



**Question 1****Marks**

a) Solve  $\frac{1}{4}(2x-3) + x = 2$

/2

b) Simplify  $\frac{x+2}{2} - \frac{x-1}{3}$

/2

- c) Divide the product of  
 $2.9384 \times 10^{12}$  and  $1.58 \times 10^{11}$  by  $2.068 \times 10^{10}$   
Give your answer correct to 3 significant figures

/2

- d) Find the values of  $a$  and  $b$  if

$$\frac{3}{\sqrt{5+2}} = a + \sqrt{b}$$

/2

- e) Brian pays \$34.50 for a shirt which was originally marked at \$45.00  
Calculate the percentage discount correct to one decimal place

/2

- f) Find the exact value of  $\tan 300^\circ$

/1

**Question 2****Marks**

a) Find the range of values of  $p$  for which  $p - 6x - x^2$  is negative definite /2

b) Use the general quadratic formula to solve:  $2x^2 - 2x - 5 = 0$   
Leave your answer in simplified surd form. /2

c) (i) For the function:  $f(x) = x^2 + 2$  write down the simplified expression for /2

$$\frac{f(x+h) - f(x)}{h}$$

(ii) Hence or otherwise find: /1

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

d) Evaluate: /2

$$\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x - 3}$$

e) Sketch the function  $y = |x| - 3$  showing all important features including intercepts /2

**Question 3****Marks**

a) Solve:  $|2x + 3| = 7$

/2

b) Differentiate with respect to  $x$ 

(i)  $y = (3x - 5)^4$

/2

(ii)  $y = \frac{2x-3}{x+5}$

/2

(iii)  $y = 2x^2(5x + 2)^3$

/2

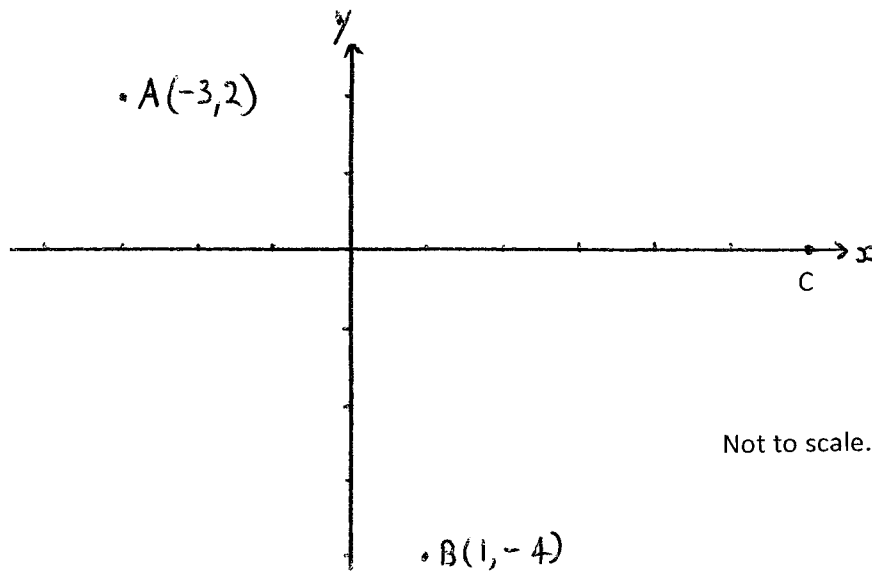
c) Find the equation of the normal to the curve  $y = 3x^2 - 5x + 2$  at the point  $(1, 0)$ 

/3

Question 4

Marks

a)



A, B and C are the vertices of a right angled triangle with  $\angle ABC = 90^\circ$

(i) Find the length AB (exact value)

/1

(ii) Find the gradient of AB

/1

(iii) Find the equation of BC in general form.

