SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS

YEAR 11 YEARLY EXAMINATION

PRELIMINARY HSC ASSESSMENT TASK 3

SEPTEMBER 2010

General Instructions:

- Working time allowed – 120 minutes
- Write using black or blue pen
- Approved calculators may be used
- All necessary working should be shown
- Start each question on a new page
- Attempt all questions
- Questions are of equal value
- Full marks may not be awarded if working is poorly set out or difficult to read.

Name:_____ **Teacher:**_____

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Question 7	Question 8	Total
/10	/10	/10	/10	/10	/10	/10	/10	/80

Question 1 **Marks**

a) Simplify $\frac{a^4(b^2)^5}{a^3b^4}$ 2

b) Evaluate $\frac{15.73-8.27}{\sqrt[3]{2.43}}$ correct to 3 significant figures. 2

c) Find the values of a and b if $\frac{2}{2+\sqrt{3}} = a - \sqrt{b}$ 2

d) If $tan\theta = \frac{-8}{15}$, find $sin\theta$, if $90 < \theta < 180$ (leave in fraction form) 2

e) Simplify $\frac{9x^2-4}{6x^2-x-2}$ 2

Question 2 (Start on a new page)

a) Solve $|3x + 2| \geq 8$ 2

b) i) Sketch the graph $y = |x - 1|$ 1

State its

ii) domain 1

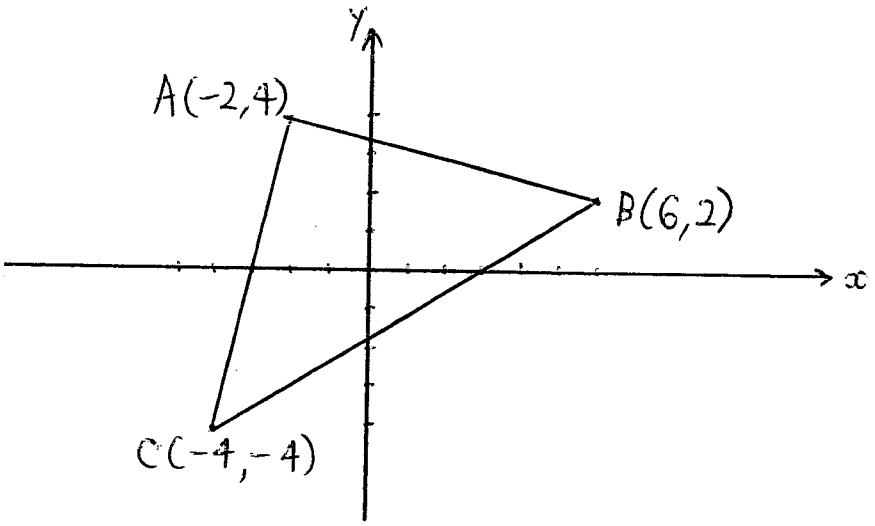
iii) range 1

c) Solve $x^6 + 7x^3 - 8 = 0$ using the substitution $V = x^3$ 3

d) Find $\lim_{x \to 2} \frac{x^2-4}{x-2}$ 2

Question 3 (Start on a new page) **Marks**

a) The diagram below shows the points $A(-2,4), B(6,2)$ and $C(-4,-4)$



i) Calculate the length of the interval BC 1

ii) Find the gradient of the line BC. 1

iii) Find the coordinates of M, the midpoint of BC. 1

iv) Show that the equation of l, the perpendicular bisector of BC is $5x + 3y - 2 = 0$. 2

v) Show that l passes through A. 1

vi) Given the equation of BC *is* $3x - 5y - 8 = 0$ find the perpendicular distance of A from BC 2

vii) Hence or otherwise find the area of ΔABC. 1

b) Find the angle sum of a 15 sided polygon. 1

Question 4 (Start on a new page)

a) Differentiate

i) $-3x^4$ 1

ii) $(4x - 9)^5$ 1

iii) $4x^2(6x-5)^5$ 2

iv) $\frac{4x-3}{3x+4}$ 2

b) i) Find the point on the curve $y = x^2 + 2$ where the tangent has a gradient of -4. 2

ii) Find the equation of the normal at this point and give your answer in general form. 2

