Name: .... Maths Class: ....

SYDNEY TECHNICAL HIGH SCHOOL

YEAR 11 PRELIMINARY HSC COURSE

Mathematics

September 2008

TIME ALLOWED: **120 minutes**

Instructions:

- Write your name and class at the top of this page, and on all your answer sheets.
- Hand in your answers attached to the rear of this question sheet.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied at the time of marking

(FOR MARKERS USE ONLY)

1	2	3	4	5	6	7	8	TOTAL
/14	/13	/13	/12	/13	/12	/12	/11	/100

QUESTION 1: (14 Marks)

Marks

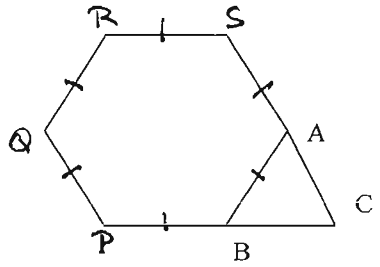
(a) Fully factorise $5x^2 - 45$ **2**

(b) Rationalise the denominator and simplify: $\dfrac{\sqrt{5}}{\sqrt{5}-1}$ **2**

(c) Simplify $7\sqrt{3} - 2\sqrt{27}$ **2**

(d) Expand and simplify $(x-2)(x^2+2x+4)$ **1**

(e) The diagram below is of a regular hexagon with a triangle ABC attached made by extending two of its sides.



(i) Find the size of each of the internal angles of the hexagon **1**

(ii) Giving all reasons, identify the type of triangle ABC drawn **3**

(f) Find the equation of the line through the point (3, -4) and parallel to the line $x + 2y - 2 = 0$. Give your answer in general form. **3**

QUESTION 2: (13 Marks)

Marks

(a) Find $\lim\limits_{x \to \infty} \dfrac{3x^2 - 2x + 3}{2x^2 - 5}$ **1**

(b) If $\cos\theta = \dfrac{5}{7}$ and $0^o \le \theta \le 90^o$ find $\tan\theta$ as a surd in simplified terms **2**

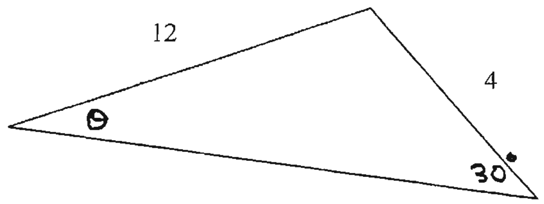
(c) Differentiate the following with respect to x:

(i) $y = (4 + x)(2 - x)$ **1**

(ii) $y = \dfrac{3}{x^2}$ **1**

(iii) $y = (5x^2 - 4)^4$ **1**

(d)



(i) Find the value of θ to the nearest minute **3**

(ii) $\theta = 170°24'$ is also a solution to the problem above. Explain why this "solution" is not an answer to the question. **1**

(e) Solve $|2x - 3| \le 5$ and plot the solution on a number line **3**

QUESTION 3: (13 Marks):

Marks

(a) Find the point(s) on the line $y = x^3 - 12x + 1$ where the tangent is parallel to the x-axis **3**

(b) The functions *f(x)* and *g(x)* are defined as:

$$f(x) = 1 - x^2 \quad \text{and} \quad g(x) = \sqrt{x}$$

(i) On a set of neat axes, sketch $y = f(x)$ *showing all intercepts with the co-ordinate axes.* **2**

(ii) Find $f(-3)$ **1**

(iii) Find the Range of $f(x)$ **1**

(iv) What is the natural domain for $g(x)$? **1**

(v) Find $f(g(x))$ **1**

(c) Differentiate the following with respect to x:
(give your answers without negative indices)

(i) $y = \dfrac{2 - x^2}{x^2 + 2}$ **2**

(ii) $y = \sqrt{1 - x^2}$ **2**

QUESTION 4: (12 Marks)

Marks

(a) Given that α and β are the roots to $2x^2 - 3x + 2 = 0$, find the value of the following:

(DO NOT ATTEMPT TO SOLVE THE EQUATION)