/2

(iv) Find the coordinates of C

/1

(v) Find the area of  $\triangle ABC$

/2

b) Find the shortest distance between the parallel lines

/2

$3x+2y+1=0$  and  $3x+2y-1=0$ ? (leave answer as a surd)

c) Solve  $|2x - 1| = x - 7$

/2

**Question 5****Marks**a) Solve for  $0 \leq \theta \leq 360^\circ$ 

(i)  $2\cos \theta + 1 = 0$

/2

(ii)  $\operatorname{cosec}^2 \theta = 2$

/2

b) Find the quadratic equation whose sum of roots is 5 and product of roots is  $-3$ 

/2

c) Find the values of  $k$  in  $3x^2 - kx + 3 = 0$  if the quadratic equation has no real roots.

/2

d) If  $\alpha$  and  $\beta$  are the roots of the quadratic equation  $x^2 + 3x - 5 = 0$ , find

(i)  $\alpha + \beta$

/1

(ii)  $\alpha\beta$

/1

(iii)  $\alpha^2 + \beta^2$

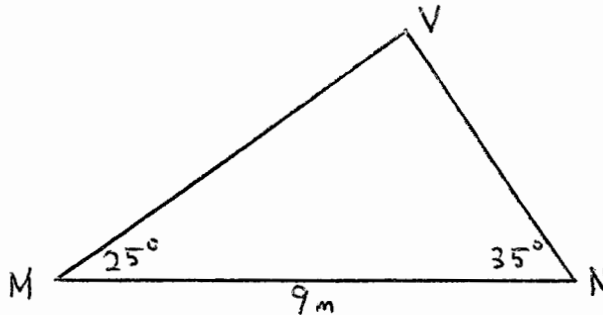
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**Question 6****Marks**

a) Prove that  $\frac{1}{\cot^2 \theta + 1} = (1 - \cos \theta)(1 + \cos \theta)$

/3

b) The framework of a roof MNV is shown in the diagram below



The frame spans a horizontal distance of 9m with beams VM and VN inclined at angles  $25^\circ$  and  $35^\circ$  respectively to the horizontal.

i. Calculate the length of the beam VN to the nearest centimetre.

/2

ii. Hence or otherwise find the area enclosed by the frame (correct to 1 decimal place)

/2

c) Draw a diagram showing the region described by the following inequalities:

/2

$$y \leq 4 - x^2 \quad y \geq 0 \quad x \leq 0$$

d) Solve  $2^{3x-1} = \frac{1}{4}$

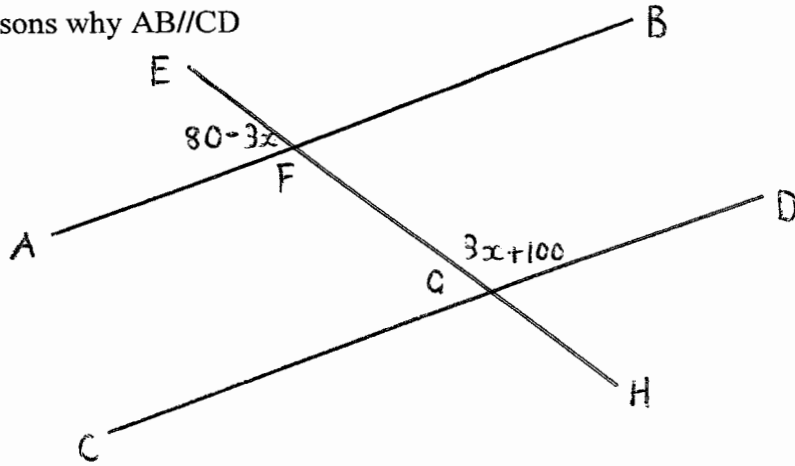
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Question 7

