SYDNEY TECHNICAL HIGH SCHOOL

YEAR 11 END OF COURSE EXAMINATION

MATHEMATICS

2005

Time allowed: 120 minutes

Directions to Candidates

- Attempt all questions
- Start each question on a new page
- All necessary working should be shown
- Unless otherwise specified, answers must be given in their simplest form
- Approved calculators may be used in all sections.
- Use a ruler when drawing straight lines
- Marks may be deducted for careless or poorly arranged work.
- Marks shown are approximate and may be varied.
- These questions must be attached to the top of your solutions.

Name:_____ Class:_____

1	2	3	4	5	6	7	8	**TOTAL**
/12	**/13**	**/13**	**/13**	**/12**	**/12**	**/13**	**/12**	**/100**

QUESTION 1: (12 Marks)

1 (a) Express $2\sqrt{3}\times\sqrt{27}$ as an integer

1 (b) (i) If $v^2 = u^2 + 2as$, find the value of a if $v = 5$, $u = -2$ and $s = 10$.

1 (ii) Write 23.8249 correct to 2 decimal places

2 (c) (i) Fully factorise $16x^2 - 4$

1 (ii) Fully factorize $6m^2 - 5m - 4$

2 (d) Simplify $\frac{2x-y}{4} - \frac{x+y}{6}$

2 (e) Find $\lim_{x\to 3}\left(\frac{x^2-9}{x-3}\right)$

2 (f) Express $\frac{2}{\sqrt{5}-\sqrt{3}}$ with a rational denominator

QUESTION 2: (13 Marks)

3 (a) Solve $x(x-2) = 2$ leaving your answer as a surd in simplest form.

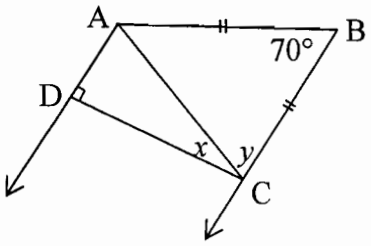
3 (b) Solve the following inequality and plot your answer on a number line:

$$|x-2| \geq 4$$

2 (c) Solve $x^2 < 3x$

3 (d) In the diagram at right, you are given that

$AD \parallel BC$
$AB = BC$
$\angle ABC = 70°$
$AD \perp DC$



SHOWING ALL WORKING, and giving all reasons, calculate the size of $\angle ACD$

2 (e) Prove that $\frac{\sin^3\theta}{\cos\theta} + \sin\theta\cos\theta = \tan\theta$

QUESTION 3: (13 Marks)

1 (a) (i) Find the midpoint, **R**, of the line joining the points A(3, 5) to B(7, -3)

3 (ii) Find the equation of the line through **R** in part (i) and parallel to the line $3y + 2x = 7$. Give your answer in general form.

(b) For the parabola $y = 2x^2$

3 (i) Find the equation of the tangent to the curve at the point P where $x = -1$

3 (ii) Find the equation of the normal to the curve at the point P where $x = -1$

3 (iii) The tangent and the normal above cut the y-axis at T and N respectively. Find the length of TN.

QUESTION 4: (13 Marks)

(a) Differentiate with respect to x:

3 (i) $y = 3x^2 + 2x - \frac{5}{x}$

1 (ii) $y = 3\sqrt{x}$

1 (iii) $y = (3x^2 - 1)^5$

2 (iv) $y = \frac{5}{x-2}$

2 (v) $y = \frac{x-1}{2x+3}$ (Give answer in simplest form)

1 (b) If $f(x) = 5x^2 + 2$, find $f'(2)$

3 (c) If $y = (x^2 - x + 1)^3$, show that $x = \frac{1}{2}$ is the ONLY solution to $\frac{dy}{dx} = 0$

QUESTION 5: (12 Marks)

3 (a) State the largest possible domain for the function $f(x) = \sqrt{25 - x^2}$

2 (b) Determine if the function $f(x) = \dfrac{1}{x^2 - 4}$ is odd, even or neither, showing clearly all reasoning

3 (c) What is the equation of the circle which touches the x-axis at (2, 0) and the y-axis at (0, 2)

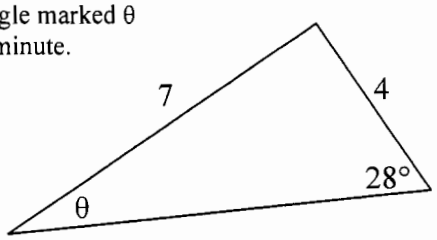
4 (d) Sketch the function $y = f(x)$ which is defined by :

$$f(x) = \begin{cases} x - 2, & x \leq 2 \\ (x-2)^2, & x > 2 \end{cases}$$

QUESTION 6: (12 Marks)

2 (a) Find the equation of the line which passes through the point (3, -1) and makes equal intercepts on the two positive co-ordinate axes.

