SYDNEY TECHNICAL HIGH SCHOOL

(Est 1911 - celebrating 50 years at Bexley)

Mathematics

YEAR 11 YEARLY EXAMINATION
PRELIMINARY HSC ASSESSMENT TASK 3
SEPTEMBER 2006

General Instructions

- Working time allowed – 120 minutes.
- Write using black or blue pen.
- Approved calculators may be used.
- All necessary working should be shown.
- Start each question on a new page.
- Attempt all questions.
- Questions are of equal value.
- Full marks may not be awarded if working is poorly set out or difficult to read.

NAME :________________________

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Question 7	TOTAL

Question 1 (13 marks)

Marks

a) Solve $x^2 - 5x + 6 = 0$ — 2

b) Simplify $\frac{2x-5}{2} - \frac{2x-1}{5}$ — 2

c) Find $\frac{d}{dt}(5t^3 + 1)$ — 1

d) Simplify $\sqrt{128} + \sqrt{2}$ — 1

e) Find the exact value of

i) $\sec 30°$ — 1

ii) $\cot 90°$ — 1

f) Find the gradient of the curve $y = x^2 - 4x$ at the point $(1, -3)$. — 2

g) i) Write down the discriminant of $2x^2 - 3x + k$ — 1

ii) For what values of k is the expression $2x^2 - 3x + k$ positive for all values of x ? — 2

Question 2 (**13 marks)** (Start a new page) **Marks**

a) Evaluate $\sqrt{\dfrac{284.6}{8.3 \times 6.2}}$ correct to 2 significant figures. 2

b) Solve $\tan\theta = 0.3$ for $0 \le \theta \le 360°$ 2

giving answers correct to the nearest degree

c) The price of an item increases from \$1.50 to \$3.60. 2

What percentage increase is this ?

d) Solve $|x - 3| \le 10$ 2

e) State the domain of the function $y = 4\sqrt{x-5}$ 1

f) If α and β are the roots of the equation $x^2 - 2x - 7 = 0$

find the value of i) $\alpha + \beta$ 1

ii) $\alpha \times \beta$ 1

iii) $\alpha^3\beta + \alpha\beta^3$ 2

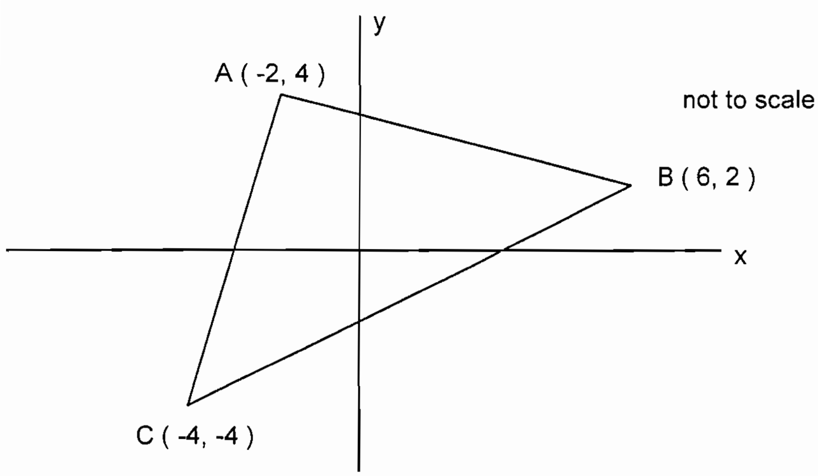
Question 3 **(13 marks)** (Start a new page) **Marks**

a) If $\dfrac{2}{2-\sqrt{3}} = a + \sqrt{b}$ find the value of a and b given they are both rational. 3

b) Find the perpendicular distance from the point $(2,3)$ 1

to the line $3x + y + 2 = 0$.

c) The diagram below shows the points $A(-2,4)$, $B(6,2)$ and $C(-4,-4)$



i) Calculate the length of the interval BC. 1

ii) Find the gradient of the line BC. 1

iii) Find the coordinates of M, the midpoint of BC. 1

iv) Show that the equation of *l*, the perpendicular 2

bisector of BC is $5x + 3y - 2 = 0$.