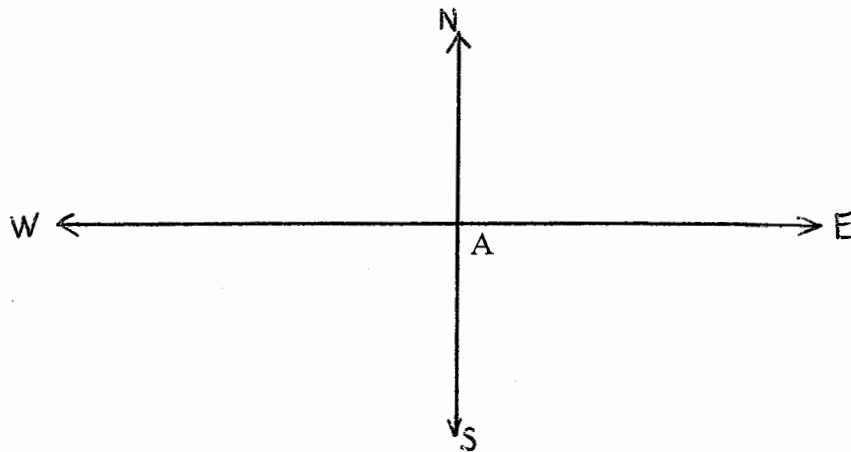
Marks

a) Give reasons why  $AB \parallel CD$

/2



b) From town A a plane flies on a bearing of  $310^\circ$  for 16km to town B. It then changes direction and flies at a bearing of  $200^\circ$  for 20km to town C.



i. Copy the diagram above including the given information

/1

ii. Find the distance of town A from town C (correct to 2 decimal places)

/2



- c) Find the  $x$  coordinate of the point on the curve  $y = x^2 + 2$  where the tangent has a gradient of  $-2$ . /1

d) For the function 
$$f(x) = \begin{cases} x^3 + 1 & \text{if } x > 2 \\ 2x & \text{if } -1 \leq x \leq 2 \\ 5 & \text{if } x < -1 \end{cases}$$

Find  $f(-3) - f(5) + f(2)$  /2

e) Solve  $25 - n^2 \geq 0$  /2

f) Find the angle sum of a 16 sided polygon. /1

**Question 8****Marks**

a) Solve  $(2^x)^2 - 6(2^x) + 8 = 0$

/2

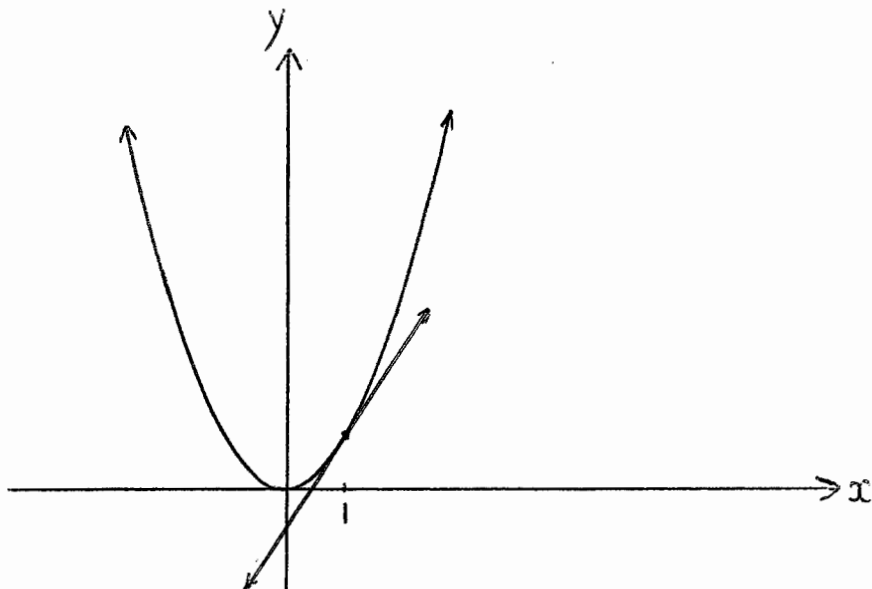
b) (i) State the domain and range of  $y = \frac{1}{x-1} + 1$

/2

(ii) Sketch the curve showing all important features including intercept

/2

c)



- i. Find the equation of the tangent to  $y = x^2$  at  $x = 1$  /2
- ii. Find where this tangent crosses the  $x$  and  $y$  axes. /2
- iii. Find the area of the triangle enclosed by the tangent and the  $x$  and  $y$  axes. /1

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# Solutions 2011 2 Unit Preliminary Final

