

Name: ..... Maths Class: .....

# SYDNEY TECHNICAL HIGH SCHOOL



## Year 11 Mathematics

Preliminary HSC Course

Yearly Exam

September, 2016

*Time allowed: 2 hours*

### ***General Instructions:***

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided

Section 1	Multiple Choice Questions 1-10 10 Marks
Section II	Questions 11-18 72 Marks
Total	82 marks

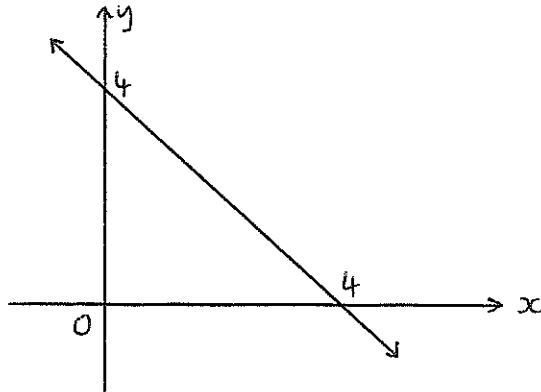
**SECTION 1 -- MULTIPLE CHOICE (10 marks)**

**QUESTION 1**

What is the gradient of a line parallel to the line  $2x + 3y - 1 = 0$  ?

- A. 2            B. -2            C.  $\frac{3}{2}$             D.  $-\frac{2}{3}$

**QUESTION 2**



- The equation of the line above is: A.  $x - y + 4 = 0$             B.  $x + y - 4 = 0$   
C.  $x + y + 4 = 0$             D.  $x - y - 4 = 0$

**QUESTION 3**

A function is given by  $f(x) = \sqrt{9 - x^2}$ . What is its natural domain?

- A.  $x < 3$             B.  $x \leq 3$             C.  $-3 \leq x \leq 3$             D.  $-9 \leq x \leq 9$

**QUESTION 4**

The function in Question 3 above is:

- A. even            B. odd            C. neither            D. cannot be determined

**QUESTION 5**

What is the minimum value of  $x^2 - 4x + 6$  ?

- A. 2            B. 4            C. 6            D. 8

**QUESTION 6**

If  $a^b = 5$ , what is the value of  $2a^{3b}$ ?

- A. 30      B. 250      C. 500      D. 1000

**QUESTION 7**

If  $3^{x-4} = 9^{2x}$ , then  $x = ?$

- A.  $\frac{3}{4}$       B.  $\frac{4}{3}$       C.  $-\frac{3}{4}$       D.  $-\frac{4}{3}$

**QUESTION 8**

If  $2x^2 - 12x + 11$  is expressed in the form  $2(x - b)^2 + c$ , what is the value of  $c$ ?

- A. -25      B. -7      C. 2      D. 29

**QUESTION 9**

$\frac{\sin(180^\circ - \theta)}{\cos(90^\circ - \theta)}$  simplifies to:

- A. 1      B. 2      C.  $\tan \theta$       D.  $\cot \theta$

**QUESTION 10**

If  $a > b$ , which of the following is always true?

- A.  $a^2 > b^2$       B.  $\frac{1}{a} < \frac{1}{b}$       C.  $-a > -b$       D.  $2^a > 2^b$

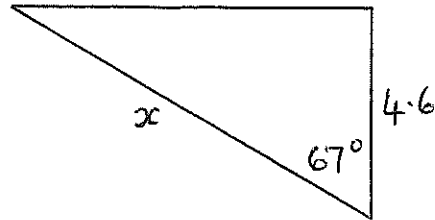
END OF SECTION 1

## SECTION 2

### QUESTION 11 (9 marks)

- a) Evaluate  $13.6 \sin 42^\circ 15'$  correct to 2 significant figures. 1
- b) Expand and simplify  $(2\sqrt{3} - 1)(\sqrt{3} + 4)$  2
- c) Write the exact value of  $\operatorname{cosec} 60^\circ$ . 1
- d) Simplify  $\frac{x-3}{x^2-4x+3}$  1
- e) Find the value of  $x$ , correct to 1 decimal place. 2