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **2**

(iv) $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$ **2**

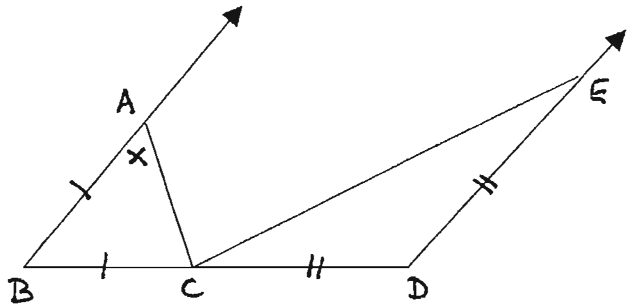
(b) In the diagram at right,

AB || ED

AB = BC and CD = DE

B, C and D are collinear

∠BAC = x



Give reasons for all of your answers to the questions below

(i) Find ∠ACB in terms of x **1**

(ii) Find ∠ABC in terms of x **1**

(iii) Why is ∠CDE = $2x$? **1**

(iv) Hence find an expression for ∠DCE **2**

(v) What can you conclude about the size of ∠ACE ? **1**

QUESTION 5: (13 Marks)

Marks

(a) Find the equation of the tangent to the curve $y = x^3 - 2x + 1$ at the point where x = 2 **3**

(b) The point P (2, 2) lies on the curve $y = 2x - x^2 + 2$
Find the equation of the normal to this curve at P **3**

(c) (i) Sketch $y = |2x - 1|$ showing all intercepts **2**

(ii) Draw another line on your sketch which could be used to solve $|2x - 1| = 3$ and label it. **1**

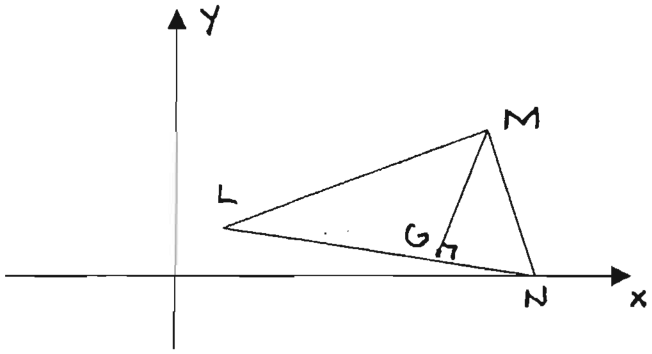
THERE IS NO NEED TO SOLVE THE INEQUALITY

(d) Find the values of k for which the equation $4x^2 - 4(k-3)x + 1 = 0$ has no real roots. **4**

QUESTION 6: (12 Marks)

Marks

(a) The points L (1, 1), M (4, 2), and N (5, 0) shown below form a triangle. MG is an altitude of this triangle.



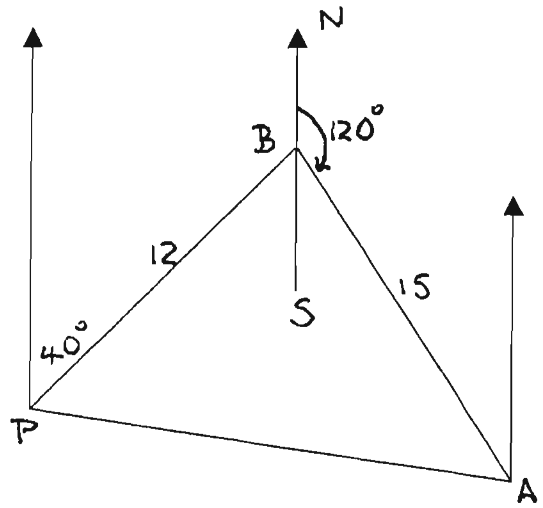
(i) Find the length of LN **1**

(ii) Find the equation of LN **2**

(iii) Find the height MG **2**

(iv) Find the area of Δ LMN **2**

(b) The diagram below shows the course of a ship, which sails from a port P on a bearing of 040^0 for 12 km before changing course to a bearing of 120^0 and travelling a further 15 km to a destination A.



(i) Explain why $\angle PBA = 100^0$ **1**

(ii) Find the distance of A from P to the nearest km **2**

(iii) Find the bearing of P from A to the nearest degree. **2**

QUESTION 7: (12 Marks)

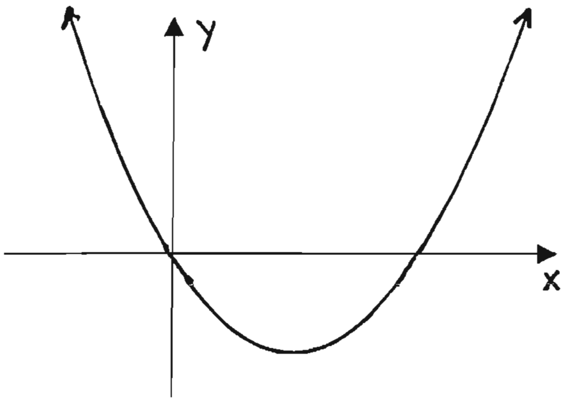
Marks

(a) Solve $5^{1-2x} = 25$ **1**

(b) The point $(k, 1)$ is equidistant from the points A(2, 3) and B(-1, 4). Find the value of k. **3**