## Question 1

$$\begin{aligned}
 \text{a) } \frac{1}{4}(2x-3) + x &= 2 \\
 2x-3 + 4x &= 8 \\
 6x-3 &= 8 \\
 6x &= 11 \\
 \underline{x = \frac{11}{6}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \frac{x+2}{2} - \frac{x-1}{3} \\
 \frac{3(x+2) - 2(x-1)}{6} \\
 \frac{3x+6-2x+2}{6} \\
 \underline{\underline{\frac{x+8}{6}}}
 \end{aligned}$$

$$\text{c) } 2 \cdot 25 \times 10^{13}$$

$$\begin{aligned}
 \text{d) } \sqrt[3]{5+2} &= a + \sqrt{b} \\
 \frac{3}{\sqrt{5+2}} \times \frac{\sqrt{5-2}}{\sqrt{5-2}} \\
 &= \frac{3\sqrt{5-6}}{1} \\
 &= -6 + \sqrt{45} \\
 \underline{\underline{a = -6 \quad b = 45}}
 \end{aligned}$$

$$\begin{aligned}
 \text{e) Discount} &= \$10.50 \\
 \frac{10.50}{45} \times \frac{100}{1} \\
 &= \underline{\underline{23.3\%}}
 \end{aligned}$$

$$\begin{aligned}
 \text{f) } -\tan 60 \\
 &= -\frac{\sqrt{3}}{1} \\
 &= \underline{\underline{-\sqrt{3}}}
 \end{aligned}$$

## Question 2

$$\begin{aligned}
 \text{a) } -x^2 - 6x + p \\
 \Delta < 0 \\
 36 - 4 \times 1 \times p < 0 \\
 36 + 4p < 0 \\
 4p < -36 \\
 \underline{\underline{p < -9}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } 2x^2 - 2x - 5 &= 0 \\
 \frac{2 \pm \sqrt{(-2)^2 - 4 \times 2 \times (-5)}}{2 \times 2} \\
 x &= \frac{2 \pm \sqrt{44}}{4} \\
 &= \frac{2 \pm 2\sqrt{11}}{4} \\
 \underline{\underline{x = \frac{1 \pm \sqrt{11}}{2}}}
 \end{aligned}$$

$$\begin{aligned}
 \text{e) (i) } \frac{f(x+h) - f(x)}{h} \\
 = \frac{(x+h)^2 + 2 - (x^2 + 2)}{h} \\
 = \frac{x^2 + 2xh + h^2 + 2 - x^2 - 2}{h}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) } \lim_{h \rightarrow 0} 2x+h \\
 = \underline{\underline{2x}}
 \end{aligned}$$

$$= \underline{\underline{2x+h}}$$

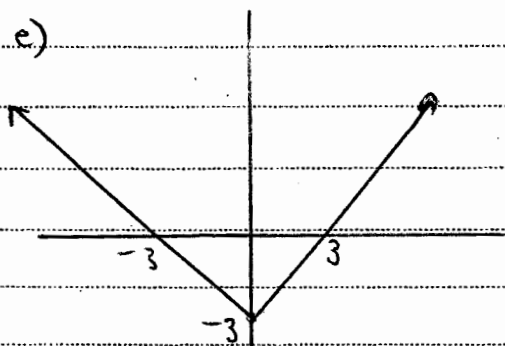
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$$d) \lim_{x \rightarrow 3} \frac{(x+1)(x-3)}{(x-3)}$$

$$= \underline{4}$$

e)