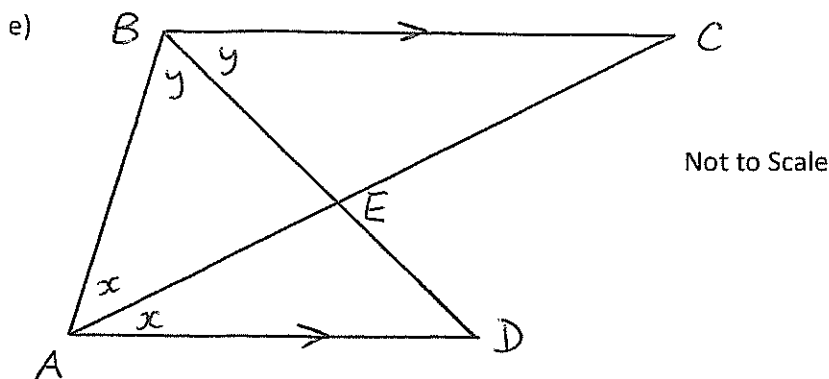
Not to Scale



- f) Solve  $(x + 1)^2 = 5$ , giving answers correct to 1 decimal place. 2

### QUESTION 12 (9 marks) Start a new page.

- a) Solve  $|3x - 6| < 12$  2
- b) Find  $\theta$  to the nearest degree if  $\cos \theta = 0.4$  and  $0^\circ \leq \theta \leq 360^\circ$ . 1
- c) Fully simplify  $\frac{a+b}{\frac{1}{a} + \frac{1}{b}}$  2
- d) Find derivatives of: i)  $y = 3x^2 - 4 + 7x$  1
- ii)  $f(x) = \frac{4}{x^2}$  1



$AD \parallel BC$ .  $AC$  and  $BD$  intersect at  $E$ .  $\angle BAD$  and  $\angle ABC$  are bisected as shown.

Prove that  $\angle BEA = 90^\circ$ .

2

**QUESTION 13** (9 marks) Start a new page.

a) Factorise  $x^3 - 27$ . 1

b) Simplify  $\sin \theta(1 + \cot^2 \theta) \tan \theta$ . 2

c) Differentiate i)  $(x^2 + 5)^4$  1

ii)  $x\sqrt{x}$  1

d) Find the gradient of the curve  $y = \frac{2x}{x+3}$  when  $x = -2$ . 2

e) The solutions of a quadratic equation are  $x = \frac{1 \pm \sqrt{5}}{2}$ . Write a quadratic equation with these solutions. 2

**QUESTION 14** (9marks) Start a new page.

a) Find the coordinates of the vertex of the parabola  $y = (x + 3)^2 + 4$ . 1

b) Given  $f(x) = x^2 + \frac{x}{2}$ , evaluate  $f(2) + f'(2)$ . 2

c) Solve for  $\theta$ , given  $0^\circ \leq \theta \leq 360^\circ$ :

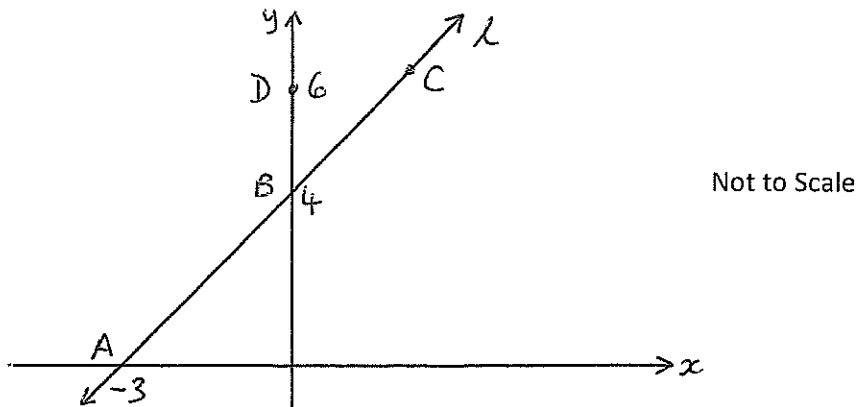
i)  $(\sin \theta + 1)(\cos \theta - 1) = 0$  2

ii)  $3 \tan^2 \theta - 1 = 0$  2

d) If  $\cos \theta = -\frac{2}{3}$  and  $\sin \theta > 0$ , find the exact value of  $\tan \theta$ . 1