Question 5 (Start on a new page) **Marks**

a) Solve $x^2 - 7x + 12 > 0$ and sketch your solution on a number line. 3

b) Solve $|2x + 6| = 3x - 1$ and show that only one solution is valid. 3

c) i) Find the discriminant of $2x^2 - 3x + K$ 1

ii) For what values of K is the expression $2x^2 - 3x + K$ positive definite? 2

iii) For what value of K does $2x^2 - 3x + K$ have equal roots? 1

Question 6 (Start on a new page)

a) Prove that $\frac{1}{1+cot^2\theta} = (1 - cos\theta)(1 + cos\theta)$ 3

b) Find the values of A, B *and* C if $3x^2 - 7x + 5 \equiv Ax(x - 1) + Bx + C$ 3

c) If α and β are the roots of the equation $x^2 - 2x - 7 = 0$ find the value of

i) $\alpha + \beta$ 1

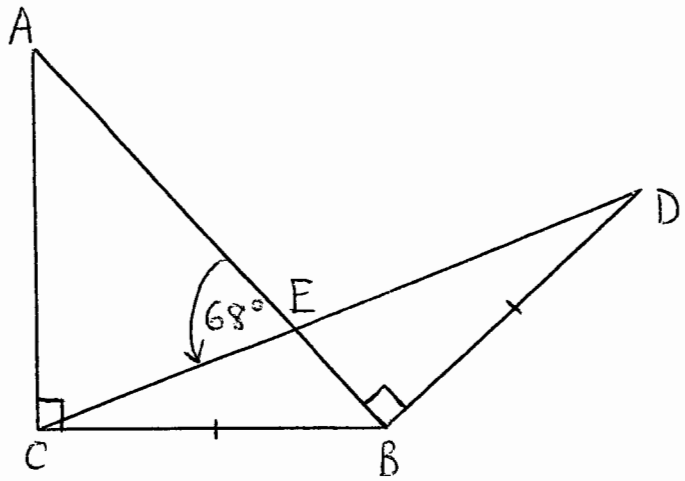
ii) $\alpha\beta$ 1

iii) $\alpha^3\beta + \alpha\beta^3$ 2

Question 7 (Start on a new page)

a) If $f(x) = (x+1)(x+2)$ find in simplest terms an expression for $f(x+1) - f(x)$. 2

b) 3



Copy the above diagram into your answer booklet.

In the diagram, above find $\angle DCB$ giving reasons.

