SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS

YEAR 11 YEARLY EXAMINATION
PRELIMINARY HSC ASSESSMENT TASK 3
SEPTEMBER 2009

General Instructions:

- Working time allowed – 120 minutes

- Write using black or blue pen

- Approved calculators may be used

- All necessary working should be shown

- Start each question on a new page

- Attempt all questions

- Questions are of equal value

- Full marks may not be awarded if working is poorly set our or difficult to read

Name: _____   **Teacher:**_____

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Question 7	Question 8	TOTAL
/10	/10	/10	/10	/10	/10	/10	/10	/80

Question 1   (10 marks)

a)   Evaluate $3^{2.5}$ correct to 3 significant figures   2

b)   Factorise completely $ab - a - bx + x$ 2

c)   Find integers a and b such that $\frac{1}{\sqrt{3}+2} = a\sqrt{3} + b$ 2

d)   Solve $tan\theta = \frac{-1}{\sqrt{3}}$ for $0° \leq \theta \leq 360°$ 2

e)   Write $1 - \frac{a-b}{a+b}$ as a single fraction in simplest form   2

Question 2   (10 marks)   Start a new page

a)   i)   Sketch $y = 2 - (x + 1)^2$ clearly label the <u>vertex</u> and the y <u>intercept</u>   2

      ii)   State the range for $y = 2 - (x + 1)^2$ 1

b)   The points A, B and C have coordinates (1,5), (6,0) and (5,7) respectively.
      Plot these points on a number plane.

      i)   Show that the length of AB is $5\sqrt{2}$ units.   1

      ii)   Show that the triangle ABC is isosceles by finding the length of BC.   1

      iii)   Find the equation of the line AB.   2

      iv)   BA is produced to meet line $y = 7$ at P.

            Find the coordinates of P.   1
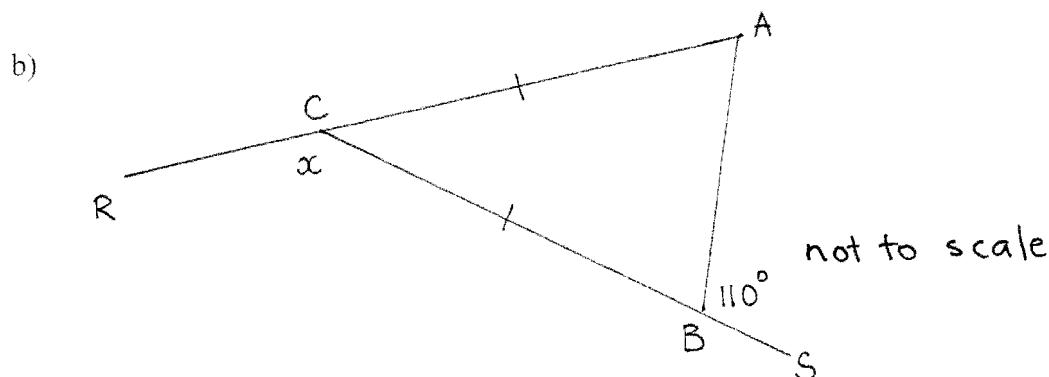
      v)   Find the area of triangle PAC.   2

Question 3   **(10 marks)**   **(Start a new page)**

a)   Express $0.3 + 0.\overset{\bullet}{3}$ in the form $\frac{a}{b}$ where   2

a and b are integers

b)   Differentiate the following

i) $y = \frac{2}{\sqrt{x}}$ 1

ii) $y = \frac{3}{x-1}$ 2

iii) $y = \frac{2x+1}{1-x}$ 2

c)   If α and β are the roots of the quadratic equation $x^2 - 5x + 2 = 0$ find the value

of   3

i) $\alpha + \beta$

ii) $\alpha\beta$

iii) $\alpha^2 + \beta^2$

Question 4   **(10 marks)**   **(Start a new page)**

a)
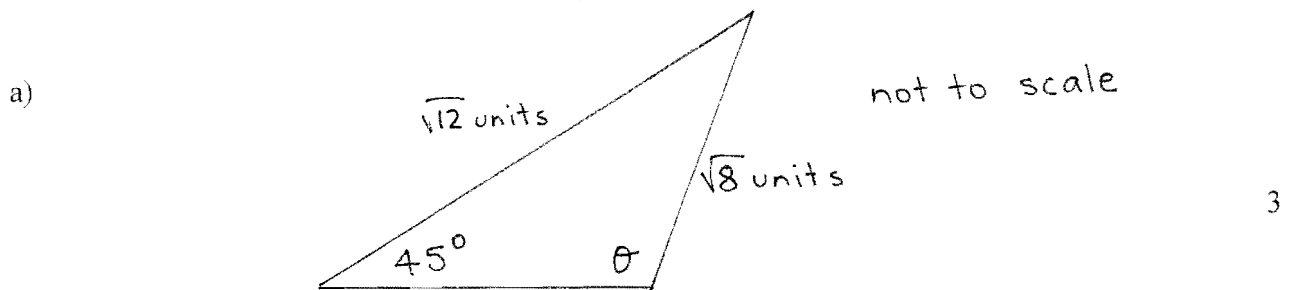
Use the sine rule to find the value of θ, where θ is obtuse   3

b)

In the diagram above $AC = BC, A\hat{B}S = 110°$ and $R\hat{C}B = x$.

