Name..

Teacher...

2012
Preliminary Course
FINAL EXAMINATION

Mathematics

General Instructions

- Working Time - 2 hours.
- Write using a blue or black pen.
- Approved calculators may be used.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks – 90

| Section I | Pages 3–4 | 10 marks |

- *Attempt Questions 1–10*
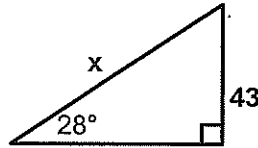- *Allow about 15 minutes for this section*

| Section II | Pages 5–12 | 80 marks |

Attempt Questions 11–16

Allow about 1 hours 45 minutes for this section

1.   The number 147.658 correct to two significant figures is

(a) 15   (b) 150   (c) 147.65   (d) 147.66

2.   The value of x is given by

(a) $43 \times \cos 28°$ (b) $43 \times \sin 28°$

(c) $\dfrac{43}{\cos 28°}$ (d) $\dfrac{43}{\sin 28°}$

3.   If $4y^2 - 12y + P = (2y + Q)^2$ then

(a) $P = 9 \quad Q = 3$ (b) $P = 9 \quad Q = -3$

(c) $P = -9 \quad Q = -3$ (d) $P = -9 \quad Q = -3$

4.   The gradient of any line perpendicular to the line $3x - 2y + 12 = 0$ is

(a) $\dfrac{3}{2}$ (b) $-\dfrac{3}{2}$ (c) $\dfrac{2}{3}$ (d) $-\dfrac{2}{3}$

5. $\dfrac{2}{\sqrt{2}} + \dfrac{\sqrt{2}}{2} =$

(a) 1   (b) 2   (c) $\dfrac{3\sqrt{2}}{2}$ (d) $\dfrac{2 + \sqrt{2}}{2\sqrt{2}}$

6.   Simplify $\dfrac{x^2 - 1}{(x - 1)^2}$

(a) 1   (b) $\dfrac{-1}{-2x + 1}$ (c) $\dfrac{x + 1}{x - 1}$ (d) $\dfrac{x - 1}{x + 1}$

Question 11   (10 Marks)   Use a Separate Sheet of paper   **Marks**

(a) Find the value of $\dfrac{4.23}{\sqrt{6.14-1.78}}$, giving your answer correct to 2   **1**

decimal places.

(b)   If $s = \dfrac{a}{1-r}$ find s when $a = 7, r = \dfrac{1}{3}$.   **1**

(c)   If $\sqrt{12} + \sqrt{27} = \sqrt{a}$, find the value of a.   **2**

(d)   Express $\dfrac{2}{5-2\sqrt{5}}$ as a fraction with a rational denominator.   **2**

(e) Fully factorise the following expressions fully

    (i) $16x^3y - 2y^4$ **2**

    (ii) $mx^2 + my^2 - nx^2 - ny^2$ **2**

Question 13   (10 Marks)   Use a Separate Sheet of paper   **Marks**

(a)   A function is defined by the rule $g(x) = \begin{cases} x+1, & \text{if } x \geq 1 \\ -1, & \text{if } -2 < x < 1 \\ 1-x, & \text{if } x \leq -2 \end{cases}$

Find

(i) $g(1)$ 1

(ii) $g(-3) + g(0)$ 1

(b)   Sketch the graphs of the following, **stating the domain and range of each**.

(i) $y = \dfrac{2}{x}$ 2

(ii) $x^2 + y^2 = 25$ 2

(iii) $3(x+2) - y = 0$ 2

(c)   Show that the function $f(x) = \dfrac{1-x^2}{x}$ is an odd function.   2

Question 15   (10 Marks)   Use a Separate Sheet of paper   **Marks**

The points A(2,0), B(8,4) ,C(4,6) and D (x_1, y_1) form the 4 vertices of a parallelogram ABCD.

(a)   Draw a number plane and plot the points A, B & C.   1
         (USE A RULER)

(b)   Find the gradient of line AB 1

(c)   Show that the equation of the line l parallel to AB and passing   2
         through C is $2x - 3y + 10 = 0$

(d)   Find the point $D(x_1, y_1)$ and mark this point on your diagram.   2

(e)   Find the angle θ to the **nearest degree** that the line AB makes   2
         with the positive x-axis

(f)   Find the perpendicular distance between the line l and A.   2

Question 17   (10 Marks)   Use a Separate Sheet of paper   **Marks**

(a) Find the derivative of the following: (You do not need to simplify your
     answers after finding the derivative.)