(c) Solve $2\cos^2 x - 1 = 0$ for $0^0 \le x \le 360^0$ **4**

(d) The diagram below represents $y = kx^2 - 4kx$



(i) Find the intercepts with the x-axis **1**

(ii) Find the co-ordinates of the vertex in terms of k **2**

(iii) What does the result to part (ii) above imply about the number of roots to the equation $kx^2 - 4kx + A$ when $A>4k$? **1**

QUESTION 8: (11 Marks)

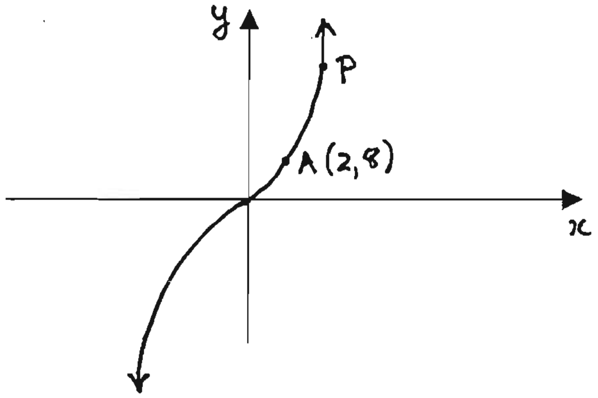
Marks

(a) On a set axes draw the graph of $(x-1)^2+(y+2)^2=9$ — **3**

and on it shade the region satisfying $(x-1)^2+(y+2)^2 \le 9$

(b) Show that $\frac{(1-\cos\theta)(1+\cos\theta)}{\sin\theta\cos\theta}=\tan\theta$ — **2**

(c) The curve given below is $y=x^3$. The point A on it is (2, 8).



(i) The point P has an x value of $2+h$.
Give the y co-ordinate of point P.

(You may use the formula $(a+b)^3=a^3+3a^2b+3ab^2+b^3$*)* — **1**

(ii) Showing all working, prove that the slope of the chord PA is given by — **3**

$$12+6h+h^2$$

(iii) Find the limit of the slope of chord PA as P approaches A along the curve (ie as $h \to 0$) — 1

(iv) Give an interpretation of the answer you obtained in part (iii) above — **1**

END OF EXAMINATION PAPER

2 UNIT YEARLY – YEAR 11 2008.

Teacher's Name: **Student's Name/N°:**

(1) (a) $5(x+3)(x-3)$ ← (1) (1)

(b) $\frac{\sqrt{5}(\sqrt{5}+1)}{4} = \frac{5+\sqrt{5}}{4}$ ← (1) ← (1)

(c) $7\sqrt{3} - 6\sqrt{3} = \sqrt{3}$ ← (1) (1)

(d) $x^3 - 8$

(e) (i) Angle sum $= 720°$

$\therefore$ Each angle is $120°$ (1)

(ii) Since $\angle SAC = 180°$

$\angle BAC = 60°$ } (1)

Similarly, $\angle ABC = 60°$

$\therefore \angle ACB = 60°$ (angle sum of $\triangle ABC$) (1)

$\therefore \triangle ABC$ is equilateral (1)

(f) EITHER:

Slope $= -\frac{1}{2}$. (1)

$\therefore y + 4 = -\frac{1}{2}(x-3)$

$2y + 8 = -x + 3$

$\therefore x + 2y + 5 = 0$ (2)

[subtract 1 mark if not in general form]

OR

$x + 2y + k = 0$ (1)

substitute $(3, -4)$

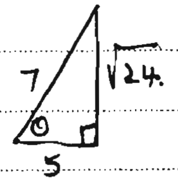
$\therefore 3 - 8 + k = 0$

$k = 5$ (1)

$\therefore x + 2y + 5 = 0$ (1)

(2) (a) $\frac{3}{2}$

(b)



← (1)

$\tan\theta = \frac{\sqrt{24}}{5}$
$= \frac{2\sqrt{6}}{5}$ } either (1)

(c) (i) $-2 - 2x$

(ii) $-\frac{6}{x^3}$ OR $-6x^{-3}$

(iii) $40x(5x^2-4)^3$

(d) (i) $\frac{\sin\theta}{4} = \frac{\sin 30°}{12}$ (1)