3 (b) Find the size of the angle marked θ correct to the nearest minute.



(c) Two adjacent sides of a parallelogram have sides of 8 cm and 10cm, while the longer diagonal is 14 cm

(i) Draw the diagram neatly onto your answer sheet (NO MARKS)

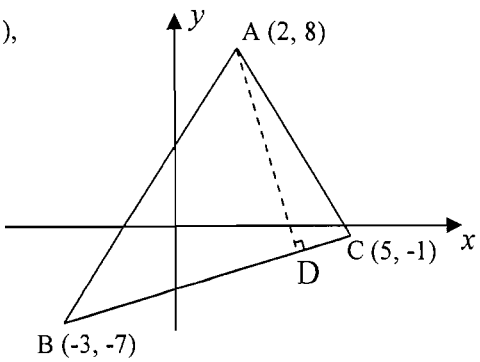
2 (ii) Calculate the size of the larger angle of the parallelogram, correct to the nearest minute.

3 (iii) Calculate the length of the shorter diagonal correct to 2 decimal places.

2 (d) Give the exact value of $2\sin 45° \cos 60°$ giving your answer with a rational denominator.

QUESTION 7: (13 Marks)

(a) The triangle ABC shown at right, has vertices of A(2, 8), B(-3, -7) and C(5, -1). AD is an altitude.



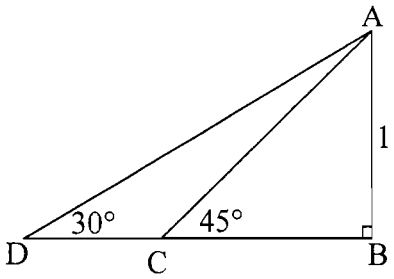
1 (i) Find the length of BC

2 (ii) Find the equation of BC

2 (iii) Find the distance of A to BC

1 (iv) Find the area of Δ ABC

3 (b) (i) In the diagram at right, find the exact length of DC



1 (ii) Find the size of $\angle DAC$

3 (iii) Hence prove, using the figure given, that the exact value of

$\sin 15°$ is $\dfrac{\sqrt{3}-1}{2\sqrt{2}}$

QUESTION 8: (12 Marks)

3 (a) Find $\dfrac{d}{dx}\left[x^2(1-2x)^3\right]$ (Give answer in simplest factored form)

5 (b) Find the equation of the normal to the semi-circle $y=\sqrt{100-x^2}$ at the point (6, 8) on it. Give the equation in general form.

4 (c) Solve the equation $4^x - 3.2^x - 4 = 0$

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N 1

$\sqrt{27} = 2\sqrt{3} \times 3\sqrt{3}$

$= 18$

$5 = 4 + 20a$

$1 = 20a$

$= \frac{21}{20}$

$23 \cdot 82$

$(4x^2 - 1) = 4(2x-1)(2x+1)$

$m \quad -4$

$2m \quad +1$

$3m-4)(2m+1)$

$\frac{x-y}{4} - \frac{x+y}{6}$

$\frac{3(2x-y) - 2(x+y)}{12}$

$\frac{6x - 3y - 2x - 2y}{12}$

$\frac{4x - 5y}{12}$

$m \quad \frac{(x-3)(x+3)}{(x-3)} = 6$

$\to 3$

$\frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$\frac{2\sqrt{5} + 2\sqrt{3}}{2}$

$\sqrt{5} + \sqrt{3}$

ion 2

$-2x - 2 = 0$

$\frac{2 \pm \sqrt{4 - 4 \times 1 \times -2}}{2}$

$\frac{2 \pm \sqrt{12}}{}$

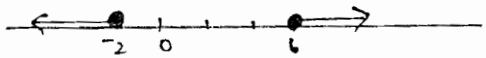
$= \frac{2 \pm 2\sqrt{3}}{2}$

$= 1 \pm \sqrt{3}$

b) $|x-2| \geq 4$

$x - 2 \geq 4 \qquad x - 2 \leq -4$

$x \geq 6 \qquad x \leq -2$

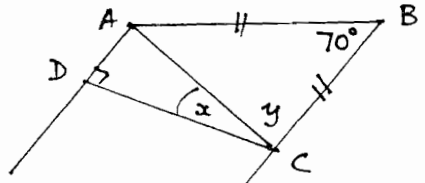


c) $x^2 - 3x < 0$

$x(x-3) < 0$

$0 < x < 3$

d)



$y = 55°$ (opposite equal sides in isosceles triangle)