     (i) $x^4 - 3x^3 + 2$ 1

     (ii) $\sqrt{x^3}$ 1

     (iii) $\dfrac{1}{3x^4}$ 1
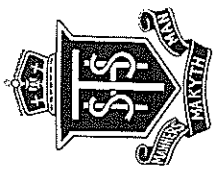
(b) Find $f'(2)$ for $f(x) = \left(3x^2 - 5x\right)^5$.   2

(c) Given $y = \dfrac{x^2 - 1}{x^2 + 1}$ find $\dfrac{dy}{dx}$ 2

(d) Find the equation of the normal to the curve $y = 2x^3 - 4x^2$ at the point (1, -2)   3

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Sydney Technical High School
Preliminary Examination
Mathematics 2012

Multiple Choice Answer Sheet

Name _____

Teacher _____

Completely fill the response oval representing the most correct answer.

1.   A ◯   B ●   C ◯   D ◯
2.   A ◯   B ◯   C ◯   D ●
3.   A ◯   B ●   C ◯   D ◯
4.   A ◯   B ◯   C ◯   D ●
5.   A ◯   B ◯   C ●   D ◯
6.   A ◯   B ◯   C ●   D ◯
7.   A ●   B ◯   C ◯   D ◯
8.   A ◯   B ●   C ◯   D ◯
9.   A ◯   B ●   C ◯   D ◯
10. A ◯   B ◯   C ◯   D ●

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Question 11

a) $2 \cdot 03$ (2 d.p)

b) $S = \dfrac{a}{1-r}$

$S = \dfrac{7}{1-\frac{1}{3}}$

$S = 10\frac{1}{2}$

c) $\sqrt{12} + \sqrt{27} = \sqrt{a}$

$2\sqrt{3} + 3\sqrt{3} = \sqrt{a}$

$5\sqrt{3} = \sqrt{a}$

$\sqrt{25} \times \sqrt{3} = \sqrt{a}$

$a = 75$

d) $\dfrac{2}{5-2\sqrt{5}} \times \dfrac{5+2\sqrt{5}}{5+2\sqrt{5}}$

$= \dfrac{10 + 4\sqrt{5}}{25 - 20}$

$= \dfrac{10 + 4\sqrt{5}}{5}$

e) i) $2y(8x^3 - y^3) = 2y(2x-y)(4x^2 + 2xy + y^2)$

ii) $mx^2 + my^2 - nx^2 - ny^2 = m(x^2+y^2) - n(x^2+y^2)$

$= (m-n)(x^2+y^2)$

Question 14

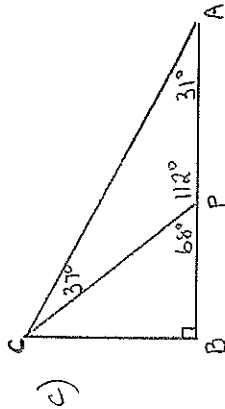a) i) $\cos 135° = \dfrac{-1}{\sqrt{2}}$ or $-\dfrac{\sqrt{2}}{2}$

   ii) $\cot 210° = \sqrt{3}$

b) $2\sin^2\theta - 1 = 0$

$\sin^2\theta = \dfrac{1}{2}$

$\sin\theta = \pm\sqrt{\dfrac{1}{2}}$

$\theta = 45°, 135°, 225°, 315°$

c)