e) Fully factorise  $x^2 + 8x + 16 - y^2$ . 1

**QUESTION 15** (9 marks) Start a new page.

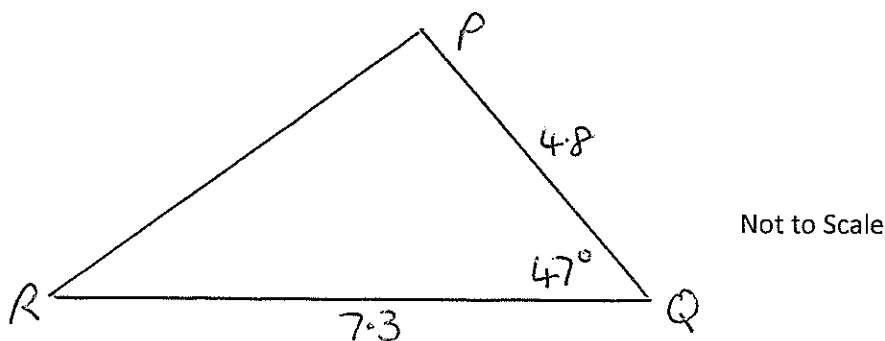


In the diagram above, line  $l$  cuts the  $x$  axis at  $A(-3,0)$  and the  $y$  axis at  $B(0,4)$ .  $D$  has coordinates  $(0,6)$  and point  $C$  is on  $l$ .

- Find the gradient of line  $l$ . 1
- Show that line  $l$  has equation  $4x - 3y + 12 = 0$ . 1
- $B$  is the midpoint of  $AC$ . Find the coordinates of  $C$ . 1
- Find the perpendicular distance from  $D$  to the line  $l$ . 1
- Find the area of  $\triangle BDC$ . 2
- Find the equation of the perpendicular bisector of  $AB$ . Leave your answer in general form. 3

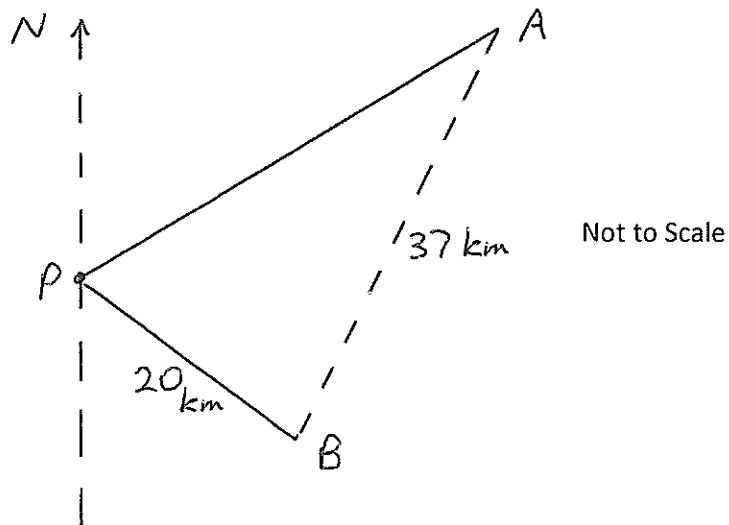
**QUESTION 16** (9 marks) Start a new page.

- Find  $\frac{d}{dr} \left( \frac{4}{3} \pi r^3 \right)$  1
- Find the point(s) on the curve  $y = x^3 - 3x^2 + 3x$  where the tangent is horizontal. 2
- 



- Find the area of  $\triangle PQR$ . 1
- Find the length of  $RP$ , correct to 1 decimal place. 2

d)



Ship A leaves port P and sails on a compass bearing of  $N50^\circ E$ . Ship B also leaves port P and sails 20 km on a compass bearing of  $S55^\circ E$ . The two ships are now 37 km apart.

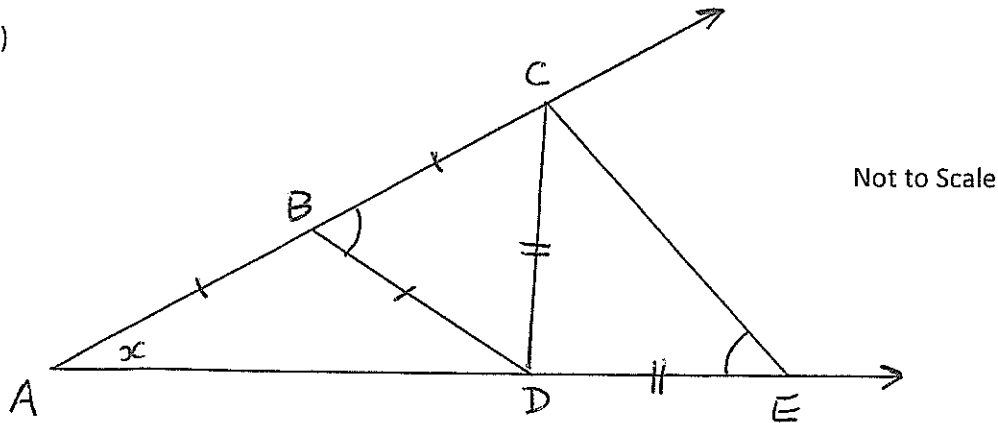
- i) Find  $\angle APB$ . 1
- ii) Find  $\angle PAB$  to the nearest minute. 2

