Name:_____ Maths Class:_____

SYDNEY TECHNICAL HIGH SCHOOL

Year 11

MATHEMATICS

Term 3 Examination

September 2003

Time allowed: 2 hours

Instructions:

- Write your name and class at the top of this page.
- At the end of the examination this examination paper must be attached to the front of your answers
- All questions are of equal value and may be attempted
- All necessary working must be shown. Marks will be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Total
								/80

Question 1 (10 marks)

a) Evaluate $|-2|-|-4|$ (1)

b) Write $49^{-3/2}$ as a simple fraction (1)

c) Find cot 102°13' correct to 3 decimal places. (2)

d) Factorise fully $x^4 - 4x^2$ (2)

e) Solve $5 = \frac{2}{5}(w+4)$ (2)

f) Find the values of a and b if $(2+\sqrt{3})^2 = a+\sqrt{b}$ (2)

Question 2 (10 marks)

a) For $f(x) = \frac{2}{x+1}$

i) Write down the domain of the function (1)

ii) Find $f(\frac{1}{a})$ as a simple fraction (2)

b) Solve $x^2 = 2x$ (1)

c) Solve $\sin\theta = \frac{-\sqrt{3}}{2}$ for $0° \leqslant \theta \leqslant 360°$ (2)

d) i) Simplify $(2x+h)^2 - 4x^2$ (1)

ii) Hence evaluate $\lim_{h \to 0} \frac{(2x+h)^2 - 4x^2}{h}$ (1)

e) Solve $\frac{|x|}{2} < 1$ (2)

Question 3 (10 marks)

a) Differentiate the following

i) $y = \frac{1}{3}x^4 + k$ (1)

ii) $y = \frac{4x^3 + x^4}{x^2}$ (2)

b) The points (2, 7) and (-4, -5) are the end points of the diameter of a circle

i) Find the coordinates of the centre of the circle (1)

ii) Find the length of the radius (2)

c) If $x^2 + 2x + m = 0$ has roots α *and* β

i) Without finding the roots, find the value of

I) $\alpha + \beta$ (1)

II) $\alpha\beta$ (1)

ii) If $\beta = 2\alpha$. Find the value of m (2)

Question 4 (10 marks)

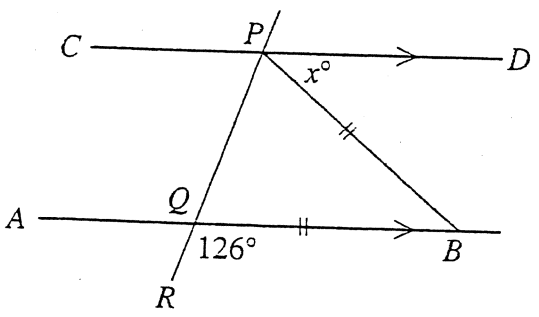
a) Find the equation of the tangent to the curve $y = 3x^2 + x$ *at* $x = 1$ (2)

b) i) Explain why the lines $y = 2x - 1$ *and* $6x - 3y + 5 = 0$ are parallel. (2)

ii) If $(a,5)$ lies on $y = 2x - 1$, find the value of a (1)

iii) Hence find the distance between the parallel lines in part (i) (2)

c)



NOT TO SCALE

In the diagram CD is parallel to AB and PB = QB.

i) Find the size of $< PQB$ in degrees (1)

ii) Find the value of x giving reasons (2)

Question 5 (10 marks)

a) i) Write $\sqrt[3]{x}$ in index form (1)

ii) If $f(x) = \sqrt[3]{x}$. Find $f'(8)$ as a fraction (2)

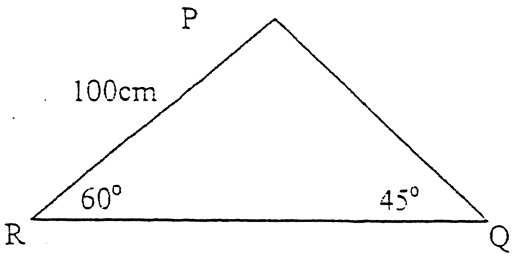
b) For the parabola $y = x^2 - 6x + 4$

i) Find the coordinates of the vertex (2)

ii) Sketch the parabola showing the vertex and y – intercept (1)

iii) Use your graph or otherwise determine the smallest value of k so that $x^2 - 6x + k$ is positive for all values of x (2)

c) In ΔPQR (2)