Copy the diagram onto your answer sheets. Find the value of x giving reasons.   2

c)
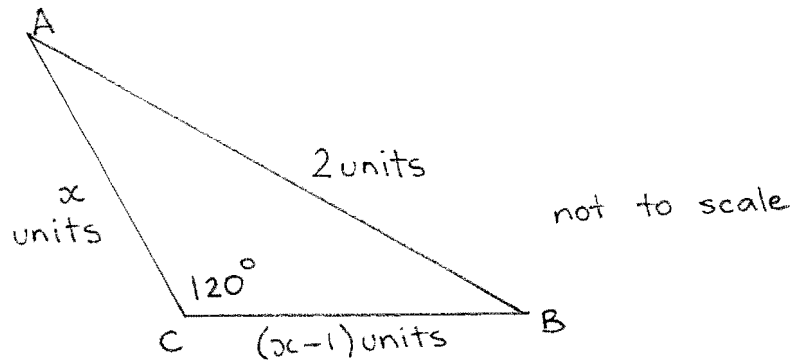
In the diagram above, ABC is a triangle in which $AC = x$ units, $BC = (x - 1)$ units and $AB = 2$ units, $ACB = 120°$.

Find the <u>exact</u> value of x 3

d)   Solve the pair of simultaneous equations
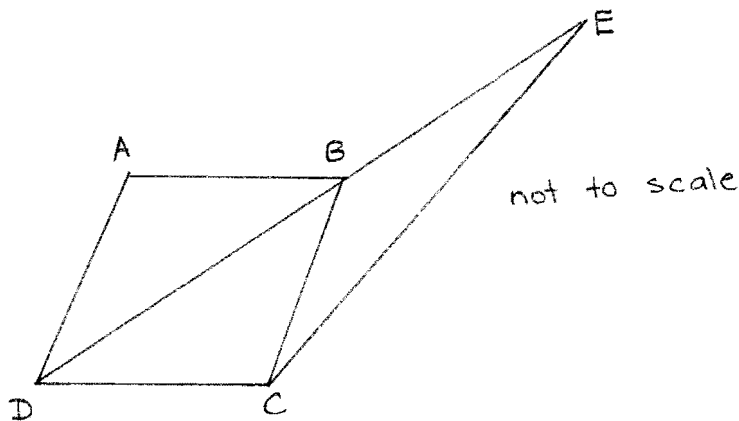
$$3x - y = 15$$
$$x + 2y = -2$$ 2

Question 5   (10 marks)   (Start a new page)

a)

In the diagram above, $ABCD$ is a rhombus.
DB is produced to E such that $D\hat{E}C = 10°, B\hat{D}C = 30°$

Copy the diagram showing the above information.
Calculate the size of $B\hat{C}E$ giving reasons   3

b) Solve $2\cos^2 x - 3\cos x - 2 = 0$ for $-180° \le x \le 180°$ 3

c) Prove $\dfrac{1}{\cot^2\theta} + 1 = \sec^2\theta$ 2

d) Prove that $5x - 12y + 52 = 0$ is a tangent to the circle with centre the origin and
    radius 4   2

Question 6   (10 marks)   (Start a new page)

a) i)   On the same diagram sketch $y = x - 4$ and $xy = 5$ (use a ruler for your
         axes)   Label each function clearly   2
    ii) Find the <u>points</u> of intersection of these graphs   2

b) A piecemeal function is defined as $f(x) = \begin{cases} 1 - x, x < 0 \\ x^2 + 2, x \ge 0 \end{cases}$

    i)   Sketch the function   2
    ii) Find the value of $f(-2) + f(0) + f(3)$ 1
    iii) For what value(s) of x is this function not differentiable   1

c) Find the value of k if the equation $x^2 - 6x + 4k = 0$ has real roots   2

**Question 7   (10 marks)   (Start a new page)**

a) Differentiate $y = x^2(1 - 2x)^3$ 2

b) Find $\displaystyle\lim_{x \to 2}\left(\dfrac{x^2 - 4}{x - 2}\right)$ 1

c) If $f(x) = x^2 - 3x + 5$, for what value of x is $f'(x) = 9$ 2

d) The curve $y = ax + \dfrac{b}{x^2}$ cuts the x axis at $x = 2$, and the gradient of the
    tangent to the curve at this point is 3. Find the values of a and b.   3

e) By first substituting m for x^2, solve the equation $x^4 - 10x^2 + 9 = 0$ for x 2