**QUESTION 17** (9marks) Start a new page.

- a) The interior angle of a regular polygon is  $165^\circ$ . How many sides does the polygon have? 1
- b) Find the centre and radius of the circle  $x^2 + y^2 - 4y - 1 = 0$ . 2
- c) i) On the same axes, neatly sketch the functions  $y = \frac{1}{x+2}$  and  $y - x = 2$ . Use a ruler, label any asymptotes and all  $x$  and  $y$  intercepts. 2
- ii) Find the points of intersection of the two graphs. Show working. 2
- iii) Find the equation of the normal to the curve  $y = \frac{1}{x+2}$  at the point where it crosses the  $y$  axis. 2

QUESTION 18 (9 marks) Start a new page.

a)



Rays AC and AE enclose isosceles triangles ABD, BCD and CDE as shown above.

i) If  $\angle A = x$ , find  $\angle CBD$  in terms of  $x$ , giving reasons.

2

ii) Hence, find the size of  $\angle DEC$ . Reasons are not required.

1

b) Simplify  $\lim_{h \rightarrow 0} \frac{3(x+h)^2 - 3x^2}{h}$ . Show full working.

2

c) Prove that  $\sec \theta - \sin \theta \tan \theta = \cos \theta$ .

2

d) Differentiate  $y = x^2(x^3 - 1)^4$ . Leave your answer in fully factored form.

2

END OF TEST



# SOLUTIONS Yr 11 ~~Maths~~ Zunit

1. D    2. B    3. C    4. A    5. A    6. B  
 7. D    8. B    9. A    10. D

11 a) 9.1

b)  $6 + 8\sqrt{3} - \sqrt{3} - 4 \leftarrow \textcircled{1}$

$= 2 + 7\sqrt{3} \leftarrow \textcircled{1}$

c)  $\frac{2}{\sqrt{3}}$

d)

$\frac{1}{x-1}$

e)  $\cos 67^\circ = \frac{4.6}{x} \leftarrow \textcircled{1}$

$x = \frac{4.6}{\cos 67^\circ} \leftarrow \textcircled{1}$

$= 11.8 \text{ (1 dec.)} \leftarrow \textcircled{1}$

f)  $x+1 = \pm\sqrt{5} \leftarrow \textcircled{1}$

$x = -1 \pm\sqrt{5}$

$= 1.2 \text{ or } -3.2 \leftarrow \textcircled{1}$   
 (1 dec.)

12 a)  $3x-6 < 12$  or  $-(3x-6) < 12$

$3x < 18$

$-3x+6 < 12$

$x < 6$

$-3x < 6$

must have

$x > -2$

$\therefore$  solution is  $-2 < x < 6$

b)  $\theta = 66^\circ$  or  $29.4^\circ$  need both

c)  $\frac{a+b}{\frac{b+a}{ab}} = \frac{a+b}{ab} \times \frac{ab}{b+a}$

$= ab \leftarrow \textcircled{1}$

d) i)  $y' = 6x+7$

ii)  $f(x) = 4x^{-2}$

$f'(x) = -8x^{-3} \leftarrow \textcircled{1}$

or  $-\frac{8}{x^3}$

$\textcircled{1}$

e)  $2x+2y = 180^\circ$  (co-interior angles parallel lines)

$\therefore x+y = 90^\circ$

$\therefore \angle BEA = 90^\circ$  (angle sum  $\triangle BEA$ )

13 a)  $(x-3)(x^2+3x+9)$

b)  $\sin \theta \times \operatorname{cosec}^2 \theta \times \tan \theta$

$= \cancel{\sin \theta} \times \frac{1}{\cancel{\sin^2 \theta}} \times \frac{\cancel{\sin \theta}}{\cos \theta} \leftarrow \textcircled{1}$

$= \frac{1}{\cos \theta} = \sec \theta \leftarrow \textcircled{1}$

c) i)  $y' = 4(x^2+5)^3 \times 2x$   
 $= 8x(x^2+5)^3$

ii)  $y = x^{3/2}$

$y' = \frac{3}{2} x^{1/2}$

or  $\frac{3\sqrt{x}}{2}$

(13) d)  $y' = \frac{2(x+3) - 1(2x)}{(x+3)^2}$   
 $= \frac{6}{(x+3)^2} \leftarrow \textcircled{1}$