v) Show that *l* passes through A. 1

vi) What does the result in part v) tell us about triangle ABC. 1

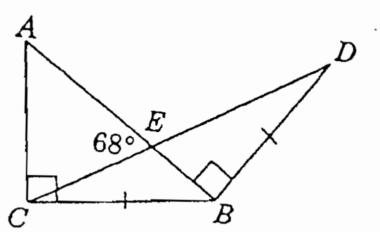
vii) Hence or otherwise find the area of triangle ABC. 2

Question 4 **(13 marks)** (Start a new page) **Marks**

a) Find the minimum value of $x^2 - 4x + 14$ 2

b) On a number plane sketch the region described by $y \geq 4x - x^2$ 2

c)



ABC is a right angled triangle in which $\angle ACB = 90°$. 2
Triangle CDB is isosceles, in which CB=DB.
$\angle AEC = 68°$ and $\angle EBD = 90°$.
Find $\angle DCB$, giving reasons.

d) Differentiate the following

i) $\dfrac{5}{x^2}$ 1

ii) $\sqrt{2x+7}$ 2

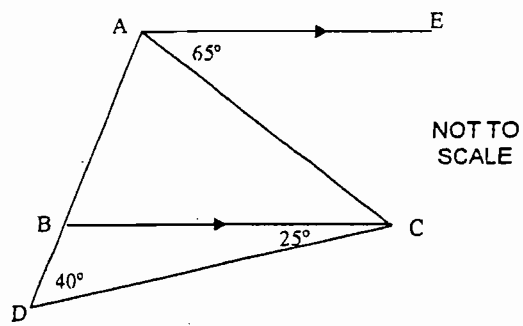
iii) $\dfrac{2x-1}{x+1}$ 2

iv) $3x^2(x-2)^4$ 2

Question 5 **(13 marks)** (Start a new page) **Marks**

a) Evaluate $\lim_{x \to 4} \dfrac{x^2 + 2x - 24}{x - 4}$ 2

b)



In the diagram above, AE is parallel to BC, $\angle BCD = 25°$, $\angle BDC = 40°$ and $\angle EAC = 65°$. 3

Copy the diagram onto your answer sheet and show that triangle ABC is isosceles.

c) Find the equation of the normal to $y = x^3 - 6x - 2$ 3

at the point $(1, -7)$

d) Simplify $\dfrac{1 + \cos\theta}{1 - \sin\theta} \div \dfrac{1 + \sin\theta}{1 - \cos\theta}$ 3

e) If $f(x) = x(x+1)(x+2)$ 2

find in simplest terms an expression for $f(x+1) - f(x)$

Question 6 **(13 marks)** (Start a new page) **Marks**

a) The quadratic expression $Q(x)$ is given by $Q(x) = x^2 - (2+k)x + 4$.

i) For what values of k is $x = -2$ a root of $Q(x) = 0$. 2

ii) For what value of k does $Q(x) = 0$ have real roots. 3

b) Find a quadratic equation which has roots $1+\sqrt{3}$ and $1-\sqrt{3}$. 2

Express your answer in the form $ax^2 + bx + c = 0$ where a, b and c are real.

c) Find the coordinates of the point on the curve $y = x^2 + 6x + 2$ 3

at which the tangent to the curve is parallel to the line $y = 2x + 3$.

d) A triangle has sides of length 8 cm , 11 cm and 16 cm.

i) Find the size of the largest angle (nearest degree) 2

ii) Find the area of the triangle (correct to 1 decimal place) 1

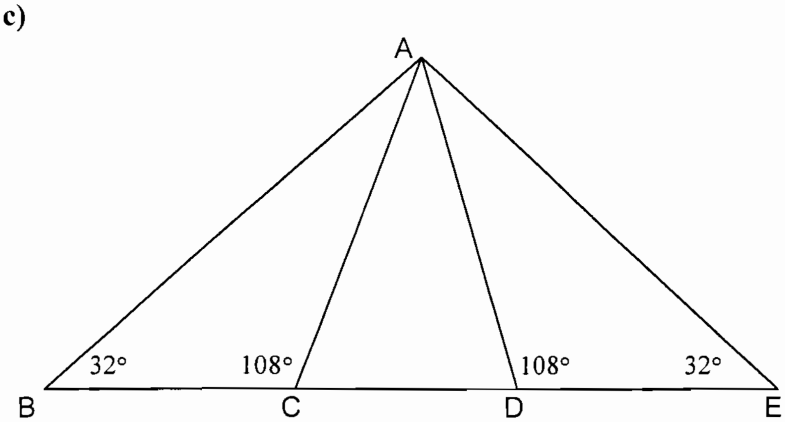
Question 7 **(13 marks)** (Start a new page) **Marks**

a) Solve $x^6 + 7x^3 - 8 = 0$ 3

b) Find the values of A, B and C if 3

$$3x^2 - 7x + 5 \equiv Ax(x-1) + Bx + C$$

c)



