SYDNEY TECHNICAL HIGH SCHOOL

PRELIMINARY HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 3

SEPTEMBER 2013

Mathematics

General Instructions

- Working time - 120 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in questions 11 to 18
- Start each question on a new page

Total marks - 82

Section 1 - 10 marks

Attempt Questions 1 – 10.
Allow about 15 minutes for this section.

Section 2 - 72 marks

Attempt Questions 11 – 18.
Allow about 105 minutes for this section.

Name :____________________________

Teacher :____________________________

Section 1

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet in your answer booklet for Questions 1 – 10.
Do not remove the multiple-choice answer sheet from your answer booklet.

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1. If $x = -4$ is a root of the equation $2x^2 + kx + 4 = 0$, what is the value of k ?

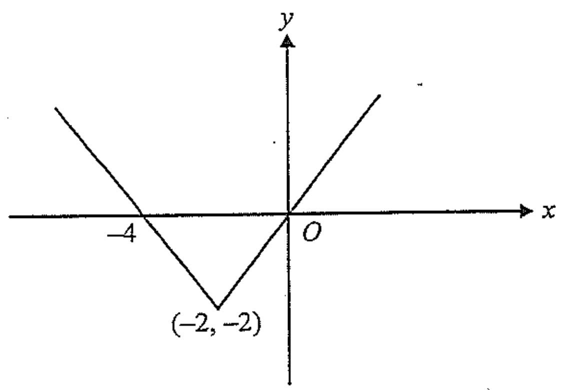
(A) 7

(B) 8

(C) 9

(D) 10

2.



The rule of the function whose graph is shown is

(A) $y = |x| - 4$

(B) $y = |x - 2| + 2$

(C) $y = |x + 2| - 2$

(D) $y = |2 - x| - 2$

3. If $\sqrt{12} + \sqrt{3} = \sqrt{b}$ then

(A) $b = \sqrt{15}$

(B) $b = 3\sqrt{3}$

(C) $b = 15$

(D) $b = 27$

4. The x coordinates of the points of intersection of $y = x^2$ and $x + y = 6$ are the solutions of

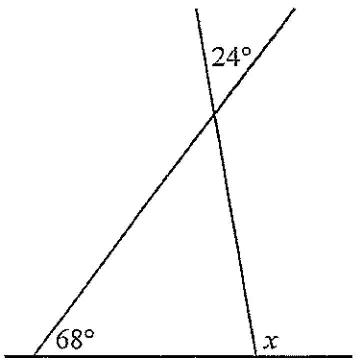
(A) $x^2 - x - 6 = 0$

(B) $x^2 + x - 6 = 0$

(C) $x^2 - x + 6 = 0$

(D) $x^2 + x + 6 = 0$

5.



The size of the angle x is

(A) 68°

(B) 88°

(C) 92°

(D) 112°

6. Given $y = a\,x^n$ then $\dfrac{dy}{dx} = ?$

(A) $a \times n \times x^{n-1}$

(B) $a \times n \times x^{n+1}$

(C) $n \times x^{n-1}$

(D) $a \times x^{n-1}$

7. Find the values of m for which $24 + 2m - m^2 \leq 0$

(A) $\mathrm{m} \leq -4$ or $\mathrm{m} \geq 6$

(B) $\mathrm{m} \leq -6$ or $\mathrm{m} \geq 4$

(C) $-4 \leq \mathrm{m} \leq 6$

(D) $-6 \leq \mathrm{m} \leq 4$

8. For $y = (4x + 1)(x + 2)^3$, $\dfrac{dy}{dx}$ is equal to

(A) $12(\mathrm{x} + 2)^2$

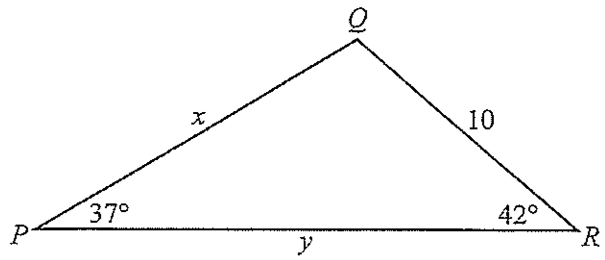
(B) $(x + 2)^2(16x + 11)$

(C) $3(x + 2)^2(4x + 1)$

(D) $(x + 2)^2(12x + 7)$

9. PQR is a triangle with side lengths x , 10 and y, as shown below.

In this triangle, angle RPQ = 37° and angle QRP = 42°.



Which one of the following expressions is correct for triangle PQR ?

