Sydney Technical High School

Name: ....

Maths Class: ....

Year 11
Mathematics
Preliminary Course Final Exam
September 2017

Time allowed: 120 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A reference sheet is provided at the rear of this Question Booklet, and may be removed at any time.

Section 1 Multiple Choice
Questions 1-8
8 Marks

Section II Questions 9-16
80 Marks

Total = 88 marks

SECTION 1 (10 marks)

Choose the letter corresponding to the correct answer and fill in the Answer sheet provided at the front of your answer booklet.

DO NOT REMOVE THIS SHEET

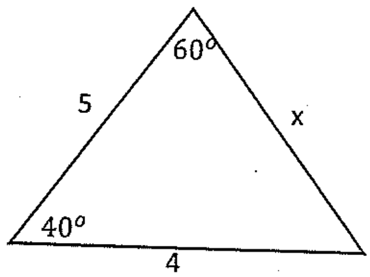
1 Which of the following is NOT always a true statement?

A. The diagonals of a rhombus bisect at right angles
B. The opposite angles of a rhombus are equal
C. The diagonals of a parallelogram bisect at right angles
D. The opposite angles of a parallelogram are equal

2 The quadratic equation $2x^2 - 4x + 5 = 0$ has:

A. No real roots
B. 1 real root
C. 2 equal roots
D. 2 distinct Real roots

3 Which statement below is true for the diagram shown?



A. $\cos 60^o = \frac{5^2 + 4^2 - x^2}{2 \times 5 \times 4}$

B. $\frac{4}{\sin 60^o} = \frac{x}{\sin 100^o}$

C. $x^2 = 25 + 16 - 2 \times 5 \times 4 \cos 60^o$

D. $\frac{5}{\sin 80^o} = \frac{x}{\sin 40^o}$

4 Find $\lim_{x \to \infty} \frac{3x^2 - 2x + 3}{2x^2 - 5}$

A. $-\frac{3}{5}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 1

5	If $= \frac{2\sqrt{3}+3}{\sqrt{3}-2} = x + y\sqrt{3}$, then A. $x = 12$ and $y = 7$ B. $x = -12$ and $y = 7$ C. $x = 12$ and $y = -7$ D. $x = -12$ and $y = -7$
6	If $y = \frac{1}{(5x-1)^2}$ then $\frac{dy}{dx} =$ A. $\frac{-10}{(5x-1)^3}$ B $\frac{-10}{(5x-1)}$ C. $\frac{-2}{(5x-1)^3}$ D. $\frac{-2}{(5x-1)}$
7	If $\cos\theta = \frac{k}{5}$ for an acute angle θ, then tanθ = A. $\frac{\sqrt{25-k^2}}{k}$ B. $\frac{\sqrt{25-k^2}}{5}$ C. $\frac{5}{\sqrt{25-k^2}}$ D. $\frac{k}{\sqrt{25-k^2}}$
8	If $5^{2x-1} = \frac{1}{125}$ then $x =$ A. 13 B. -12 C. -2 D. -1

SECTION 2

Complete all answers in your answer booklet provided

QUESTION 9: (10 Marks)

Marks

(a) Expand and simplify: $(x+3)(x^2-3x+9)$ **1**

(b) Solve the equation: $|3x-4|=5$ **2**

(c) What is the size of one of the exterior angles of a regular pentagon? **1**

(d) (i) What are the Domain and Range of the function $f(x)=\sqrt{16-x^2}$? **2**

(ii) Sketch $y=f(x)$ **2**

(e) Find the equation of the tangent to the curve $y=\frac{1}{4}x^3-4$ at the point P $(2,-3)$ **2**

QUESTION 10: (10 Marks) *Start a new page*

Marks

(a) Find the derivatives of:

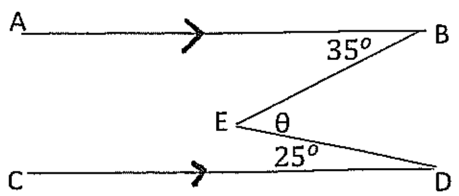
(i) $y = x^3 + 3x - 1$ 1

(ii) $y = (3x-5)^4$ 1

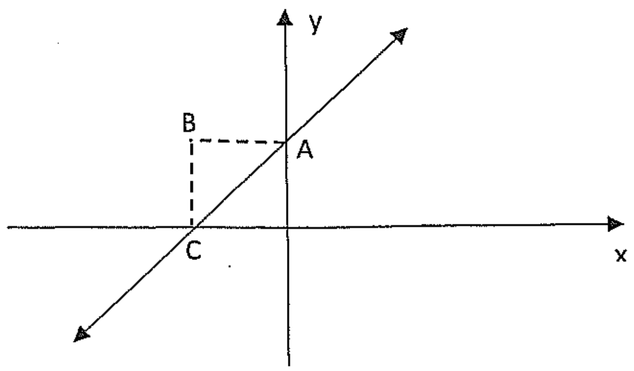
(iii) $y = \frac{2}{x}$ 1

(iv) $2\sqrt{x}$ 1

(b) Find the size of θ in the following., given AB ∥ CD, (no reasons necessary 1



(c) In the diagram below, the line AC is given as $3x - 2y + 6 = 0$



B has the same x-coordinate as C and the same y-coordinate as A

(i) Find the point B. 2

(ii) Find the equation of the line through B perpendicular to line AC 3

QUESTION 11: (10 marks) *Start a new page*

Marks

(a) For the function defined by:

