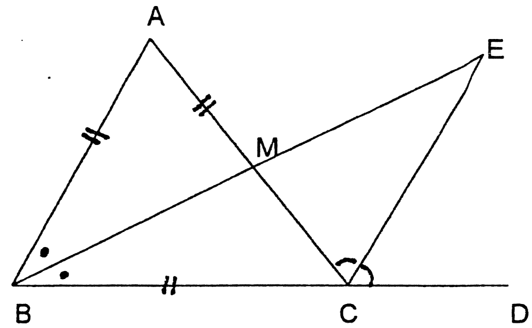


QUESTION 3: (11 Marks)

5 (a) In the diagram at right, AB // CD and BC // DE. ΔCDE is isosceles. Showing all steps and giving all reasons, show that x = y.



6 (b) In the diagram at right, ΔABC is equilateral. EB and EC bisect $\angle ABC$ and $\angle ACD$ respectively. Prove that BC=CE



QUESTION 4: (11 MARKS)

2 (a) State whether the function $f(x) = x^2 - 2$ is odd, even or neither. Give reasoning.

2 (b) If $g(x) = \frac{1}{x^2} - 4$ find the value(s) of x for which $g(x) = 0$

2 (c) Sketch the graph of $y = \sqrt{16 - x^2}$ showing all important features.

4 (d) Shade the region given by the simultaneous solution of:

$$\begin{cases} x^2 < y + 1 \\ y \le 3 \\ x \ge 0 \end{cases}$$

1 (e) Give the natural domain of:

$$y = \frac{1}{\sqrt{3 - x}}$$

QUESTION 5: (11 Marks)

3 (a) Find derivatives of:

(i) $4x^5 - 2x^2 + 3$

(ii) $\frac{5}{x^3}$

(iii) $(3x - 1)^4$

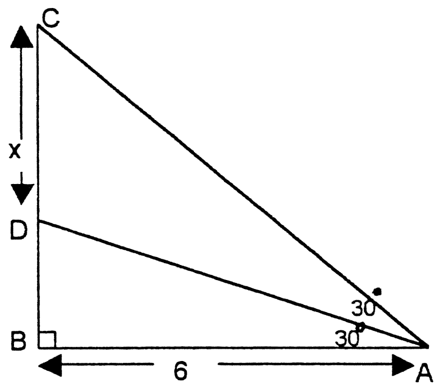
2 (b) Find the slope of the tangent to the curve $y = x^2(x - 3)$ at the point (3,0)

3 (c) Find the equation of the tangent to the curve $y = 4 - x^2$ at $x = 1$

3 (d) Differentiate $y = \frac{3x^2}{x + 5}$ and express your answer in simplest form

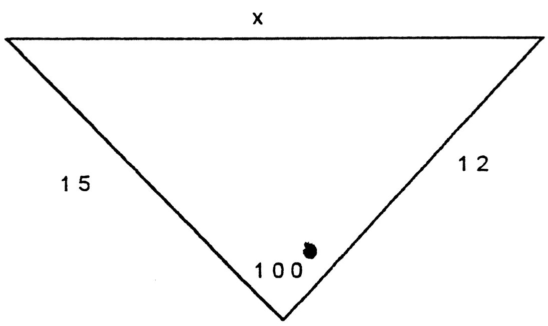
QUESTION 6: (11 marks)

3 (a)

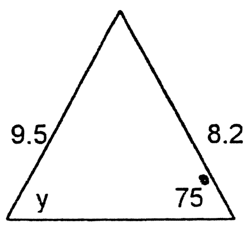


Show that $x = 4\sqrt{3}$

3 (b) Find the value of x to 2 decimal places in the following:



3 (c) In the following diagram, find the value of y to the nearest minute:



2 (d) If $\cos\theta = -5/13$ and $180° < \theta < 360°$, find $\tan\theta$ as a fraction.

QUESTION 7: (11 marks)

2 (a) The line $5x + ky = 4$ passes through the point (-2, 1). Find the value of k.

3 (b) Find the equation of the line through (-1, 4) and perpendicular to the line $3x + 4y = 5$

3 (c) Is the point (5, 4) closer to the line $x + y = 2$ or is the origin closer ? Give reasons.