$\dfrac{CP}{\sin 31°} = \dfrac{20}{\sin 37}$

$CP = \dfrac{20 \times \sin 31}{\sin 37}$

$CP = 17\cdot12$

c ii) $\cos 68° = \dfrac{BP}{17\cdot12}$

$BP = 17\cdot12 \cos 68°$

$BP = 6\cdot41$

$\approx 6 \text{ cm}$
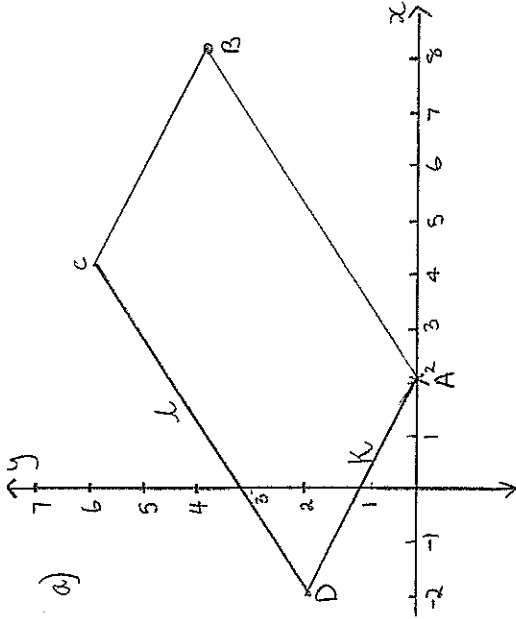
d) i) $\sin C = \dfrac{2}{3}$

Area $= \dfrac{1}{2} ab \sin C$

$= \dfrac{1}{2} \times 9\cdot8 \times 12\cdot6 \times \dfrac{2}{3}$

d ii) Area $= 41\cdot16$ units^2.

Question 15

a)

b) $M_{AB} = \dfrac{4-0}{8-2} = \dfrac{2}{3}$

c) $m_1 = m_2$ parallel

$m_1 = \dfrac{2}{3}$ $(4,6)$

$y - 6 = \dfrac{2}{3}(x-4)$

$3y - 18 = 2x - 8$

$0 = 2x - 3y + 10$

d) $D(-2,2)$ and plot on diagram

e) $\tan\theta = m$

$\tan\theta = \dfrac{2}{3}$

$\theta = 34°$

f) $d = \dfrac{|ax + by + c|}{\sqrt{a^2 + b^2}}$

$a=2, \ b=-3, \ c=10, \ x=2 \ y$

$d = \dfrac{|2\times2 - 3\times0 + 10|}{\sqrt{2^2 - (-3)^2}}$
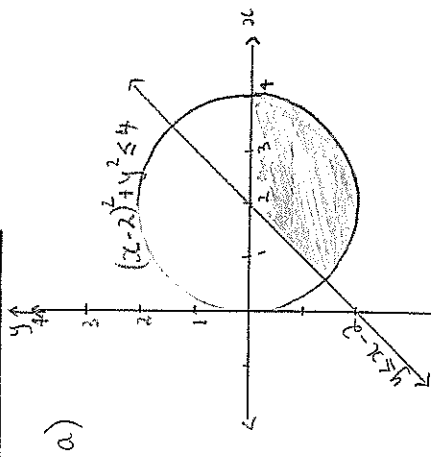
$d = \dfrac{14}{\sqrt{13}}$ or $\dfrac{14\sqrt{13}}{13}$

a)

$(x-2)^2+y^2\leq 4$

$y=2-x$

cii) $c^2 = a^2+b^2-2bc\cos A$

$(XY)^2 = 1200^2 + 1500^2 - 2\times1200\times1500\times\cos 80°$

$XY = 1750\cdot68$ m

ciii) $\dfrac{\sin\theta}{1500} = \dfrac{\sin B}{1750\cdot68}$

$\theta = 57°33'$
$= 58°$

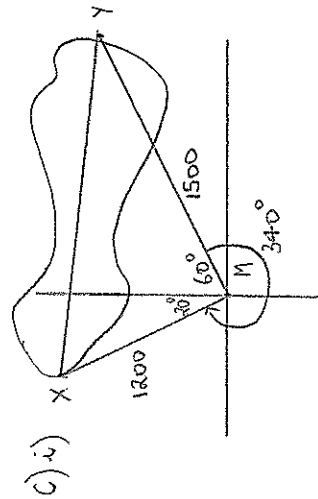$180°-(20°+58°)$
$= 102°$

Bearing of Y from X
is $102°$ T

b) $\sec\theta + \tan\theta = \dfrac{1+\sin\theta}{\cos\theta}$

$L.H.S = \sec\theta + \tan\theta$

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1+\sin\theta}{\cos\theta}$

$= R.H.S$

c)i)