$x = 35°$ (conterior angles AD || BC

e) LHS $= \frac{\sin^3\theta}{\cos\theta} + \sin\theta \cdot \cos\theta$

$= \frac{\sin^3\theta + \sin\theta\cos^2\theta}{\cos\theta}$

$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta)}{\cos\theta}$

$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$

$=$ RHS

QUESTION 3

a) i) $R(5, 1)$

ii) $3y + 2x = 7$

$y = -\frac{2x}{3} + \frac{7}{3}$

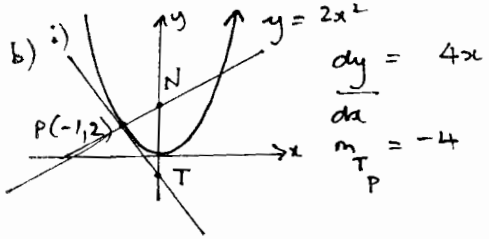
$\therefore m = -\frac{2}{3}$

eqn. $y - 1 = -\frac{2}{3}(x-5)$

$3y - 3 = -2x + 10$

$2x + 3y - 13 = 0$

b) i)



$\frac{dy}{dx} = 4x$

$m_T = -4$

P

eqn of tang: $y - 2 = -4(x+1)$

$y = -4x - 2$

ii) eqn of normal: $m_N = \frac{1}{4}$

$y - 2 = \frac{1}{4}(x+1)$

$4y - 8 = x + 1$

$x - 4y + 9 = 0$

iii) $T(0, -2)$ $\therefore TN = 4\frac{1}{4}$ units

$N(0, \frac{9}{4})$

QUESTION 4

a) i) $\frac{dy}{dx} = 6x + 2 + 5x^{-2}$

ii) $y = 3x^{1/2}$

$\frac{dy}{dx} = \frac{3}{2}x^{-1/2}$

$= \frac{3}{2\sqrt{x}}$

iii) $\frac{dy}{dx} = 5 \cdot 6x(3x^2 - 1)^4$

$= 30x(3x^2 - 1)^4$

iv) $y = 5(x-2)^{-1}$

$\frac{dy}{dx} = -5(x-2)^{-2} = \frac{-5}{(x-2)^2}$

v) $y = \frac{x-1}{2x+3}$

$u = x - 1 \qquad v = 2x + 3$

$u' = 1 \qquad v' = 2$

$\frac{dy}{dx} = \frac{2x + 3 - 2(x-1)}{(2x+3)^2}$

$= \frac{5}{(2x+3)^2}$

b) $f(x) = 5x^2 + 2$

$f'(x) = 10x$

$f'(2) = 20$

c) $y = (x^2 - x + 1)^3$

$\frac{dy}{dx} = 3(2x-1)(x^2 - x + 1)^2$

solve $3(2x-1)(x^2 - x + 1) = 0$

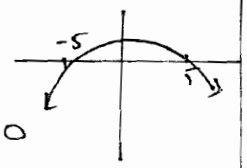
$2x - 1 = 0 \qquad x^2 - x + 1 = 0$

$x = \frac{1}{2} \qquad \Delta = 1 - 4 \times 1 \times 1$

only solution $\Delta = -3$

$\therefore$ no solutio

ION 5



$-x^2 \geq 0$

$1)(5+x) \geq 0$

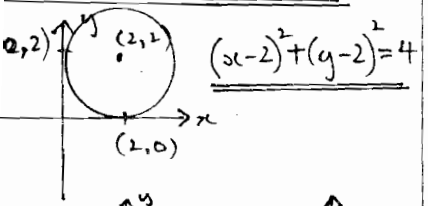
$-5 \leq x \leq 5$

$f(x) = \frac{1}{x^2-4}$

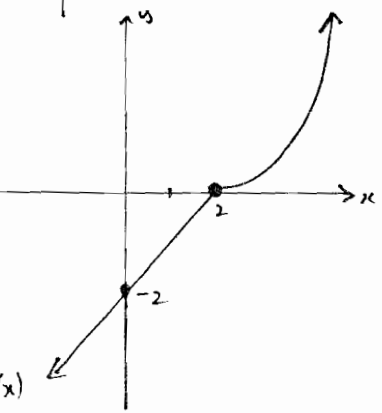
$f(-x) = \frac{1}{(-x)^2-4}$

$= \frac{1}{x^2-4}$

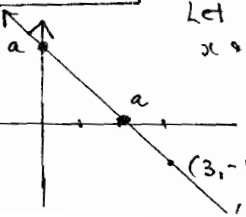
$f(x) = f(-x)$ EVEN



$(x-2)^2+(y-2)^2=4$



TION 6



Let linecut x & y axes at a

$\therefore m = -1$

eqn: $y+1 = -(x-3)$

$y+1 = -x+3$

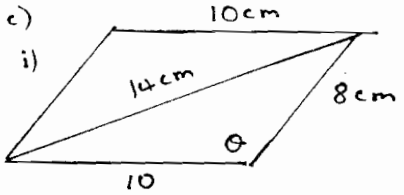
$x+y-2=0$

b) $\frac{\sin\theta}{4} = \frac{\sin 28^\circ}{7}$

$\sin\theta = \frac{4\sin 28}{7}$

$\theta = 15^\circ 34'$

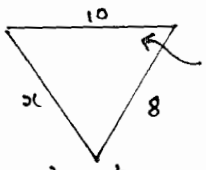
c) i)



ii) $\cos\theta = \frac{8^2+10^2-14^2}{2\times 8\times 10}$

$\theta = 101^\circ 32'$

iii)