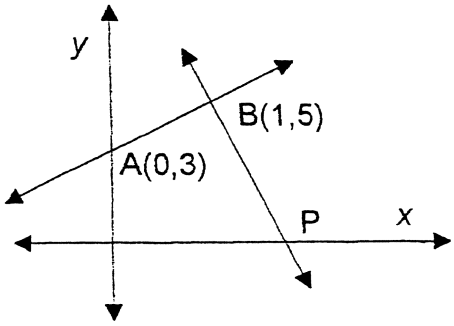


Show that the length of PQ *is* $50\sqrt{6}cm$

Question 6 (10 marks)

a) If $y = (x^2 - 5)^5$. Find $\frac{dy}{dx}$ (2)

b)



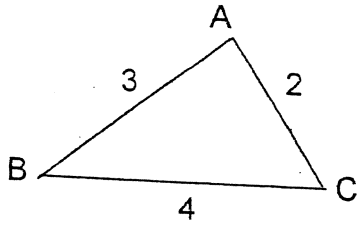
i) Find gradient of line AB (1)

ii) If PB is perpendicular to AB find the equation of PB in general form. (3)

iii) Find the coordinates of P (1)

iv) If ABPQ form a rectangle find the coordinates of Q (1)

c)



i) Use the cosine rule to find the exact value of $\cos A$ (2)

ii) Hence find the exact value of $\sin A$ (1)

Question 7 (10 marks)

a) i) Write down the discriminant of $x^2 + px + (p+3)$ (1)

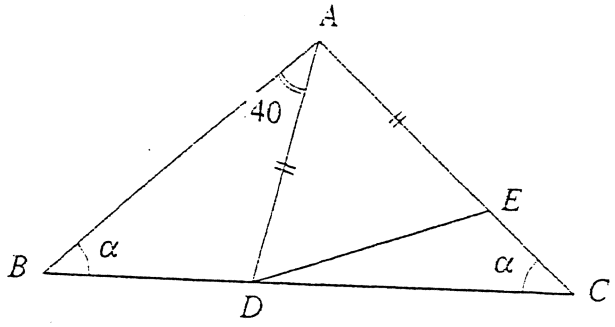
ii) If the equation $x^2 + px + (p+3) = 0$ has equal roots find the values of p. (2)

b) If $y = \dfrac{x}{x^2+1}$

i) Find $\dfrac{dy}{dx}$ (2)

ii) Find the x values of the points where $\dfrac{dy}{dx} = 0$ (1)

c)



In the isosceles triangle ABC $< ABC = < ACB = \alpha$ AD = AE

i) Explain why $< ADC = \alpha + 40$ (1)

ii) Find $< DAC$ *in terms of* α (1)

iii) Hence or otherwise find $< EDC$ giving reasons (2)

Question 8 (10 marks)

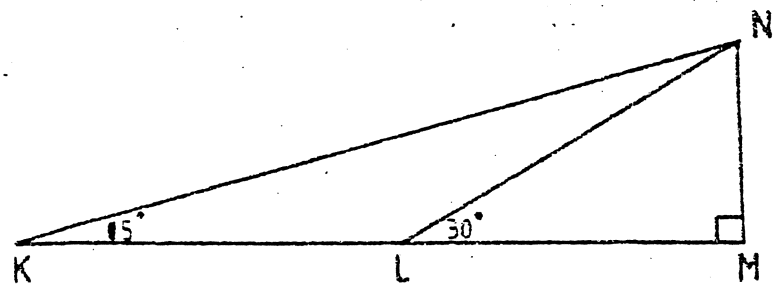
a) i) Sketch the curve $y = \frac{8}{x}$ (1)

ii) Find $\frac{dy}{dx}$ (1)

iii) Find the equation of the normal to $y = \frac{8}{x}$ *at* (4,2) (2)

iv) The normal cuts the curve again at P. Find the coordinates of P. (2)

b)



i) Explain why $KL=LN$ (1)

ii) If $NM = 1$ deduce that $\tan 15° = 2 - \sqrt{3}$ (3).