Question 8   (10 Marks)   (Start a new page)

a)   For what values of a will $ax^2 + 5x + a$ be positive definite   2

b)
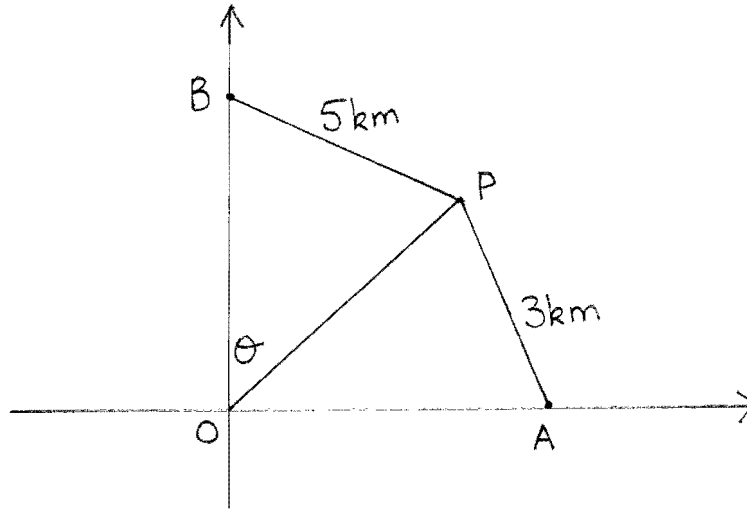
In the diagram above, A is a point due east of O and B is a point due north of O. P is on a bearing of $120°$ from B and $345°$ from A. AP is 3 km and BP is 5 km. The bearing of P from O is θ.

i)   Find the size of $O\hat{B}P$ and $O\hat{A}P$ 2

ii)   Prove that $OP = \dfrac{5\sqrt{3}}{2sin\theta}$ 2
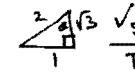
iii)   Prove that $OP = \dfrac{3sin75°}{cos\theta}$ 2
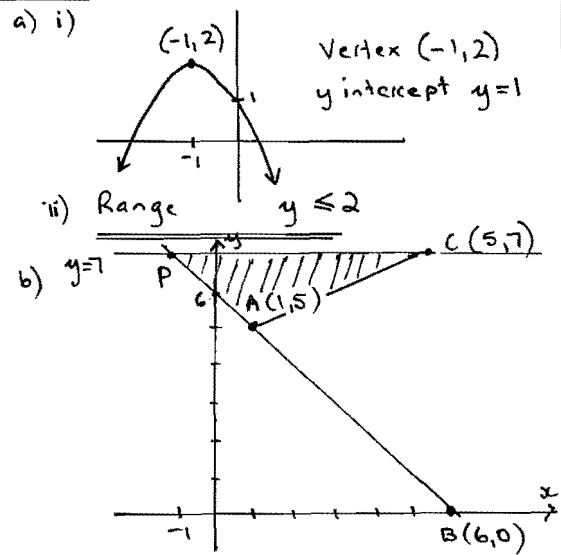
iv)   Using parts ii) and iii) find θ to the nearest degree   2

Question 1

a) $3^{2.5} = \underline{15.6}$ (3 sig fig)

b) $ab - a - bx + x$
$= a(b-1) - x(b-1)$
$= \underline{(b-1)(a-x)}$

c) $\dfrac{1}{\sqrt{3}+2} \times \dfrac{\sqrt{3}-2}{\sqrt{3}-2} = \dfrac{\sqrt{3}-2}{3-4}$
$= -\sqrt{3}+2$
$\therefore \underline{a=-1 \quad b=2}$

d) $\tan\theta = -\dfrac{1}{\sqrt{3}}$

acute $\theta = 30°$
$\therefore \underline{\theta = 150°, 330°}$

e) $1 - \dfrac{a-b}{a+b} = \dfrac{(a+b)-(a-b)}{a+b}$
$= \underline{\dfrac{2b}{a+b}}$

Question 2

a) i)