When  $x = -2, M_T = 6 \leftarrow \textcircled{1}$

e)  $a = 1, b = -1, c = -1$   
 $\therefore x^2 - x - 1 = 0 \leftarrow \text{must have } \textcircled{1}$

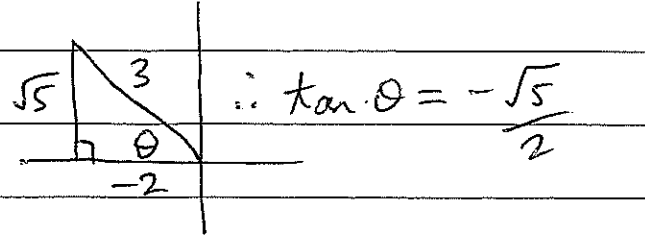
(14) a)  $V(-3, 4)$   
 b)  $f(2) = 5$

$f'(x) = 2x + \frac{1}{2}$   
 $\therefore f'(2) = 4\frac{1}{2} \leftarrow \textcircled{1}$   
 $\therefore f(2) + f'(2) = 9\frac{1}{2} \leftarrow \textcircled{1}$

c) i)  $\sin \theta = -1$  or  $\cos \theta = 1$   
 $\therefore \theta = 270^\circ, 0^\circ, 360^\circ$   
 $\textcircled{1} \quad \textcircled{1}$

ii)  $\tan^2 \theta = \frac{1}{3} \leftarrow \textcircled{1}$   
 $\therefore \tan \theta = \pm \frac{1}{\sqrt{3}}$   
 $\therefore \theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$   
 $\text{all 4 } \textcircled{1}$

d) 2nd quadrant



e)  $(x+4)^2 - y^2$   
 $= (x+4+y)(x+4-y)$

(15) a)  $M_x = \frac{4}{3}$   
 b) use  $B(0, 4)$   
 $\therefore y - 4 = \frac{4}{3}(x - 0)$   
 $\therefore 3y - 12 = 4x$   
 $\therefore 4x - 3y + 12 = 0$  as reqd.

c)  $C(3, 8)$

d)  $D(0, 6)$  and  $4x - 3y + 12 = 0$   
 $\therefore \text{p.d.} = \frac{|0 - 18 + 12|}{\sqrt{4^2 + 3^2}}$   
 $= \frac{6}{5} \leftarrow \textcircled{1}$

e)  $B(0, 4) C(3, 8) \Rightarrow BC = \sqrt{9 + 16}$   
 $= 5 \leftarrow \textcircled{1}$

$\therefore \text{area } \Delta BDC = \frac{1}{2} \times 5 \times \frac{6}{5}$   
 $= 3 \text{ u}^2 \leftarrow \textcircled{1}$

f)  $MP(AB) = (-1\frac{1}{2}, 2) \leftarrow \textcircled{1}$   
 $M_{AB} = \frac{4}{3} \Rightarrow M_{\perp} = -\frac{3}{4} \leftarrow \textcircled{1}$

$\therefore \text{eqn is } y - 2 = -\frac{3}{4}(x + 1\frac{1}{2})$   
 $4y - 8 = -3x - 4\frac{1}{2}$   
 $\therefore 6x + 8y - 7 = 0$

$\leftarrow \textcircled{1}$



(16) a)  $4\pi r^2$

b)  $\frac{dy}{dx} = 0$

$\therefore 3x^2 - 6x + 3 = 0$   
 $\therefore 3(x^2 - 2x + 1) = 0$  } ①

$3(x-1)^2 = 0$  } ①

$\therefore x = 1 \Rightarrow (1, 1)$  on curve

c) i)  $A = \frac{1}{2} \times 7 \cdot 3 \times 4 \cdot 8 \times \sin 47^\circ$   
 $\doteq 12.81 \text{ cm}^2$  } ①

ii)  $RP^2 = 7 \cdot 3^2 + 4 \cdot 8^2 - 2 \times 7 \cdot 3 \times 4 \cdot 8 \times \cos 47^\circ$   
 $= 28.53555$

$\therefore RP \doteq 5.3$  (1 dec.)