Given that $BE = 30\,metres$ find the length of AC 3
(correct to 1 decimal place)

d) The roots of the equation $ax^2 + bx + c = 0$ differ by 4. 4

Show that $b^2 = 4ac + 16a^2$

Teacher's Name: **Student's Name/N°:**

Q1

a. $(x-2)(x-3)=0$

$x = 2, 3$

b. $\frac{5(2x-5)-2(2x-1)}{10}$

$= \frac{6x-23}{10}$

c. $15t^2$

d. $8\sqrt{2}+\sqrt{2}$

$= 9\sqrt{2}$

e. i) $\frac{2}{\sqrt{3}}$

ii) 0

f. $y' = 2x-4$

when $x=1$

$m = -2$

g. i) $\Delta = 9-8k$

ii) $\Delta < 0 \quad (a>0)$

$9-8k<0$

$k > \frac{9}{8}$

Q2

a. 2.4

b. $\theta = 17^\circ, 197^\circ$

d. $-10 \leq x-3 \leq 10$

$-7 \leq x \leq 13$

e. domain $x \geq 5$

f. i) $\alpha+\beta = 2$

ii) $\alpha\beta = -7$

iii) $\alpha\beta(\alpha^2+\beta^2)$

$= \alpha\beta\left[(\alpha+\beta)^2 - 2\alpha\beta\right]$

$= -7\left[2^2 - 2\times -7\right]$

$= -126$

Q3

a. $\frac{2}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{4+2\sqrt{3}}{4-3}$

$= 4+2\sqrt{3}$

$= 4+\sqrt{12}$

$\therefore a=4 \quad b=12$

b. $d = \frac{|2\times3+3\times1+2|}{\sqrt{3^2+1^2}}$

$= \frac{11}{\sqrt{10}}$ units

Teacher's Name: Student's Name/N°:

c. i) $d = \sqrt{(6--4)^2 + (2--4)^2}$

$= \sqrt{136}$ units

ii) $m = \dfrac{2--4}{6--4}$

$= \dfrac{6}{10}$

$= \dfrac{3}{5}$

iii) $M = (1,-1)$

iv) $m_l = -\frac{5}{3}$

$\therefore\ y+1 = -\frac{5}{3}(x-1)$

$3y + 3 = -5x + 5$

$5x + 3y - 2 = 0$

v) sub. $x = -2, y = 4$

$5(-2) + 3(4) - 2 = 0$

$\therefore\ 0 = 0$

$\therefore$ A lies on l

vi) ΔABC is isosceles

(or $AB = AC$)

vii) $d_{AB} = \sqrt{(-2-1)^2 + (4--1)^2}$

$= \sqrt{9 + 25}$

$= \sqrt{34}$

$\therefore$ Area $= \frac{1}{2} \times \sqrt{136} \times \sqrt{34}$

Q4

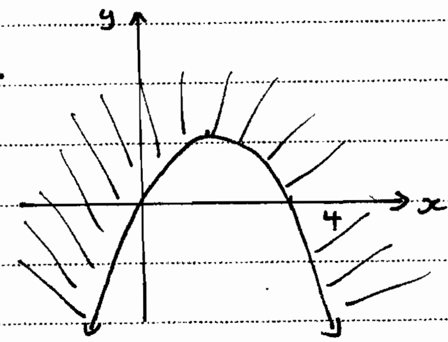
a. $x = \dfrac{4}{2}$

$= 2$

$\therefore$ min $= 2^2 - 4 \times 2 + 14$

$= 10$

b.



c.

