SYDNEY TECHNICAL HIGH SCHOOL

(Est 1911)

MATHEMATICS

YEAR 11
YEARLY EXAMINATION

SEPTEMBER 2004

Time allowed : 2 hours

Instructions :

- **Show all necessary working in every question.**
- **Start each question on a new page.**
- **Attempt all questions.**
- **All questions are of equal value.**
- **Full marks may not be awarded for careless or badly arranged work.**
- **Approved calculators may be used.**
- **These questions are to be handed in with your answers.**
- **Answers must be written in blue or black pen.**

Name : _____

Class : _____

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Question 7	Question 8	TOTAL

Question 1

a) Find the midpoint of the interval from $A(-3,6)$ to $B(4,0)$. 1

b) Factorise fully $12a^3 - 4a^2 - a$ 2

c) Find $\frac{d}{dx}(6x^2 - 3x - 1)$ 1

d) Find the exact value of $\sin 240°$ 1

e) Solve $|x-3| = |2x+1|$ 2

f) Lenny paid \$207 for a meal in a restaurant. This included a 15% tip. 2

What was the cost of the meal without the tip ?

g) Solve $6x^2 = x$ 2

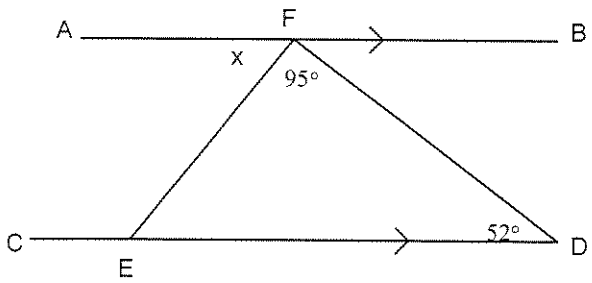
Question 2 (Start a new page)

a) Evaluate the following correct to 3 significant figures. 2

$$\sqrt{\frac{4^2+18^2}{23^2-18^2}}$$

b) Solve $x-\frac{x+4}{4}=2$ 2

c) Find x, giving reasons 2



d) i) Sketch $y=\sqrt{9-x^2}$ 2

ii) State the range of $y=\sqrt{9-x^2}$ 1

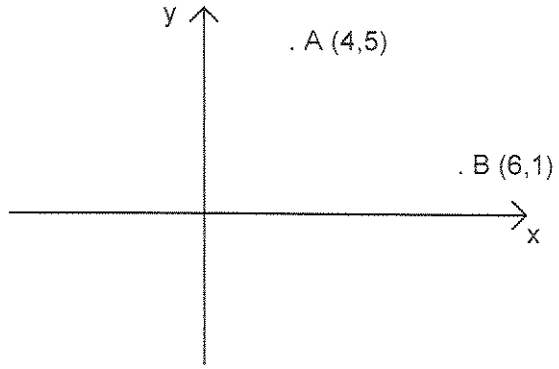
e) Solve simultaneously for x and y 2

$2x-y=7$

$x-2y=-1$

Question 3 (Start a new page)

a)



i) Show that the equation of the line AB is given by 2

$$2x + y - 13 = 0$$

ii) The line AB is produced to meet the x axis at T. 1

Find the coordinates of T.

iii) Find the distance AB 1

iv) Find the perpendicular distance of the point $(-1,5)$ 2

to the line AB.

v) ABC is a right angled isosceles triangle with $\angle BAC = 90°$ 2

Find all the possible coordinates of the point C.

b) Sketch the region satisfied simultaneously by the inequalities 3

$x + y + 2 \leq 0$ and $x - 2y \geq 0$

(Your diagram must be neat and clearly labelled)

Question 4 (Start a new page)

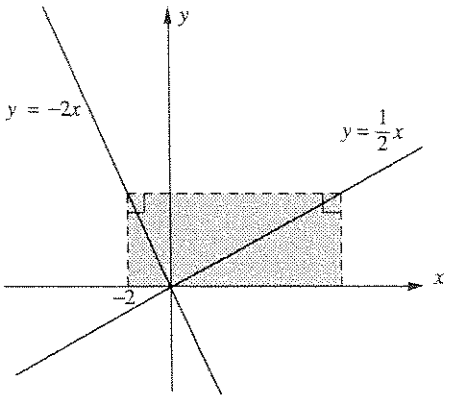
a) Find the equation of the line that passes through the point $(2,4)$ and is parallel to the x axis. 1

b) If $\sin\theta = \frac{1}{5}$ and θ is obtuse, find the exact value of $\cos\theta$. 2