c) i) Sketch $y = x + 1$ and $y = x - 2$ on the same graph. 1

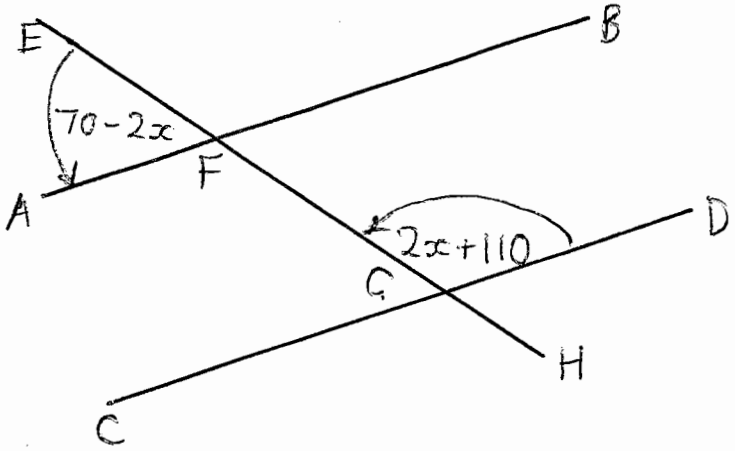
ii) Shade the region $y \geq x + 1$ 1

iii) What inequality describes the region between and including the two lines? 1

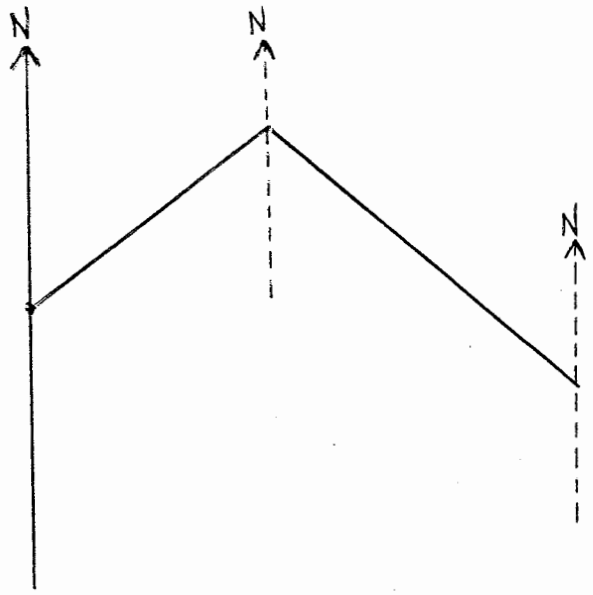
d) Sketch $y = \frac{2}{x-2}$ showing all important features. 2

Question 8 (Start on a new page) **Marks**

a) Prove that AB and CD are parallel. 2



b)



A ship sails for 270 n. miles from Sydney on a bearing of $055°T$. It then turns and sails for 360 n. miles on a bearing of $120°T$.

i) Copy the above diagram onto your answer sheet and show the information given. 1

ii) How far is the ship from Sydney to the nearest nautical mile? 2

iii) Find the bearing of Sydney from the ship to the nearest degree 2

c) Solve $cos^2\theta = \frac{3}{4}$ for $0° \leq \theta \leq 360°$. 3

Teacher's Name: Student's Name/N°:

Year 11 Mathematics Yearly 2010 Solutions

Question 1

a) $\frac{a^4 b^{10}}{a^3 b^4}$

$= ab^6$

b) $5 \cdot 55$

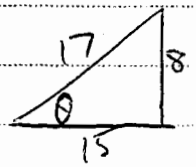
c) $\frac{2}{2+\sqrt{3}} = a - \sqrt{b}$

$\frac{2}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{4-2\sqrt{3}}{1} = a - \sqrt{b}$

$4 - \sqrt{12} = a - \sqrt{b}$

$a = 4 \quad b = 12$

d)



$\sin\theta = \frac{8}{17}$

e) $\frac{(3x-2)(3x+2)}{(3x-2)(2x+1)}$

$= \frac{3x+2}{2x+1}$

Question 2

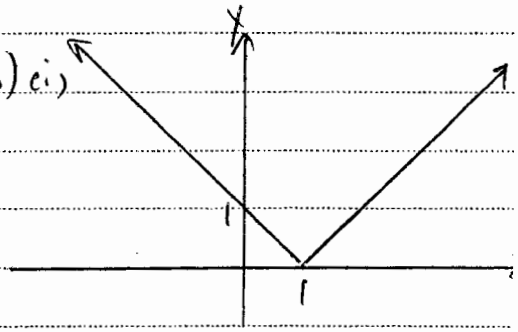
a) $|3x+2| \geq 8$

$3x+2 \geq 8 \quad$ or $\quad 3x+2 \leq -8$

$3x \geq 6 \quad$ or $\quad 3x \leq -10$

$x \geq 2 \quad$ or $\quad x \leq -\frac{10}{3}$

b) (i)



(ii) D: All real x

(iii) R: $y \geq 0$

c) $x^6 + 7x^3 - 8 = 0$

$V^2 + 7V - 8 = 0$ if $V = x^3$

$(V-1)(V+8) = 0$

$V = 1$ or -8

$x^3 = 1$ or $x^3 = -8$

$x = 1$ or $x = -2$

d) $\lim_{x \to 2} \frac{x^2-4}{x-2}$

$\lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)}$

$= 4$

Question 8

a) $\angle BFG = 70 - 2x$ (vertically opp.)