3 (d) The points A(0,0), B(2,3), C(3,5) and D complete a parallelogram.

(i) Find the co-ordinates of the point D

(ii) Find the length of the diagonal BD.

QUESTION 8: (11 Marks)

1 (a) Find an expression for $1 - \sin^2 \theta$

5 (b) For the parabola $y = 3\cancel{x^2 + 5x -} 2$ find $3x^2 - 12x + 9$

(i) the intercepts on the x-axis

(ii) the y-intercept

(iii) the equation of the axis of symmetry

(iv) the co-ordinates of the vertex

2 (c) Find $\dfrac{d}{dx}(3\sqrt{x})$

3 (d) Sketch the graph of y = f(x) where f(x) is defined by:

$$f(x) = \begin{cases} 1 & x < 0 \\ 1 - x & 0 \le x \le 2 \\ 3 - x^2 & x > 2 \end{cases}$$

YEAR 11 2 UNIT YEARLY 2002

SOLUTIONS

1

(a) (i) $(2x-1)(x+3)$ 1

(ii) $(a+b)(a-b)-(a-b)$ 1

$= (a-b)(a+b-1)$. 1

(b) $6\sqrt{2} - 4\sqrt{2} = 2\sqrt{2}$ 1

(c) 0.72

(d) $f(x) = x^2+3$

(i) $f(-2) = 7$ 1

(ii) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{(x+h)^2+3-(x^2+3)}{h}$ 1

$= \dfrac{x^2+2xh+h^2+3-x^2-3}{h}$

$= 2x+h$

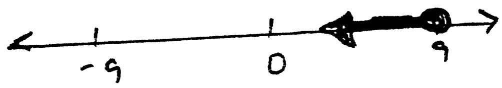
(e) $M = (2,0)$ 1

(f) $\dfrac{5\sqrt{3}-1}{2-\sqrt{3}} \times \dfrac{2+\sqrt{3}}{2+\sqrt{3}} = \dfrac{10\sqrt{3}-2+15-\sqrt{3}}{1}$ 1

$= 9\sqrt{3}+13$ 1

2

(a) $x \leq 9$ 2 (lose 1 for open circle)



(b) $x = \frac{1}{3}$ or $x = -\frac{5}{2}$

1 1

(c) $x = \dfrac{4 \pm \sqrt{16-8}}{4}$

$= \dfrac{2 \pm \sqrt{2}}{2}$ 2

(d) $(2x+5)^2 = (x+1)^2 + 9x^2$ 1

$4x^2+20x+25$

$= x^2+2x+1+9x^2$

$6x^2-18x-24=0$

$x^2-3x-4=0$ 1

$(x-4)(x+1)=0$

$x=4$ or $x=-1$ 1

ONLY SOLUTION is $x=4$ 1

3

(a) $\angle ABC = \angle BCD = x^\circ$ (alternate angles $AB \parallel CD$) 2

$\angle BCD = \angle CDE = x^\circ$ (alternate angles $BC \parallel DE$) 2

Because $\triangle CDE$ is isosceles, base angles are equal

$\therefore x = y$. 1

(b) Because $\triangle ABC$ is equilateral, 1

all angles are 60°

$\therefore \angle MBC = \angle ABM = 30^\circ$ (BE bisects $\angle ABC$) 1

also $\angle ACD = 120^\circ$ (supplementary to $\angle BCM$)

$\therefore \angle MCE = \angle ECD = 60^\circ$ (CE bisects $\angle MCD$) 1

$\therefore \angle BCE = \angle BCM + \angle MCE = 120^\circ$

$\therefore \angle MEC = 30^\circ$ (angle sum of $\triangle BCE$) 1

$\therefore$ Since $\angle MEC = \angle MBC = 30^\circ$

$\triangle BCE$ is isosceles 1

$\therefore BC = CE$ 1

(4) (a) $f(a) = a^2 - 2$

$f(-a) = (-a)^2 - 2$

$= a^2 - 2$ (1)

$\therefore f(a) = f(-a)$

$\therefore$ EVEN FN. (1)