$\therefore \sin\theta = \frac{2}{12}$ (1)

$\theta = 9°36'$ (1)

(subtract 1 if not to nearest minute)

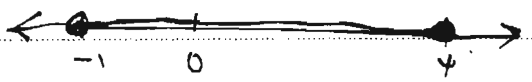
(ii) The angle sum of the triangle would exceed $180°$

(e) EITHER

$2x - 3 \le 5$ and $2x - 3 \ge -5$

either (2) { $x \le 4$ and $x \ge -1$

$-1 \le x \le 4$ }



(1)

OR $2x - 3 = 5$ or $2x - 3 = -5$

$x = 4$ or $x = -1$ (1)

(1)

If they use this method they MUST write $-1 \le x \le 4$ to get the extra mark.

Teacher's Name: Student's Name/N°:

(3)

(a) $\frac{dy}{dx} = 3x^2 - 12$

To be parallel with x-axis

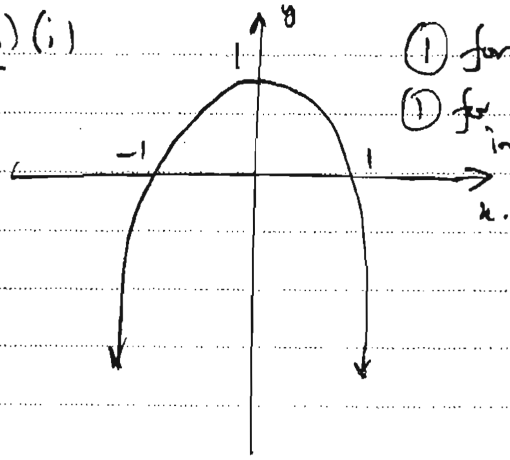
$\frac{dy}{dx} = 0$ ← (1) (if you see this anywhere)

$\therefore \quad x^2 = 4$

$\therefore \quad x = \pm 2$

Pts are $(2, -15)$ or $(-2, 17)$

(b) (i)



(1) for shape
(1) for all intercepts

(ii) $f(-3) = -8$

(iii) Range $y \leq 1$.

(iv) $x \geq 0$ (no mark if the equals sign is missing)

(v) $1 - (\sqrt{x})^2 = 1 - x$

(c) $\frac{dy}{dx} = \frac{(x^2+2)(-2x) - (2-x^2)(2x)}{(x^2+2)^2}$ ← (1)

$= \frac{-8x}{(x^2+2)^2}$ ← (1)

(ii) $\frac{dy}{dx} = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)$ (1)

$= \frac{-x}{\sqrt{1-x^2}}$ (1)

(4)

(a) (i) $\alpha + \beta = \frac{3}{2}$

(ii) $\alpha\beta = 1$

(iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ (1)

$= \frac{1}{4}$ (1)

(iv) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$ (1)

$= \frac{1/4}{1}$

$= \frac{1}{4}$ (1)

(b) (i) $\angle ACB = x°$ (base angles of isosceles $\triangle ABC$)

(ii) $\angle ABC = 180 - 2x$ (angle sum of $\triangle ABC$)

(iii) Because it is co-interior to $\angle ABC$ and they must sum to 180°

(iv) $\angle DCE + \angle DEC = 180 - 2x$ and they are equal (angle sum of isosceles $\triangle DCE$)

$\therefore \quad \angle DCE = 90 - x$.

(v) It is 90°

Teacher's Name: ______________________ Student's Name/N°: ______________________

(5) (a) $y = x^3 - 2x + 1$

$\frac{dy}{dx} = 3x^2 - 2$

At $x = 2$

$m_T = 10$ (1)

$y = 5$ (1)

$y - 5 = 10(x - 2)$

$y - 5 = 10x - 20$

$y = 10x - 15$ (1)

(b) $\frac{dy}{dx} = 2 - 2x$

At $(2, 2)$ $m_T = -2$ ← (1)

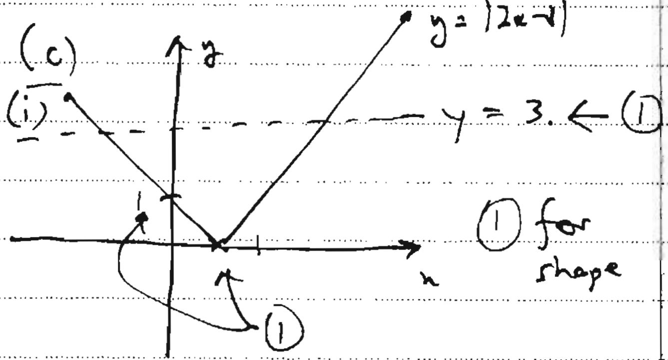
$m_N = \frac{1}{2}$ ← (1)