$\angle BFG + \angle FGD$

$70 - 2x + 2x + 110 = 180$

$\therefore AB \parallel CD$ (cointerior angles are supplementary)

Teacher's Name: Student's Name/N°:

Question 3

a) c) i) $BC = \sqrt{(6--4)^2+(2--4)^2}$
$= \sqrt{10^2+6^2}$
$= \sqrt{136}$

ii) $M_{BC} = \frac{2--4}{6--4}$
$= \frac{6}{10}$
$= \frac{3}{5}$

iii) $M_{BC}: \left(\frac{-4+6}{2}, \frac{-4+2}{2}\right)$
$= (1,-1)$

iv) $m = -\frac{5}{3}$, $(1,-1)$
$y--1 = -\frac{5}{3}(x-1)$
$3y+3 = -5x+5$
$5x+3y-2=0$

v) $(-2,4)$ satisfies $5x+3y-2=0$
if $5\times-2+3\times4-2=0$
$-10+12-2=0$
$2-2=0$ ✓
∴ A lies on line.

vi) $d = \frac{|3\times-2+-5\times4-8|}{\sqrt{3^2+(-5)^2}}$
$= \frac{34}{\sqrt{34}} = \sqrt{34}$

vii) $A = \frac{1}{2}bh$
$= \frac{1}{2}\times\sqrt{136}\times\sqrt{34}$
$A = 34$ units^2

b) Angle Sum $= (n-2)\times180$
$= 13\times180$
$= 2340^\circ$

Question 4

a) i) $-3x^4$
$\frac{d}{dx} = -12x^3$

ii) $(4x-9)^5$
$\frac{d}{dx} = 5(4x-9)^4\times4$
$= 20(4x-9)^4$

iii) $4x^2(6x-5)^5$
$\frac{d}{dx} = 4x^2\times5(6x-5)^4\times6 + 8x(6x-5)^5$
$= 120x^2(6x-5)^4 + 8x(6x-5)^5$

iv) $\frac{4x-3}{3x+4}$

$\frac{d}{dx} = \frac{(3x+4)4-(4x-3)3}{(3x+4)^2} = \frac{25}{(3x+4)^2}$

Teacher's Name: **Student's Name/N°:**

b.(i) $y = x^2 + 2$

$\frac{dy}{dx} = 2x$

$-4 = 2x$

at $x = -2$ ∴

Pt. is $(-2, 6)$

(ii) $m = \frac{1}{4}$ $(2, 6)$

$y - 6 = \frac{1}{4}(x + 2)$

$4y - 24 = x + 2$

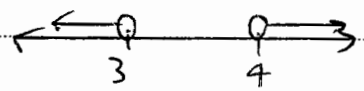
$x - 4y + 26 = 0$

Question 5

a) $x^2 - 7x + 12 > 0$

$(x - 4)(x - 3) > 0$

$x > 4, \quad x < 3$



b) $|2x + 6| = 3x - 1$

$2x + 6 = 3x - 1$ or $2x + 6 = -(3x$

$7 = x$ or $5x = -5$

$x = -1$

Sub. $x = 7$ ✓ Sub. $x = -1$

$|14 + 6| = 21 - 1$ ✓ $|-2 + 6| \neq -4$ X

∴ only $x = 7$ valid

c)(i) $\Delta = b^2 - 4ac$

$= (-3)^2 - 4 \times 2 \times K$

$= 9 - 8K$

(ii) If $\Delta < 0$

$9 - 8k < 0$

$k > \frac{9}{8}$

(iii) $\Delta = 0$ when $k = \frac{9}{8}$

Question 6

a) $\frac{1}{1 + \cot^2\theta} = (1 - \cos\theta)(1 + \cos\theta)$

$\frac{1}{\operatorname{cosec}^2\theta} = (1 - \cos\theta)(1 + \cos\theta)$

$\sin^2\theta = "$

$1 - \cos^2\theta = "$

$(1 - \cos\theta)(1 + \cos\theta) =$ RHS

b) $3x^2 - 7x + 5 \equiv Ax(x - 1) + Bx + C$

When $x = 0$

$5 = C$

When $x = 1$,

$1 = B + C \quad \therefore B = -4$

When $x = 2$

$3 = 2A - 8 + 5$

$A = +3$

Teacher's Name: **Student's Name/N°:**

c) (i) $\alpha + \beta = \frac{-b}{a}$ (ii) $\alpha\beta = \frac{c}{a}$ (iii) $\alpha^3\beta + \alpha\beta^3$