$x^2 = 10^2 + 8^2 - 2\times 10\times 8\cos 78^\circ 28'$

$x = 11.49$ cm

d) $2\sin 45^\circ \cos 60^\circ$

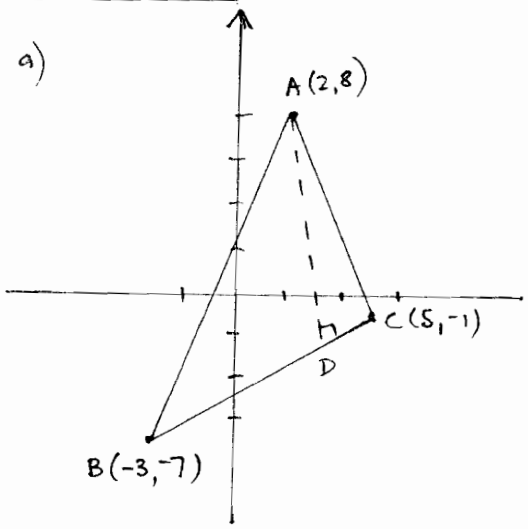
$2 \times \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{\sqrt{2}}{2}$

QUESTION 7

a)



i) $BC = \sqrt{(5+3)^2+(-1+7)^2}$

$= \sqrt{64+36}$

$= 10$ units

ii) $m_{BC} = \frac{-1+7}{5+3} = \frac{6}{8} = \frac{3}{4}$

eqn $y+1 = \frac{3}{4}(x-5)$

$4y+4 = 3x-15$

$0 = 3x-4y-19$

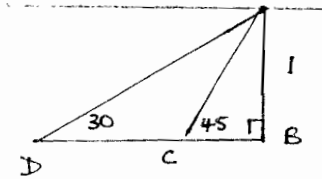
iii) $p = \left|\frac{3\times 2 - 4\times 8 - 19}{\sqrt{9+16}}\right|$

$= \left|\frac{-45}{5}\right|$

$= 9$ units

iv) Area $\triangle ABC = \frac{1}{2} \times 10 \times 9$

b) i)



$BC = 1$ (isosceles $\triangle$)

$\tan 30 = \frac{1}{DB}$

$\frac{1}{\sqrt{3}} = \frac{1}{DB}$

$\therefore DB = \sqrt{3}$

$\therefore DC = (\sqrt{3}-1)$ units

ii) $D\hat{A}C = 15^\circ$ (ext angle of $\triangle$)

iii) $AC = \sqrt{2}$ units

sine Rule: $\frac{\sin 15}{\sqrt{3}-1} = \frac{\sin 30}{\sqrt{2}}$

$\sin 15^\circ = \frac{(\sqrt{3}-1)\cdot 1/2}{\sqrt{2}}$

$= \frac{(\sqrt{3}-1)}{2\sqrt{2}}$

QUESTION 8

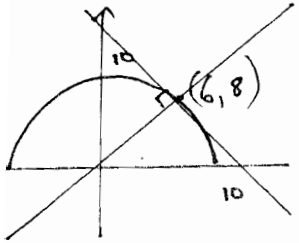
a) $u = x^2$ $\quad v = (1-2x)^3$

$u' = 2x$ $\quad v' = -6(1-2x)^2$

$\frac{dy}{dx} = 2x(1-2x)^3 - 6x^2(1-2x)^2$

$= 2x(1-2x)^2[(1-2x) - 3x]$

$= 2x(1-2x)^2(1-5x)$



$\sqrt{100-x^2}$

$(100-x^2)^{1/2}$

$= \frac{1}{2}x - 2x(100-x^2)^{-\frac{1}{2}}$

$= \frac{-x}{\sqrt{100-x^2}}$

$= \frac{-6}{\sqrt{100-36}} = \frac{-6}{8} = -\frac{3}{4}$

$\frac{4}{3}$

rmal: $y-8 = \frac{4}{3}(x-6)$

$3y - 24 = 4x - 24$

$4x - 3y = 0$

$^x - 3.2^x - 4 = 0$

$u = 2^x$

$^2 - 3u - 4 = 0$

$- 4)(u - 1) = 0$

$u = 4 \qquad u = 1$

$2^x = 4 \qquad 2^x = 1$

$x = 2 \qquad x = 0$