(A) $x = \frac{10}{\sin 37°}$

(B) $x = 10 \times \frac{\sin 42°}{\sin 37°}$

(C) $y = 10 \times \frac{\sin 37°}{\sin 101°}$

(D) $10^2 = x^2 + y^2 - 2xy \cos 42°$

10. For $y = \sqrt{1 - f(x)}$, $\frac{dy}{dx}$ is equal to

(A) $\frac{2\, f'(x)}{\sqrt{1 - f(x)}}$

(B) $\frac{-1}{2\sqrt{1 - f(x)}}$

(C) $\frac{1}{2}\sqrt{1 - f'(x)}$

(D) $\frac{-\, f'(x)}{2\sqrt{1 - f(x)}}$

Section 2

72 marks
Attempt Questions 11 – 18
Allow about 105 minutes for this section
Start each question on a new page

Question 11 (9 marks)

a) Simplify $\frac{3p^2-3q^2}{6p-6q}$ 2

b) If $\frac{\sqrt{2}}{9} = 2^m \times 3^n$ find the values of m and n. 2

c) Solve $\frac{x+1}{x+3} = 5$ 2

d) Differentiate $y = 4x^3 - 3x^2 - x + 2$ 2

e) Find the exact value of $\tan 330°$ 1

Question 12 (9 marks) Start a new page

a) The points $A(1\,,7)$, $B(-3\,,5)$ and $C(4\,,-1)$ lie on a number plane.

i) Find the length of AC. 1

ii) Find the gradient of AC. 1

iii) Find the equation of the line AC. 1

iv) Find the perpendicular distance of B from the line AC. 1

v) If ABCD is a parallelogram, find the coordinates of D. 1

b) Find the exact solution of $2x^2 + 4x - 5 = 0$ 2

c) Simplify $\sqrt{60} + (\sqrt{5} - \sqrt{3})^2$ 2

Question 13 (9 marks) Start a new page

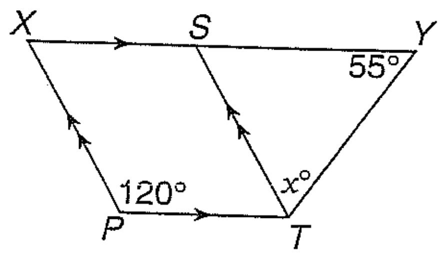
a) Find the area bounded by the line $4x - y = 8$, the x axis and the y axis. 2

b) If $f(x) = x^2 + 2x$ find $\frac{f(x+h)-f(x)}{h}$ in simplest form. 2

c) Find the equation of the tangent to $y = (x-3)^3$ at the point $(1,-8)$ 3

d) The diagram shows XY parallel to PT, XP parallel to ST,

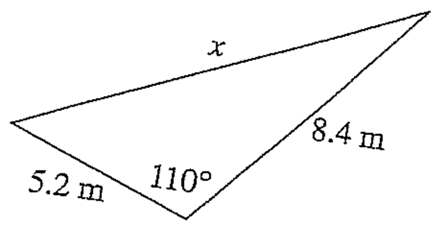
angle XPT = 120° and angle SYT = 55°.



Find , with reasons, the value of x. 2

Question 14 (9 marks) Start a new page

a) Find the value of x, correct to 1 decimal place. 2



b) Solve $|2x - 4| < 2$ 2

c) Evaluate $\lim_{x \to 2} \frac{3x^2-5x-2}{x-2}$ 2

d) Indicate, by shading, the region where points simultaneously satisfy the inequalities $y \le x^2 - 1$ and $x^2 + y^2 \le 4$ 3

Question 15 (9 marks) Start a new page

a) If $\sin\theta = \frac{2}{3}$ and $\cos\theta < 0$ find the exact value of $\tan\theta$. 2

b) Differentiate the following with respect to x

i) $y = (5x-3)^4$ 1

ii) $y = \frac{6}{x^2}$ 1

iii) $y = 12\sqrt{x^3}$ 1

c) If the quadratic equation $ax^2 + bx + c = 0$ has a discriminant equal to 4, 2

what does this tell us about the nature of the roots of the equation ?

d) If the lines $2x - 5y + 3 = 0$ and $ax + 4y + 12 = 0$ 2

are perpendicular, find the value of a.