$y - 2 = \frac{1}{2}(x - 2)$

$2y - 4 = x - 2$

$x - 2y + 2 = 0$ ← (1)

(c) (i)



(d) No real roots ⇒ $\Delta < 0$ (1)

$\therefore [4(k-3)]^2 - 4 \cdot 4 < 0$ (1)

$\therefore k^2 - 6k + 9 - 1 < 0$

$k^2 - 6k + 8 < 0$

$(k-4)(k-2) < 0$

$k = 4$ or $k = 2$ (2)

(6)

(a)(i) $LN = \sqrt{4^2 + 1^2}$

$= \sqrt{17}$. (1)

(ii) $m_{LN} = -\frac{1}{4}$. (1)

$y - 1 = -\frac{1}{4}(x - 1)$

$4y - 4 = -x + 1$

$x + 4y - 5 = 0$ (1)

(iii) $p = \left| \frac{4 + 8 - 5}{\sqrt{17}} \right|$ (1)

$= \frac{7}{\sqrt{17}}$ (1)

(iv) Area $= \frac{1}{2} \times \frac{7}{\sqrt{17}} \times \frac{\sqrt{17}}{1}$

$= \frac{7}{2}\ u^2$

(b) By alternate angles $\angle PBS = 40°$

Also, ~~by alternate~~ by straight angles,

$\angle ABS = 60°$

$\therefore \angle PBA = 100°$

(ii) $PA^2 = 12^2 + 15^2 - 2 \times 12 \times 15 \cos 100°$ ← (1) for cosine rule

$PA = 20.77$

≈ 21 km ← (1) (no penalty if not rounded)

(iii) $\frac{\sin\theta}{12} = \frac{\sin 100°}{20.77}$

$\therefore \theta = 34°41'$

$\approx 35°$

$\therefore$ Bearing is S 85° W or 265°

Teacher's Name: Student's Name/N°:

(7) (a) $5^{1-2x} = 5^2$

$1-2x = 2$

$2x = -1$

$x = -\tfrac{1}{2}$

(b) $d_{PA} = \sqrt{(2-k)^2 + 4}$

$d_{PB} = \sqrt{(k+1)^2 + 9}$ } (1)

(1)

$\therefore 4 - 4k + k^2 + 4 = k^2 + 2k + 10$

$6k = -2$

$k = -\tfrac{1}{3}$ (1)

(c) $\cos^2 x = \tfrac{1}{2}$

$\cos x = \pm \tfrac{1}{\sqrt{2}}$ ← (2)

$\therefore x = 45°, 135°, 225°, 315°$ (2)

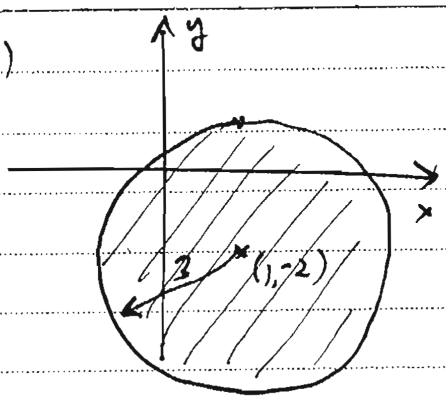
(d) (i) $x = 0$, $x = 4$

(ii) V is $(2, -4k)$

(1) (1)

(iii) there are none

(8) (a)



1 for slope

1 for centre and radius

1 for shading

(b) $LHS = \dfrac{1-\cos^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{\sin^2\theta}{\sin\theta\cos\theta}$ ← (1)

(1)

$= \tan\theta$

(c) (i) P has y co-ordinate of $(2+h)^3$ [no need to expand here]

(ii) Slope PA $= \dfrac{(2+h)^3 - 8}{2+h-2}$ (1)

the mark for the expansion may come out of part (i) → $= \dfrac{8 + 12h + 6h^2 + h^3 - 8}{h}$ (1)

$= 12 + 6h + h^2$ (1)

(iii) $\lim_{h \to 0}$ slope PA $= 12$ (1)

(iv) This is the slope of the tangent to $y = x^3$ at $(2,8)$

[OR the derivative to the curve at $x = 2$

or similar]