$= \frac{2}{1}$ $= \frac{-7}{1}$ $\alpha\beta(\alpha^2 + \beta^2)$

$= 2$ $= -7$ $-7[(\alpha+\beta)^2 - 2\alpha\beta]$

$-7[2^2 - 2\times -7]$

-126

Question 7

a) $f(x+1) - f(x)$

$(x+1+1)(x+1+2) - (x+1)(x+2)$

$x^2 + 5x + 6 - x^2 - 3x - 2$

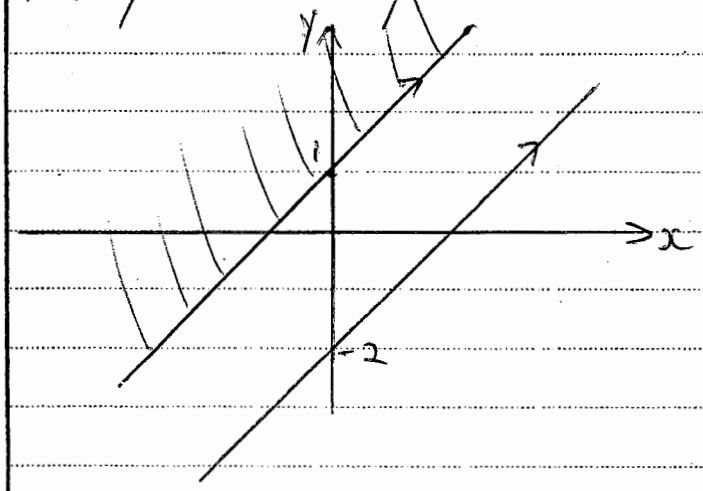
$2x + 4$

b) $\angle DEB = 68°$ (vertically opp.)

$\angle EDB = 22°$ ($\angle$ sum of $\triangle$)

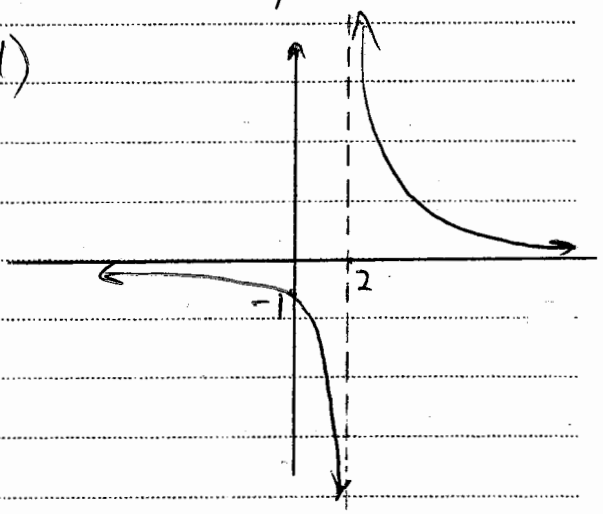
$\angle DCB = 22°$ (equal $\angle$'s opp equal sides)

c) (i) $y = x + 1$ $y = x - 2$



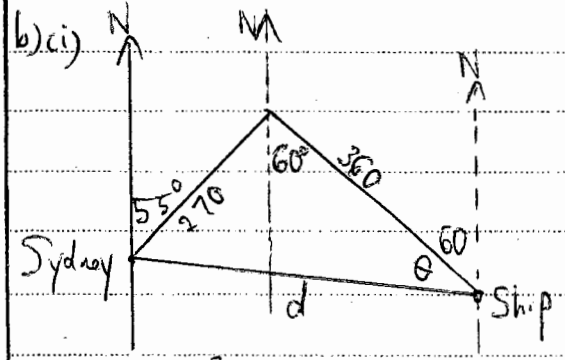
(ii) $x - 2 \leq y \leq x + 1$

d)



Question 8

b) (i)



(ii) $d^2 = 270^2 + 360^2 - 2\times 270\times 360\cos 115$

$d = 534$ n. miles

(iii) $\frac{\sin\theta}{270} = \frac{\sin 115}{534}$

$\sin\theta = 0.458$

$\theta = 27°$ (nearest degree)

$\therefore$ Bearing is 273° T or N 87° W

c) $\cos^2\theta = \frac{3}{4}$

$\cos\theta = \pm\frac{\sqrt{3}}{2}$

Working θ is 30°

$\therefore \theta = 30°, 150°, 210°, 330°$