YEAR 11 YEARLY - 2003 - 2UNIT

Question 1

a) $2 - 4 = -2$

b) $49^{-3/2} = \frac{1}{49^{3/2}}$

$= \frac{1}{343}$

c) $-0.2165.. = -0.217$

d) $x^4 - 4x^2 = x^2(x^2-4)$

$= x^2(x-2)(x+2)$

e) $5 = \frac{2}{5}(w+4)$

$25 = 2w + 8$

$17 = 2w$

$w = 8\frac{1}{2}$

f) $(2+\sqrt{3})^2 = 4 + 4\sqrt{3} + 3$

$= 7 + \sqrt{48}$

$\therefore a = 7, b = 48$

Question 2

a) i) $x \in R, x \neq -1$

ii) $\frac{2}{\frac{1}{a}+1} = \frac{2}{\frac{1+a}{a}}$

$= \frac{2a}{1+a}$

b) $x^2 - 2x = 0$

$x(x-2)$

$x = 0$ or 2.

c) $\sin\theta = -\frac{\sqrt{3}}{2}$

$\theta = 60^\circ$ (3rd, 4th)

$\theta = 240^\circ, 300^\circ$

d) i) $(2x+h)^2 - 4x^2$

$= 4x^2 + 4xh + h^2 - 4x^2$

$= 4xh + h^2$

ii) $4x$

e) $\frac{|x|}{2} < 1$

$|x| < 2$

$-2 < x < 2$

Question 3

a) i) $\frac{dy}{dx} = \frac{4}{3}x^3$

ii) $y = 4x + x^2$

$\frac{dy}{dx} = 4 + 2x$

b) i) centre $= \left(\frac{2-4}{2}, \frac{7-5}{2}\right)$

$= (-1, 1)$

ii) radius $= \frac{1}{2}\sqrt{12^2 + 6^2}$

$= \frac{1}{2}\sqrt{180}$

$= 3\sqrt{5}$

c) i) -2

ii) m

iii) $3x = -2$

$x = -\frac{2}{3}$

$2x^2 = m$

$\therefore 2 \times \frac{4}{9} = m$

$m = \frac{8}{9}$

Question 4

a) $y = 3x^2 + x$

$\frac{dy}{dx} = 6x + 1$

at $x = 1$ $y = 4$, $\frac{dy}{dx} = 7$

$\therefore y - 4 = 7(x - 1)$

$y = 7x - 3$

b) i) $y = 2x - 1$ $m = 2$

$3y = 6x + 5$ $m = 2$

they have the same gradient ($m_1 = m_2 = 2$)

ii) $5 = 2a - 1$

$a = 3$

iii) $d = \left| \frac{3 \times 6 - 3 \times 5 + 5}{\sqrt{36 + 9}} \right|$

$= \frac{8}{\sqrt{45}}$

c) i) $\angle PQB = 54°$

ii) $\angle QPB = 54°$ (base $\angle$ of isos $\triangle$)

$\angle QPD = 126°$ (corresponding $\angle$ on // lines)

$\therefore x + 54° = 126$

$x = 72°$

Question 5

a) i) $x^{1/3}$

ii) $f'(x) = \frac{1}{3} x^{-2/3}$

$f'(8) = \frac{1}{3} \times \frac{1}{8^{2/3}}$

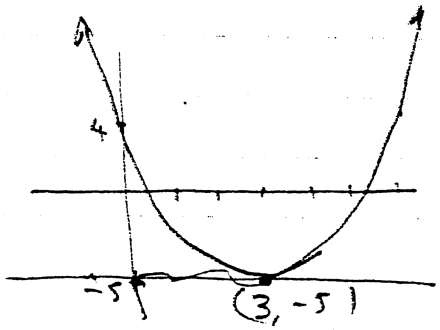
$= \frac{1}{12}$

b) $y + 4 + 9 = x^2 - 6x + 9$

$(y + 5) = (x - 3)^2$

$\therefore$ Vertex $(3, -5)$

ii)