Vertex $(-1,2)$
y intercept $y=1$

ii) Range $y \le 2$

b) $y=7$
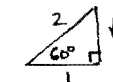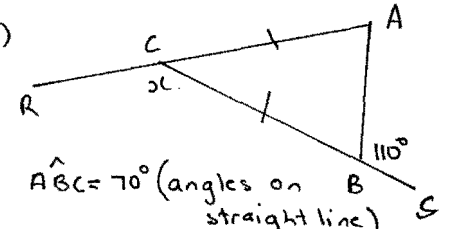

i) $AB = \sqrt{(6-1)^2 + (0-5)^2}$
$= \sqrt{25+25}$
$= \sqrt{50}$
$= \underline{5\sqrt{2} \text{ units}}$

ii) $BC = \sqrt{(6-5)^2 + (0-7)^2}$
$= \sqrt{1+49}$
$= \sqrt{50}$
$= 5\sqrt{2}$
$\underline{AB = BC \quad \therefore \quad \triangle ABC \text{ is isosceles}}$

iii) $m_{AB} = \dfrac{5-0}{1-6} = \dfrac{5}{-5} = -1$

eqn AB: $y - 5 = -1(x-1)$
$\underline{y = -x + 6}$

iv) $\underline{P(-1, 7)}$

v) area $\triangle PAC = \dfrac{6 \times 2}{2}$
$= \underline{6 \text{ units}^2}$

Question 3

a) $0.\dot{3} + 0.\dot{3}$
$\dfrac{3}{10} + \dfrac{1}{3} = \dfrac{9+10}{30}$
$= \underline{{}^{19}/_{30}}$

b) i) $y = \dfrac{2}{\sqrt{x}} = 2x^{-1/2}$
$\dfrac{dy}{dx} = 2x - \dfrac{1}{2}x^{-3/2}$
$= \dfrac{-1}{\sqrt{x^3}}$

ii) $y = 3(x-1)^{-1}$
$\dfrac{dy}{dx} = 3x - 1 (x-1)^{-2}$
$= \dfrac{-3}{(x-1)^2}$

iii) $u = 2x+1 \qquad v = 1-x$
$u' = 2 \qquad v' = -1$
$\dfrac{dy}{dx} = \dfrac{2(1-x) - -1(2x+1)}{(1-x)^2}$
$= \dfrac{2 - 2x + 2x + 1}{(1-x)^2}$
$= \dfrac{3}{(1-x)^2}$

c) i) $\alpha + \beta = \dfrac{-b}{a} = \underline{5}$

ii) $\alpha\beta = \dfrac{c}{a} = \underline{2}$

iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
$= 25 - 4$
$= \underline{21}$

Question 4

a) $\dfrac{\sin\theta}{\sqrt{12}} = \dfrac{\sin 45°}{\sqrt{8}}$
$\sin\theta = \sqrt{12} \cdot \dfrac{1}{\sqrt{2}} \cdot \dfrac{1}{\sqrt{8}}$
$\sin\theta = \dfrac{2\sqrt{3}}{\sqrt{2} \cdot 2\sqrt{2}}$
$\sin\theta = \dfrac{\sqrt{3}}{2}$
$\therefore$ acute $\theta = 60°$
$\underline{\text{obtuse } \theta = 120°}$

b)

$A\hat{B}C = 70°$ (angles on straight line)
$x = 140°$ (exterior angle of isosceles triangle)

c) $2^2 = x^2 + (x-1)^2 - 2x(x-1)\cos 120°$
$4 = x^2 + x^2 - 2x + 1 - 2x(x-1)(-\tfrac{1}{2})$
$4 = 2x^2 - 2x + 1 + x^2 - x$
$0 = 3x^2 - 3x - 3$
$0 = x^2 - x - 1$
$x = \dfrac{1 \pm \sqrt{1 - 4 \times 1 \times -1}}{2}$
$x = \dfrac{1 \pm \sqrt{5}}{2} \quad$ since $\quad x > 0$
$\therefore \underline{x = \dfrac{1+\sqrt{5}}{2} \text{ units} \quad \text{only}}$

d) $3x - y = 15 \quad$ ①
$x + 2y = -2 \quad$ ②
① $\times 2 \quad 6x - 2y = 30$
$\quad\quad\quad\quad x + 2y = -2$ $\bigoplus$
$\quad\quad\quad\quad 7x = 28$
$\therefore \underline{x = 4 \quad y = -3}$

Question 5

a)

$D\hat{C}E = 140°$ (angle sum $\triangle DCE$)
$D\hat{C}B = 120°$ (angle sum isosceles $\triangle DBC$)
$\therefore \underline{B\hat{C}E = 20°}$

b) $2\cos^2 x - 3\cos x - 2 = 0$
$(2\cos x + 1)(\cos x - 2) = 0$
$\cos x = -\tfrac{1}{2} \qquad \cos x = 2$ no sol.
acute $x = 60°$
$\therefore \underline{x = 120°, -120°}$