Question 16 (9 marks) Start a new page

a) Solve $sin^2\theta = \frac{3}{4}$ for $0 \le \theta \le 360°$. 2

b) If $f(x) = x\sqrt{2x+1}$ evaluate $f'(4)$. 2

c) Find, correct to the nearest degree, the acute angle the line 2

$3x - y - 3 = 0$ makes with the x axis.

d) Evaluate $\lim_{x \to \infty} \frac{x^2-4}{2x^2+x}$ 1

e) Simplify $\sin^3 A \sec A + \sin A \cos A$ 2

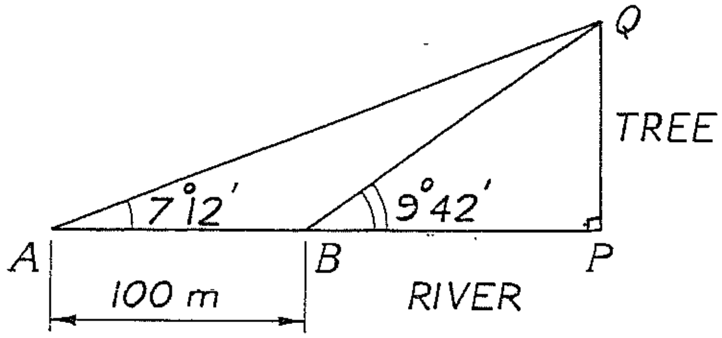
Question 17 (9 marks) Start a new page

a) State the domain of $y = \sqrt{x+4}$ 1

b) If $y = \frac{2x-1}{x+4}$ find $\frac{dy}{dx}$. 2

c) Simplify $sin(90° - \theta)\ cosec\ \theta$ 2

d)



The diagram above was sketched by a surveyor, who measured the angle of elevation of a tree top on the other side of a river to be 7°12' at the point A. At the point B, 100 metres directly towards the tree from A, the angle of elevation was 9°42'. 4

Calculate the height of the tree, correct to 3 significant figures.

Please turn over

Question 18 (9 marks) Start a new page

a) Find the gradient of the normal to the curve

$y = x^2 + 6x + 3$ at the point $(1, -2)$ 2

b) Find all values of k for which the quadratic equation 3

$kx^2 - 8x + k = 0$ has real roots.

c) The curve $y = ax + \frac{b}{x^2}$ cuts the x axis at the point $(2, 0)$ 4

and the gradient of the tangent to this curve at the point $(2, 0)$ equals 1.

Find the values of a and b.

End of paper

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1. C
2. C
3. D
4. B
5. C
6. A
7. A
8. B
9. B
10. D

11. a. $\dfrac{3(p-9)(p+9)}{6(p-9)}$

$= \dfrac{p+9}{2}$

b. $m=\frac{1}{2}$, $n=-2$

c. $x+1 = 5x+15$

$4x = -14$

$x = \dfrac{-7}{2}$

d. $y' = 12x^2 - 6x - 1$

e. $-\dfrac{1}{\sqrt{3}}$

12. a. i. $AC=\sqrt{(4-1)^2+(-1-7)^2}$

$= \sqrt{73}$ units

ii. $m = \dfrac{7--1}{1-4}$

$= \dfrac{8}{-3}$

iii. $y-7 = -\frac{8}{3}(x-1)$

$3y-21 = -8x+8$

$8x+3y-29=0$

iv. $d = \dfrac{|-3\times 8 + 5\times 3 - 29|}{\sqrt{8^2+3^2}}$

$= \dfrac{38}{\sqrt{73}}$ units

v. $D(8,1)$

b. $x = \dfrac{-4\pm\sqrt{4^2-4\times 2\times -5}}{4}$

$= \dfrac{-4\pm\sqrt{56}}{4}$

$= \dfrac{-2\pm\sqrt{14}}{2}$

c. $2\sqrt{15}+5-2\sqrt{15}+3$

$= 8$

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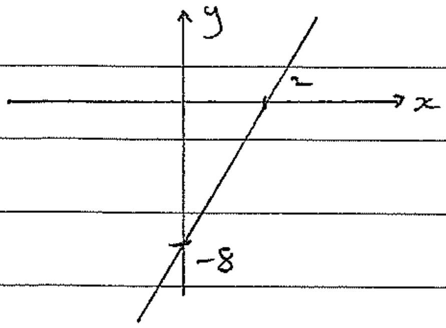
13. a.



Area $= \frac{1}{2} \times 8 \times 2$

$= 8$ sq units

b. $\frac{f(x+h) - f(x)}{h}$

$= \frac{(x+h)^2 + 2(x+h) - (x^2 + 2x)}{h}$

$= \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h}$

$= 2x + h + 2$

c. $y' = 3(x-3)^2$

sub $x = 1$

$m_T = 3(1-3)^2$

$= 12$

$\therefore\ y + 8 = 12(x-1)$

$y + 8 = 12x - 12$

$y = 12x - 20$

d. $\angle XST = 120^\circ$ (opposite angles of a parallelogram)

$\therefore\ x = 65$ (exterior angle of a triangle equals the sum of the opposite interior angles)