iii) need to add 5

$\therefore$ y-intercept $4 + 5 = 9$

$\therefore x^2 - 6x + 9$

$\therefore k > 9$

c) $\frac{PQ}{\sin 60°} = \frac{100}{\sin 45°}$

$PQ = \frac{100 \sin 60°}{\sin 45°}$

$= \frac{100 \times \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}}$

$= 50\sqrt{3} \times \sqrt{2}$

$= 50\sqrt{6}$

Question 6

a) $y = (x^2 - 5)^5$

$\frac{dy}{dx} = 5 \cdot 2x \cdot (x^2 - 5)^4$

$= 10x(x^2 - 5)^4$

b) i) $m_{AB} = \frac{5 - 3}{1 - 0}$

$= 2$

ii) $m_{\perp} = -1/2$

$y - 5 = -1/2 (x - 1)$

$2y - 10 = -x + 1$

$x + 2y - 11 = 0$

iii) $y = 0$

$x = 11$ $P(11, 0)$

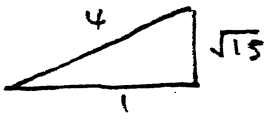
iv) $Q(18, -2)$

Q6

i) $\cos A = \frac{3^2+2^2-4^2}{2\times3\times2}$

$= -\frac{3}{12}$ $\left(-\frac{1}{4}\right)$

ii)



$\sin A = \frac{\sqrt{15}}{4}$

Question 7

a) i) $\Delta = p^2 - 4(p+3)$

$= p^2 - 4p - 12$

ii) $p^2 - 4p - 12 = 0$

$(p-6)(p+2) = 0$

$p = 6$ or -2

b) i) $\frac{dy}{dx} = \frac{(x^2+1).1 - x.2x}{(x^2+1)^2}$

$= \frac{1-x^2}{(x^2+1)^2}$

ii) $\frac{1-x^2}{(x^2+1)^2} = 0$

$\therefore x = \pm 1$

c) i) exterior $\angle$ of triangle

ii) $x + x + 40 + \angle DAC = 180°$

$\angle DAC = 140 - 2x$

iii) Now $\angle ADE = \angle DEA$ (base angles of isosceles Δ) y.

$\therefore 2y + 140 - 2x = 180°$

$y = 20 + x$.

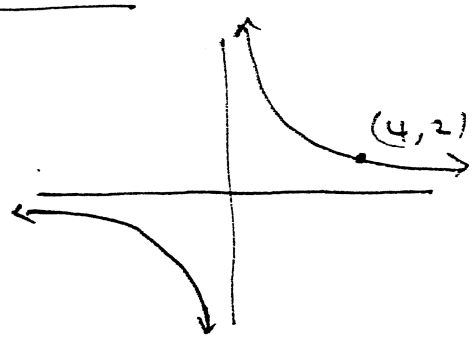
$\angle EDC = \angle ADC - \angle ADE$

$= x + 40 - (20 + x)$

$= 20°$

Question 8

i)



ii) $y = 8x^{-1}$

$\frac{dy}{dx} = -8x^{-2}$

$= -\frac{8}{x^2}$

iii) at (4,2) $\frac{dy}{dx} = -1/2$

$\therefore m_{\perp} = 2$

$y - 2 = 2(x - 4)$

$y = 2x - 6$

iv) $\therefore \frac{8}{x} = 2x - 6$

$8 = 2x^2 - 6x$

$2x^2 - 6x - 8 = 0$

$2(x-4)(x+1) = 0$

$\therefore x = 4$ or -1

$\therefore P(-1, -8)$

b) i) $\angle KNL = 15°$ (exterior $\angle$ of Δ)

$\therefore KL = LN$

ii) $\sin 30° = \frac{1}{NL}$ $\cos 30° = \frac{LM}{2}$

$\therefore NL = 2$ $LM = \sqrt{3}$

$\therefore KL = 2$

In ΔKNM

$\tan 15° = \frac{1}{KL + LM}$

$= \frac{1}{2+\sqrt{3}}$

$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= 2 - \sqrt{3}$