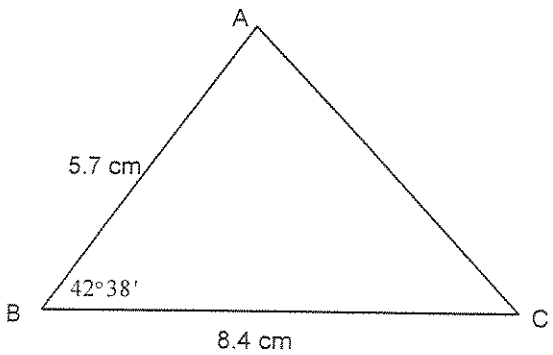
c) Evaluate $\lim\limits_{x\to 2} \dfrac{x^2 - x - 2}{x - 2}$ 1

d) If $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = x - y\sqrt{6}$ find the values of x and y. 3

e) Calculate the shaded area 2



f) Find the area of triangle ABC correct to 1 decimal place. 2



Question 5 (Start a new page)

a) Simplify $\sqrt{98}-\sqrt{8}+\sqrt{12}$ 1

b) Simplify $\dfrac{x+1}{x+2}\div\dfrac{x^2-9}{x^2+5x+6}$ 2

c) Find the equation of the line which passes through the point $(1,4)$ and is perpendicular to the line $3x-2y-4=0$, giving your answer in general form. 3

d) From a point 26 metres from the base of a tower the angle of elevation of the top of the tower is $68°$. Find the height of the tower, giving your answer correct to the nearest centimetre. 2

e) Solve $5\sin\theta+2=0$ for $0°\le\theta\le360°$ 3

(Give answers correct to the nearest degree)

Question 6 (Start a new page)

a) Find, correct to the nearest degree, the acute angle the line $3x - y - 12 = 0$ makes with the x axis. 2

b) Use the formula $\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ 2

to differentiate $y = 3x^2 + x$ by first principles.

c) A function is defined by

$$f(x) = \begin{cases} x^2 - 1 \ for \ x \geq 1 \\ |x - 1| \ for \ x < 1 \end{cases}$$

i) evaluate $f(4) + f(0)$ 1

ii) Sketch the function $y = f(x)$ 2

d) A geologist drives 30 kilometres from point A on a bearing of 324° to point B. He then drives 50 kilometres from point B, on a bearing of 040° to point C.

i) Draw a diagram showing this information 1

ii) Find the size of angle ABC. 1

iii) Find the distance of point C from point A. 2

(Give answer correct to the nearest kilometre)

Question 7 (Start a new page)

a) How many sides has a regular polygon with interior angles equal to $165°$? 2

b) Differentiate with respect to x

i) $y = (2x+1)(x-3)$ 1

ii) $y = \frac{1}{x} + x$ 2

c) Find the equation of the tangent to $y = x^3 - 2x^2 - 3$ at the point $(2,-3)$. 3

d) If $f(x) = (x+2)(x-1)^4$ evaluate $f'(2)$ 3

Question 8 (Start a new page)

a) Evaluate $\lim_{x\to\infty} \dfrac{3x^2-2x-1}{x^2-1}$ 1

b) Prove the identity $\tan\theta+\cot\theta=\dfrac{1}{\sin\theta\cos\theta}$ 3

c) Differentiate with respect to x.

i) $y=\sqrt{x^2-5}$ 1

ii) $y=\dfrac{3x}{x-3}$ 2

d) i) Find the gradient of the line $2x+5y+6=0$ 1

ii) The line $ax+by+11=0$ passes through the point $(1,4)$ 3

and is parallel to the line $2x+5y+6=0$.

Find the values of a and b.

End of Paper

Solutions

Question 1

a) $\left(\frac{1}{2}, 3\right)$

b) $a(12a^2 - 4a - 1)$

$= a(6a + 1)(2a - 1)$

c) $12x - 3$

d) $\sin 240° = -\sin 60°$

$= -\frac{\sqrt{3}}{2}$

e) $x - 3 = 2x + 1$ or $x - 3 = -(2x + 1)$

$x = -4$ or $x - 3 = -2x - 1$

$3x = 2$

$x = \frac{2}{3}$

f) $115\% \equiv \$207$

$\therefore 100\% \equiv \$180$

g) $6x^2 - x = 0$

$x(6x - 1) = 0$

$x = 0, \frac{1}{6}$

Question 2

a) $1 \cdot 29$

b) $x - \frac{x+4}{4} = 2$

$4x - (x + 4) = 8$

$3x - 4 = 8$

$3x = 12$

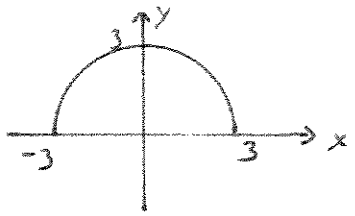
$x = 4$

c) $\angle BFD = 52°$ (alternate angles, $AB \parallel CD$)