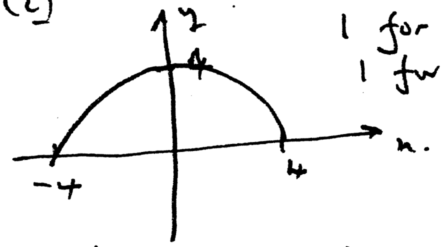
(b) $g(x) = 1/x^2 - 4$

$\therefore \frac{1}{x^2} = 4.$ (1)

$\therefore x^2 = 1/4$

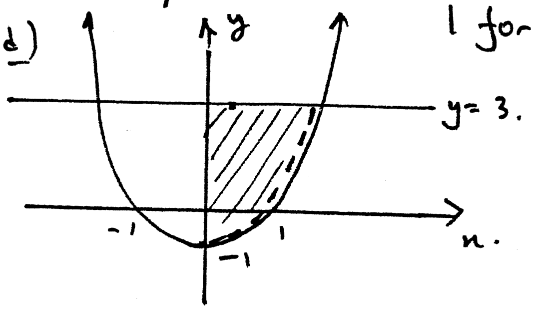
$\therefore x = \pm 1/2$ (1)

(c)



1 for semi-circle
1 for the "4"s

(d)



1 for each region Correct
1 for intersection

(e) $x < 3$ (1)

(no mark for $x \leq 3$)

(5) (a) (i) $20x^4 - 4x$ (1)

(ii) $-15/x^4$ (1)

(iii) $12(3x-1)^3$ (1)

(b) $y = x^3 - 3x^2$

$\frac{dy}{dx} = 3x^2 - 6x$ (1)

At (3, 0), $m_T = 27 - 18$

$= 9$ (1)

(c) $y = 4 - x^2$

$\frac{dy}{dx} = -2x$ (1)

At $x = 1$, $m_T = -2$. $y = 3$ (1)

Line is:

$y - 3 = -2(x-1)$

$y - 3 = -2x + 2$

$2x + y - 5 = 0$ (1)

(d) $y = \frac{3x^2}{x+5}$

$\frac{dy}{dx} = \frac{(x+5)6x - 3x^2 \cdot 1}{(x+5)^2}$ (1)

$= \frac{6x^2 + 30x - 3x^2}{(x+5)^2}$ (1)

$= \frac{3x^2 + 30x}{(x+5)^2}$ (1)

(6) (a) $BC/6 = \tan 60°$

$\therefore BC = 6\sqrt{3}$

$DB/6 = \tan 30°$

$\therefore DB = 6/\sqrt{3}$

$= \frac{6\sqrt{3}}{3}$

$= 2\sqrt{3}$

$\therefore x = 6\sqrt{3} - 2\sqrt{3}$

$= 4\sqrt{3}$

(b) $x^2 = 15^2 + 12^2 - 2.15.12.\cos 100°$

$x = 20.77$ (1) [1 off for not 2 dec pl.]

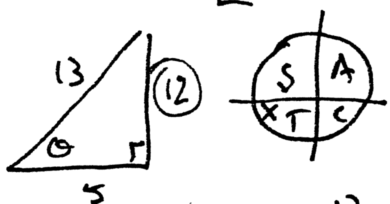
(c) $\frac{\sin y}{8.2} = \frac{\sin 75°}{9.5}$

$\sin y = \frac{8.2 \sin 75}{9.5}$ (2)

$y = 56°29'$ (1)

[1 off for not nearest minute]

(d)



$\tan\theta = {}^{12}/_{5}$ (1)

(1) for +ve.