### Question 3

$$a) 2x+3=7 \quad \text{or} \quad 2x+3=-7$$

$$2x=+4 \quad \text{or} \quad 2x=-10$$

$$x=+2 \quad \text{or} \quad -5$$

$$b) \frac{dy}{dx} = 4(3x-5)^3 \times 3$$

$$= \underline{12(3x-5)^3}$$

$$c) \frac{dy}{dx} = \frac{(x+5) \times 2 - (2x-3) \times 1}{(x+5)^2}$$

$$= \frac{2x+10-2x+3}{(x+5)^2}$$

$$= \frac{13}{(x+5)^2}$$

$$c) \frac{dy}{dx} = 2x^2 \times 3(5x+2)^2 \times 5$$

$$+ 4x(5x+2)^3$$

$$= \underline{30x^2(5x+2)^2 + 4x(5x+2)^3}$$

$$e) y = 3x^2 - 5x + 2$$

$$\frac{dy}{dx} = 6x - 5$$

$$\text{At } x=1, \quad \frac{dy}{dx} = 1$$

$$\therefore m_{\text{normal}} = -1$$

$$y - 0 = -1(x - 1)$$

$$\underline{y = -x + 1}$$

### Question 4

$$a) \text{ci) } AB = \sqrt{(-3-1)^2 + (2-(-4))^2}$$

$$= \sqrt{16+36}$$

$$= \underline{\sqrt{52} \text{ or } 2\sqrt{13}}$$

$$\text{cii) } m_{AB} = \frac{2-(-4)}{-3-1}$$

$$= \frac{6}{-4} = \underline{-\frac{3}{2}}$$

$$\text{ciii) } m_{BC} = \frac{2}{3}$$

$$y - (-4) = \frac{2}{3}(x - 1)$$

$$3y + 12 = 2x - 2$$

$$\underline{0 = 2x - 3y - 14}$$

$$\text{civ) } (7, 0) \quad \text{or} \quad x=7$$

$$\text{cv) } d_{BC} = \sqrt{(7-1)^2 + (0-(-4))^2}$$

$$= \underline{\sqrt{52}}$$

$$A_{ABC} = \frac{1}{2} \times \sqrt{52} \times \sqrt{52} = \underline{26}$$

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b)  $(0, -\frac{1}{2})$  lies on  $3x + 2y + 1 = 0$

$$d = \frac{|3 \times 0 + 2 \times (-\frac{1}{2}) + 1|}{\sqrt{3^2 + 2^2}}$$

$$= \frac{2}{\sqrt{13}} \text{ or } \frac{2\sqrt{13}}{13}$$

c)  $|2x - 1| = x - 7$

$$2x - 1 = x - 7 \text{ or } 2x - 1 = -(x - 7)$$

$$x = -6 \text{ or } 3x = 8$$

$$x = \frac{8}{3}$$

$\therefore$  No solution as RHS  $< 0$  for both answers

### Question 5

a) (i)  $2\cos\theta + 1 = 0$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = 120^\circ, 240^\circ$$

(ii)  $\operatorname{cosec}^2\theta = 2$

$$\sin^2\theta = \frac{1}{2}$$

$$\sin\theta = \pm \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

b)  $x^2 - (-\frac{b}{a})x + \frac{c}{a} = 0$

$$x^2 - 5x - 3 = 0$$

(i)  $\Delta < 0$

$$k^2 - 4 \times 3 \times 3 < 0$$

$$k^2 - 36 < 0$$

$$-6 < k < 6$$

d) (i)  $-\frac{b}{a} = -3$

(ii)  $\frac{c}{a} = -5$

(iii)  $(\alpha + \beta)^2 - 2\alpha\beta$   
 $(-3)^2 - 2 \times -5$   
 $19$

### Question 6

a)  $\frac{1}{\cot^2\theta + 1}$

$$= (1 - \cos\theta)(1 + \cos\theta)$$

$$= 1 - \cos^2\theta$$

$$= \sin^2\theta$$

$$= \frac{1}{\operatorname{cosec}^2\theta}$$

$$= \frac{1}{\cot^2\theta + 1}$$

$$= \text{LHS}$$

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b) i)  $\frac{VN}{\sin 25} = \frac{9}{\sin 120}$

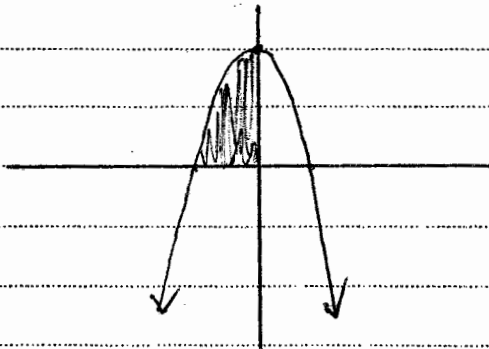
$VN = 4.39m$

ii)  $A = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 9 \times 4.39 \times \sin 35$