$\therefore x + 95 + 52 = 180$ (straight angle)

$\therefore x = 33°$

d) i)



ii) Range: $0 \leq y \leq 3$

e) $2x - y = 7$

$x - 2y = -1$

$4x - 2y = 14$

$x - 2y = -1$

subtract

$3x = 15$

$x = 5$

$\therefore y = 3$

Question 3

a) i) $m_{AB} = \frac{5-1}{4-6}$

$= -2$

$\therefore y - 5 = -2(x - 4)$

$y - 5 = -2x + 8$

$2x + y - 13 = 0$

ii) when $y = 0$ $x = 6\frac{1}{2}$

$\therefore T\left(6\frac{1}{2}, 0\right)$

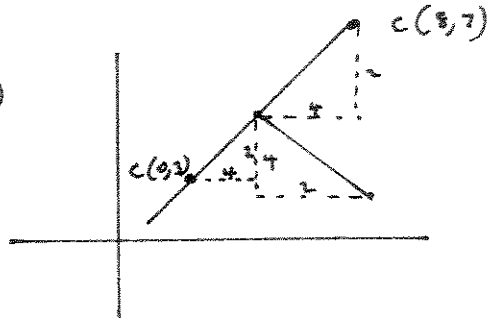
iii) $d = \sqrt{(4-6)^2 + (5-1)^2}$

$= \sqrt{20}$ units

iv) $d = \dfrac{|-1 \times 2 + 5 \times 1 - 13|}{\sqrt{2^2 + 1^2}}$

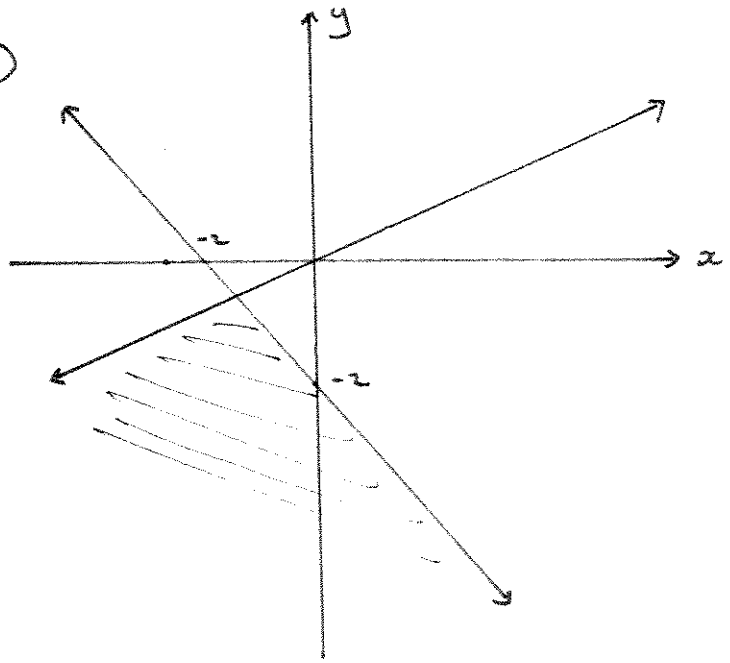
$= \dfrac{10}{\sqrt{5}}$ units

v)



C (8, 7) or C (0, 3)

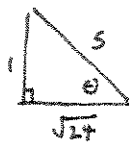
b)



QUESTION 4

a) $y = 4$

b) $\sin\theta = \frac{1}{5}$



$\cos\theta = -\frac{\sqrt{24}}{5}$ (2nd quad)

c) $\lim\limits_{x \to 2} \dfrac{(x-2)(x+1)}{x-2}$

$= 3$

d) $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \dfrac{(\sqrt{3}-\sqrt{2})^2}{1}$

$= 3 - 2\sqrt{6} + 2$

$= 5 - 2\sqrt{6}$

$\therefore x = 5, y = 2$

e) $A = 10 \times 4$

$= 40$ sq units

f) $A = \frac{1}{2} \times 8.4 \times 5.7 \times \sin 42°38'$

$= 16.2 \text{ cm}^2$

QUESTION 5

a) $7\sqrt{2} - 2\sqrt{2} + 2\sqrt{3}$

$= 5\sqrt{2} + 2\sqrt{3}$

b) $\dfrac{x+1}{x+2} \times \dfrac{(x+3)(x+2)}{(x-3)(x+3)}$

$= \dfrac{x+1}{x-3}$

c) $3x - 2y - 4 = 0$

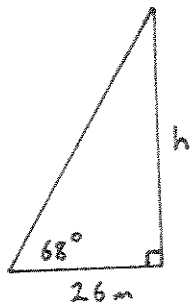
$m = \frac{3}{2}$

$\therefore m_\perp = -\frac{2}{3}$

$\therefore y - 4 = -\frac{2}{3}(x-1)$

$2x + 3y - 14 = 0$

d)