14. a. $x^2 = 5.2^2 + 8.4^2 - 2 \times 5.2 \times 8.4 \times \cos 110$

$x = 11.3$ m

b. $-2 < 2x - 4 < 2$

$2 < 2x < 6$

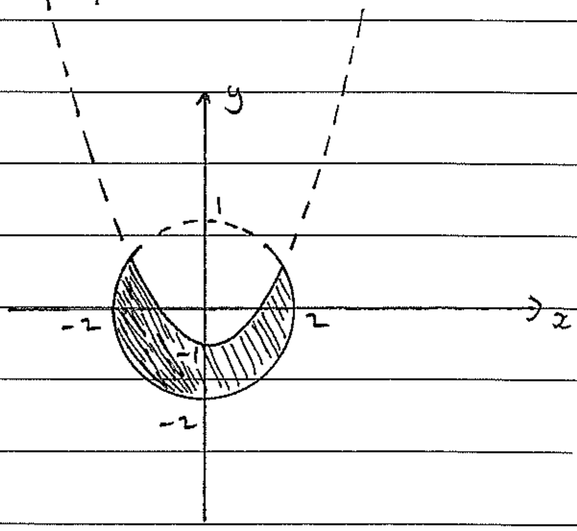
$1 < x < 3$

c. $\lim_{x \to 2} \frac{(3x+1)(x-2)}{x-2}$

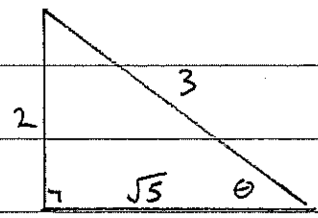
$= \lim_{x \to 2} (3x+1)$

$= 7$

d.



15. a.



2nd quadrant

$\therefore\ \tan\theta = -\frac{2}{\sqrt{5}}$

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b. i. $y' = 20(5x-3)^3$

ii. $y = 6x^{-2}$

$y' = -12x^{-3}$

$= \frac{-12}{x^3}$

iii. $y = 12x^{\frac{3}{2}}$

$y' = 12 \times \frac{3}{2} x^{\frac{1}{2}}$

$= 18\sqrt{x}$

c. roots are real, different and rational

d. $m_1 = \frac{2}{5}$ $m_2 = \frac{-a}{4}$

but $m_1 \times m_2 = -1$

$\frac{2}{5} \times \frac{-a}{4} = -1$

$2a = 20$

$a = 10$

16. a. $\sin\theta = \pm\frac{\sqrt{3}}{2}$

$\theta = 60°, 120°, 240°, 300°$

b. $f'(x) = \sqrt{2x+1} + \frac{x}{\sqrt{2x+1}}$

$f'(4) = \sqrt{9} + \frac{4}{\sqrt{9}}$

$= 4\frac{1}{3}$

c. $m = 3$

$\therefore \tan\theta = 3$

$\therefore \theta = 72°$

d. $\frac{1}{2}$

e. $\sin A\left(\frac{\sin^2 A}{\cos A} + \cos A\right)$

$= \sin A\left(\frac{\sin^2 A + \cos^2 A}{\cos A}\right)$

$= \frac{\sin A}{\cos A}$

$= \tan A$

17. a. $x \geq -4$

b. $\frac{dy}{dx} = \frac{(x+4)(2) - (2x-1)(1)}{(x+4)^2}$

$= \frac{2x+8-2x+1}{(x+4)^2}$

$= \frac{9}{(x+4)^2}$

c. $\sin(90° - \theta)\operatorname{cosec}\theta$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$

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d. $\angle AQB = 2^\circ 30'$

$\dfrac{BQ}{\sin 7^\circ 12'} = \dfrac{100}{\sin 2^\circ 30'}$

$BQ = \dfrac{100 \sin 7^\circ 12'}{\sin 2^\circ 30'}$

and $\sin 9^\circ 42' = \dfrac{h}{BQ}$

$\therefore\ h = \dfrac{100 \sin 7^\circ 12' \sin 9^\circ 42'}{\sin 2^\circ 30'}$

$= 48.4\ m$

18. a. $y' = 2x + 6$

when $x = 1$

$m_T = 8$

$\therefore\ m_N = -\frac{1}{8}$

b. real roots $\Rightarrow \Delta \geq 0$

$\therefore\ (-8)^2 - 4 \times k \times k \geq 0$

$64 - 4k^2 \geq 0$

$k^2 \leq 16$

$-4 \leq k \leq 4$

c. $(2, 0)$ satisfies $y = ax + \frac{b}{x^2}$

$\therefore\ 0 = 2a + \frac{b}{4}$

or $8a + b = 0$

$y = ax + bx^{-2}$

$y' = a - 2bx^{-3}$

when $x = 2$ $y' = 1$

$\therefore\ 1 = a - \frac{2b}{8}$

$8 = 8a - 2b$

$4 = 4a - b$

$\therefore$ Solve simultaneously

$8a + b = 0$

$4a - b = 4$ add

$12a = 4$

$a = \frac{1}{3}$

$\therefore\ b = -\frac{8}{3}$