$\angle DEB = 68^\circ$ (Vertically opposite)

$\angle EDB = 22^\circ$ (angle sum of triangle)

$\therefore\ \angle DCB = 22^\circ$ (angles opposite equal sides equal)

d. i) $-10x^{-3}$

ii) $(2x+7)^{-\frac{1}{2}}$

iii) $\dfrac{(x+1)2 - (2x-1)(1)}{(x+1)^2}$

$= \dfrac{3}{(x+1)^2}$

iv) $6x(x-2)^4 + 3x^2 \cdot 4(x-2)^3$

$= 6x(x-2)^4 + 12x^2(x-2)^3$

Teacher's Name: ________________ Student's Name/N°: ________________

Q5

a. $\lim_{x \to 4} \frac{(x-4)(x+6)}{x-4}$

$= 10$

b. $\angle ACB = 65^\circ$ (alternate angles, AE || BC)

$\angle ABC = 65^\circ$ (exterior angle equals sum of two remote interior angles)

$\therefore \triangle ABC$ is isosceles (two equal angles)

c. $y' = 3x^2 - 6$

when $x = 1$

$m_T = -3$

$\therefore m_N = \frac{1}{3}$

$\therefore y + 7 = \frac{1}{3}(x-1)$

$3y + 21 = x - 1$

$x - 3y - 22 = 0$

d. $\frac{1+\cos\theta}{1-\sin\theta} \times \frac{1-\cos\theta}{1+\sin\theta}$

$= \frac{1-\cos^2\theta}{1 - \sin^2\theta}$

$= \frac{\sin^2\theta}{\cos^2\theta}$

$= \tan^2\theta$

e. $f(x+1) - f(x)$

$= (x+1)(x+2)(x+3) - x(x+1)(x+2)$

Q6

a. i) sub $x = -2$

$(-2)^2 - (2+k)(-2) + 4 = 0$

$8 + 4 + 2k = 0$

$2k = -12$

$k = -6$

~~b.~~ ii) real roots $\Rightarrow \Delta \geq 0$

$(2+k)^2 - 4 \times 4 \geq 0$

$k^2 + 4k + 4 - 16 \geq 0$

$k^2 + 4k - 12 \geq 0$

$(k+6)(k-2) \geq 0$

$k \leq -6$, $k \geq 2$

b. $\alpha + \beta = 2$

$\alpha\beta = -2$

$\therefore x^2 - 2x - 2 = 0$

c. $y' = 2x + 6$

$m_{line} = 2$

$\therefore 2x + 6 = 2$

$2x = -4$

$x = -2$

$\therefore$ point $(-2, -6)$

d. i) $\cos A = \frac{8^2 + 11^2 - 16^2}{2 \times 8 \times 11}$

$\therefore A = 114^\circ$

Teacher's Name: Student's Name/N°:

Q7

a. $(x^3+8)(x^3-1)=0$

$x^3=-8$, $x^3=1$

$x=-2$, $x=1$

b. $3x^2-7x+5$

$\equiv Ax(x-1)+Bx+C$

$\equiv Ax^2-Ax+Bx+C$

$\therefore A=3$

$C=5$

$B-A=-7$

$\therefore B=-4$

c. $\frac{AB}{\sin 32^\circ} = \frac{30}{\sin 116^\circ}$

$\therefore AB = 17.6877$

$\frac{AC}{\sin 32^\circ} = \frac{AB}{\sin 108^\circ}$

$\therefore AC = 9.9$ m

d. let roots equal $\alpha, \alpha+4$

$\therefore 2\alpha+4 = \frac{-b}{a}$ (1)

$\alpha(\alpha+4) = \frac{c}{a}$ (2)

$\alpha = -\frac{1}{2}\left(\frac{b}{a}+4\right)$ from (1)

sub. into (2)

$-\frac{1}{2}\left(\frac{b}{a}+4\right)\left(-\frac{1}{2}\left(\frac{b}{a}+4\right)+4\right) = \frac{c}{a}$

$-\frac{1}{2}\left(\frac{b}{a}+4\right)\left(\frac{-b}{2a}-2+4\right) = \frac{c}{a}$

$-\frac{1}{2}\left(\frac{b}{a}+4\right)\left(\frac{-b}{2a}+2\right) = \frac{c}{a}$

$-\frac{1}{2}\left(\frac{-b^2}{2a^2}+\frac{2b}{a}-\frac{4b}{2a}+8\right) = \frac{c}{a}$

$\frac{-b^2}{2a^2}+\frac{2b}{a}-\frac{4b}{2a}+8 = \frac{-2c}{a}$

$-b^2+16a^2 = -4ac$

$\therefore b^2 = 4ac+16a^2$