$\tan 68° = \frac{h}{26}$

$h = 26 \times \tan 68°$

$= 64.35 m$

e) $5\sin\theta + 2 = 0$

$\sin\theta = -\frac{2}{5}$

(3rd / 4th $\Rightarrow \theta = 24°$)

$\therefore \theta = 204°, 336°$

QUESTION 6

a) $m = 3$

$\therefore \tan\theta = 3$

$\therefore \theta = 72°$

b) $y' = \lim_{h \to 0} \frac{3(x+h)^2 + (x+h) - [3x^2 + x]}{h}$

$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 + x + h - 3x^2 - x}{h}$

$= \lim_{h \to 0} \frac{6xh + 3h^2 + h}{h}$

$= \lim_{h \to 0} 6x + 3h + 1$

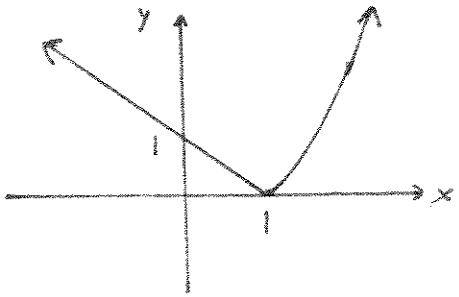
$= 6x + 1$

c) i) $f(4) + f(0)$

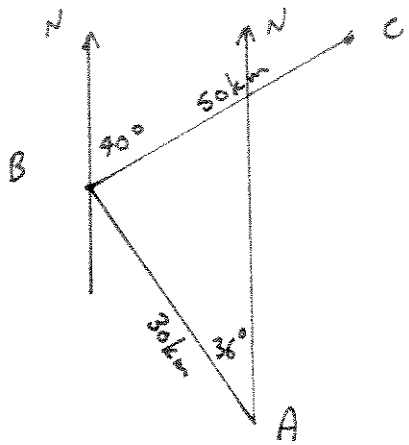
$= 15 + 1$

$= 16$

ii)



d) i)



ii) $\angle ABC = 180° - 36° - 40°$

$= 104°$

iii) $AC^2 = 30^2 + 50^2 - 2 \times 30 \times 50 \times \cos 104°$

$\therefore AC = 64 km$

QUESTION 7

a) ext angle $= 15°$

$\therefore$ no. of sides $= \frac{360}{15}$

$= 24$

b) i) $y = 2x^2 - 5x - 3$

$\therefore y' = 4x - 5$

ii) $y = x^{-1} + x$

$\therefore y' = -x^{-2} + 1$

c) $y' = 3x^2 - 4x$

when $x = 2$

$m_T = 3 \times 2^2 - 4 \times 2$

$= 4$

$\therefore y + 3 = 4(x - 2)$

$y + 3 = 4x - 8$

$y = 4x - 11$

d) $f(x) = (x+2)(x-1)^4$

$f'(x) = (1)(x-1)^4 + (x+2)4(x-1)^3$

$\therefore f'(2) = 1^4 + 4 \times 4 \times 1^3$

$= 17$

QUESTION 8

a) 3

b) RHS $= \tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}$

$= \frac{1}{\cos\theta \sin\theta}$

$=$ RHS

c) i) $y = (x^2 - 5)^{\frac{1}{2}}$

$\therefore \frac{dy}{dx} = \frac{1}{2}(x^2-5)^{-\frac{1}{2}} \cdot 2x$

$= x(x^2-5)^{-\frac{1}{2}}$

$= \frac{x}{\sqrt{x^2-5}}$

ii) $\frac{dy}{dx} = \frac{(x-3)(3) - (3x)(1)}{(x-3)^2}$

$= \frac{-9}{(x-3)^2}$

d) i) $m = -\frac{2}{5}$

ii) gradient of $ax + by + 11 = 0$

$m = -\frac{a}{b}$

as parallel $-\frac{a}{b} = -\frac{2}{5}$

$\therefore 5a = 2b$ *

sub $(1,4)$ into $ax + by + 11 = 0$

$a + 4b + 11 = 0$ *

Solve simultaneously (*)

$5a = 2b$

$a + 4b + 11 = 0$

$\therefore a + 2 \times 5a + 11 = 0$

$11a + 11 = 0$

$a = -1$

$\therefore b = -\frac{5}{2}$

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