(7)

(a) $5x + ky = 4$

Passes through $(-2, 1)$ (1)

$\therefore -10 + k = 4$

$k = 14$ (1)

(b) $3x + 4y = 5$

$y = -\frac{3x}{4} + {}^{5}/_{4}$

$\therefore$ Slope $= -{}^{3}/_{4}$ (1)

$\therefore m_p = {}^{4}/_{3}$ (1)

Equation is:

$y - 4 = {}^{4}/_{3}(x+1)$

$3y - 12 = 4x + 4$

$4x - 3y + 16 = 0$ (1)

(c) $P_1 = \frac{|lx_1 + my_1 + n|}{\sqrt{l^2 + m^2}}$

$= \frac{|1.5 + 1.4 - 2|}{\sqrt{1+1}}$

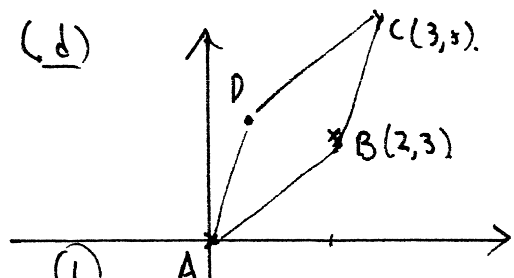
$= {}^{7}/_{\sqrt{2}} = \frac{7\sqrt{2}}{2}$ (1)

$P_2 = \frac{|1.0 + 1.0 - 2|}{\sqrt{2}}$

$= {}^{2}/_{\sqrt{2}} = \sqrt{2}$ (1)

$\therefore$ The ORIGIN is closer. (1)

(d)



(i) D is $(1,2)$ (1)

(ii) $BD = \sqrt{1^2 + 1^2}$

$= \sqrt{2}$ (2)

(8)

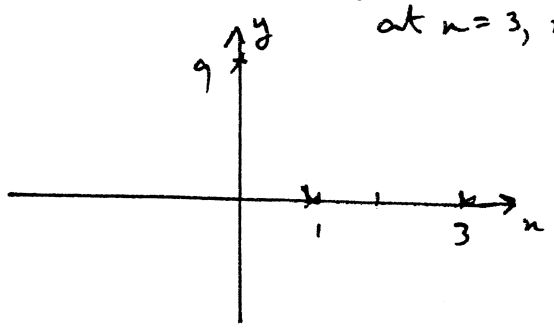
(a) $1 - \sin^2\theta = \cos^2\theta$

(b) $y = 3x^2 - 12x + 9$

(i) $y = 3(x^2 - 4x + 3)$

$= 3(x-3)(x-1)$ (2)

at $x = 3$, $x = 1$



(ii) y-intercept is 9 (1)

or $(0, 9)$.

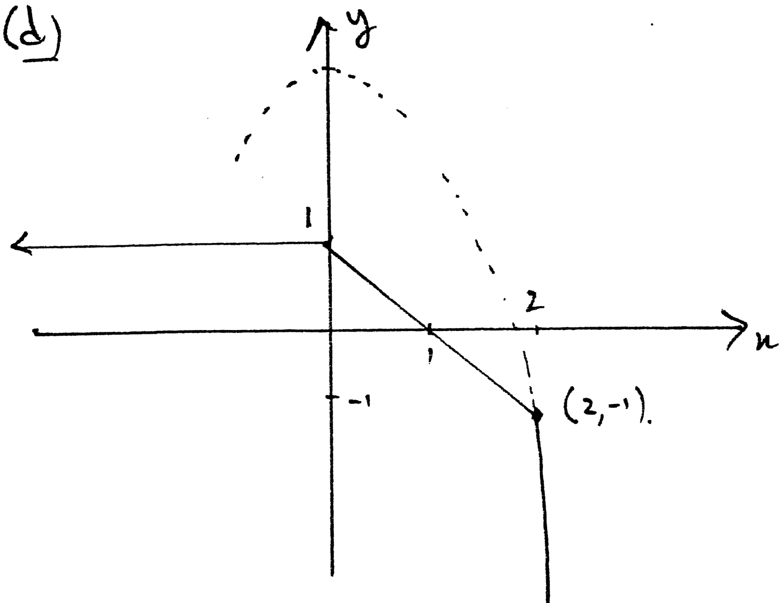
(iii) Axis is $x = 2$ (1)

(iv) Vertex is $(2, -3)$. (1)

(c) $\frac{d}{dx}\left(3x^{\frac{1}{2}}\right) = \frac{3}{2}x^{-\frac{1}{2}}$ (1)

(1) $= \frac{3}{2\sqrt{x}}$ ← not necessary.

(d)



3 MARKS

1 for each section of graph.

must indicate the point (2,-1)