$= 11.3m^2$

c)



d)  $2^{3x-1} = \frac{1}{4}$

$2^{3x-1} = 2^{-2}$

$3x-1 = -2$

$3x = -1$

$x = -\frac{1}{3}$

Question 7

a)  $\angle BFG = 80 - 3x$  (vertically opposite) ①

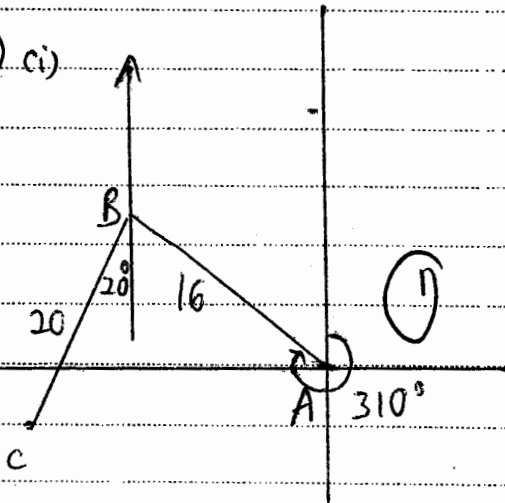
$\angle BFG + \angle FGD$

$= 80 - 3x + 3x + 100$

$= 180^\circ$

$\therefore AB \parallel CD$  (co-interior angles supplementary) ①

b) i)



ii)  $AC^2 = 20^2 + 16^2 - 2 \times 20 \times 16 \times \cos 70$

$AC = 20.91km$  ②

a)  $\frac{dy}{dx} = 2x = -2$   
at  $x = -1$  ①

d)  $5 - (5^3 + 1) + 2 \times 2$  ①  
 $= 5 - (126) + 4$   
 $= -117$  ①

e)  $25 - n^2 \geq 0$   
 $(5-n)(5+n) \geq 0$   
 $n-5 \leq n \leq 5$

②

f)  $(n-2) \times 180$   
 $14 \times 180$   
 $= 2520^\circ$  ①

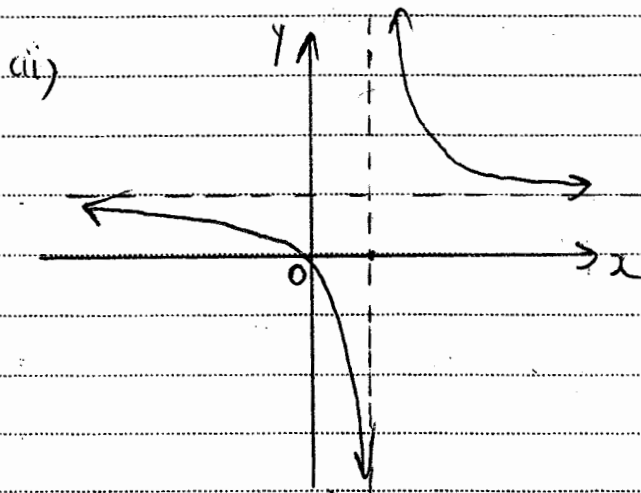
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## Question 8

a) Let  $u = 2^x$   
 $u^2 - 6u + 8 = 0$   
 $(u - 4)(u - 2) = 0$   
 $u = 4$  or  $2$   
 $2^x = 4$  or  $2^x = 2$   
 $x = 1$  or  $2$

b) i)  $D = \text{All } x \text{ except } x = 1$   
 $R = \text{All } y \text{ except } y = 1$



c) i)  $\frac{dy}{dx} = 2x$   
 At  $x = 1$

$m = 2$  (1, 1)  
 $y - 1 = 2(x - 1)$   
 $y = 2x - 1$

ii) (0, -1)  $(\frac{1}{2}, 0)$

cii)  $A = \frac{1}{2} \times 1 \times \frac{1}{2}$   
 $= \frac{1}{4} \text{ units}^2$