①

d) i)  $75^\circ$

ii)  $\frac{20}{\sin A} = \frac{37}{\sin 75^\circ}$  } ①

$\therefore \sin A = \frac{20 \sin 75^\circ}{37}$

$= 0.522 \dots$  } ①

$\therefore \angle PAB = 31^\circ 28'$

(17) a) Ext. angle =  $15^\circ$

$\therefore$  no. of sides =  $\frac{360}{15}$

= 24 sides.

b)  $x^2 + y^2 - 4y + 4 = 1 + 4$

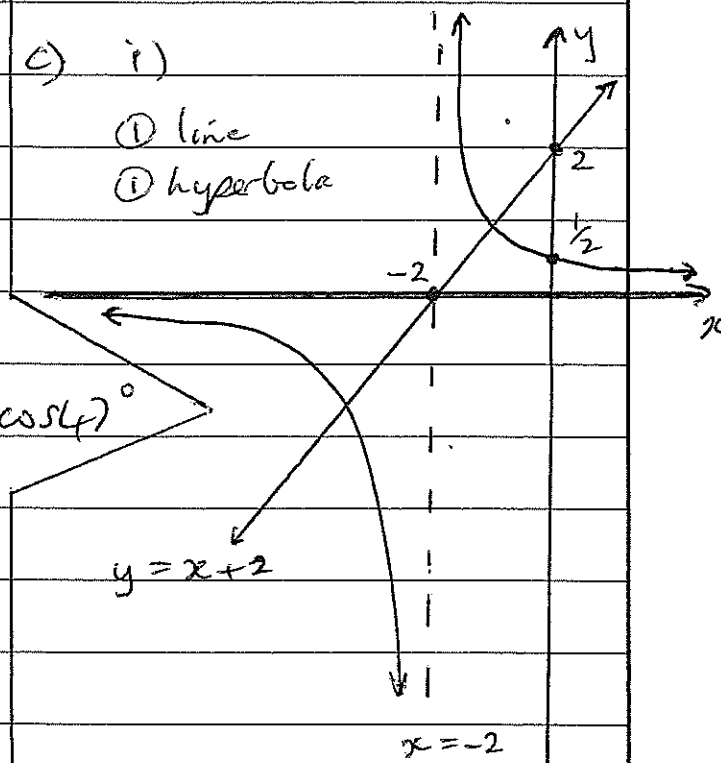
$x^2 + (y-2)^2 = 5$  } ①

$\therefore$  centre is  $(0, 2)$   
 radius is  $\sqrt{5}u$  } ①

c) i)

① line

① hyperbola



ii)  $\frac{1}{x+2} = x+2$   
 $(x+2)^2 = 1$   
 $x^2 + 4x + 3 = 0$  } ①

$(x+3)(x+1) = 0$

$\therefore x = -3, -1$

$\therefore$  pts. of intersection are  $(-3, -1)$  and  $(-1, 1)$

①

(17) c) iii)  $y = (x+2)^{-1}$   
 $y' = -1(x+2)^{-2} \times 1$   
 $= \frac{-1}{(x+2)^2}$

When  $x = 0$ ,  $y' = m_T = -\frac{1}{4}$

①

$\therefore m_N = 4$  and use  $(0, \frac{1}{2})$

$\therefore$  eqn. of normal is

$$y - \frac{1}{2} = 4(x - 0)$$

$$\therefore y = 4x + \frac{1}{2} \quad \leftarrow \text{①}$$

(or  $8x - 2y + 1 = 0$ )

(18) a) i)  $\angle BDA = x$  (base angles isosceles  $\triangle ABD$ )

$\therefore \angle CBD = 2x$  (exterior angle  $\triangle ABD$ )

ii)  $45^\circ$

$$h) = \lim_{h \rightarrow 0} \frac{3(x^2 + 2xh + h^2) - 3x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{3x^2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{K}(6x + 3h)}{\cancel{K}}$$

$= 6x$       ① method

① answer

$$c) \text{LHS} = \frac{1}{\cos \theta} - \sin \theta \times \frac{\sin \theta}{\cos \theta}$$

$$= \frac{1 - \sin^2 \theta}{\cos \theta} \quad \text{①}$$

$$= \frac{\cos^2 \theta}{\cos \theta}$$

$$= \cos \theta$$

$$= \text{RHS.} \quad \text{①}$$

$$d) y' = 2x(x^3-1)^4 \quad \text{①}$$

$$+ 4(x^3-1)^3 \times 3x^2 \times x^2$$

$$= 2x(x^3-1)^4 + 12x^4(x^3-1)^3$$

$$= 2x(x^3-1)^3 [x^3-1 + 6x^3]$$

$$= 2x(x^3-1)^3 (7x^3-1) \quad \text{①}$$