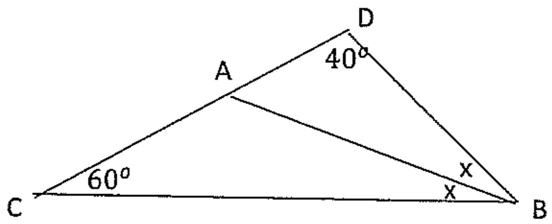
$$f(x) = \begin{cases} 2x, & x \geq 1 \\ 2 - 2x, & x < 1 \end{cases}$$

(i) Sketch $y = f(x)$ 3

(ii) Find the value of $f(-1) + f(1) + f(3)$ 1

(b) Solve simultaneously $\begin{cases} 4x - y = 19 \\ x + 2y + 2 = 0 \end{cases}$ 2

(c) In the diagram below, AB bisects $\angle DBC$, $\angle ACB = 60^o$ and $\angle CDB = 40^o$ 4



Copy the diagram into your answer booklet
Setting out a formal proof, prove that ΔCBA ||| ΔCDB

QUESTION 12: (10 marks) *Start a new page*

Marks

(a) Find the equation of the normal to the curve $y = 2x^3 - 4x^2$ at the point (1, -2) 3

(b) α and β are the roots of the quadratic equation $2x^2 - 3x + 5 = 0$
(DO NOT ATTEMPT TO FIND THESE ROOTS)

Find the value of:

(i) $\alpha + \beta$ 1

(ii) $\alpha\beta$ 1

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ 1

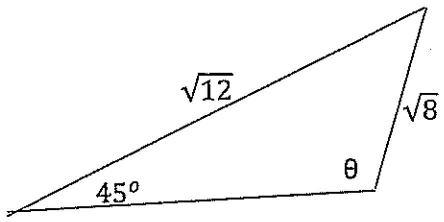
(iv) $\alpha^2 + \beta^2$ 2

(v) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ 2

QUESTION 13: (10 Marks) *Start a new page*

Marks

(a) 3



In the diagram above, find the value of θ, <u>if $90^o < \theta < 180^o$</u>

(b) (i) On the same diagram shade the region corresponding to the simultaneous solution of: 3

$$(x-3)^2 + y^2 \le 4 \quad \text{and} \quad x + y \ge 3$$

(ii) The point P lies somewhere in the shaded region described in part (i). 1

At what point in the region above is P furthest from the origin? Give the co-ordinates of this point.

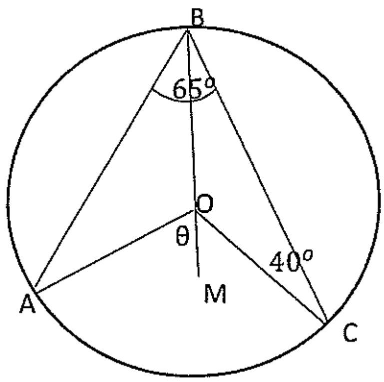
(c) If the roots of the quadratic equation $kx^2 + (k-1)x + (2k+1) = 0$ are such that one root is the reciprocal of the other, find the value of k. 3

QUESTION 14: (10 Marks) *Start a new page*

Marks

(a) For the figure below, O is the centre of the circle, $\angle BCO = 40^o$
$\angle ABC = 65^o$

BO is produced to M.

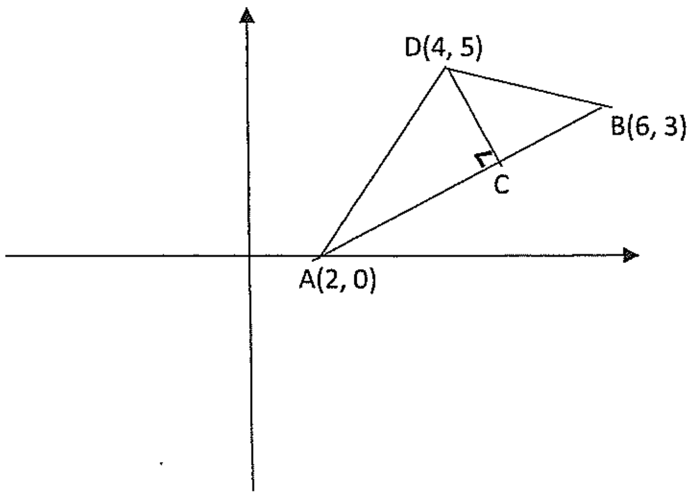


(i) Find the size of $\angle ABM$ **2**

(ii) Find the size of $\angle AOM$ **2**

You must provide reasons for each line of your proofs.

(b) The point A is (2,0) while B is (6, 3) and D (4, 5) as shown.



(i) Find the length of AB **1**

(ii) Find the equation of the line AB in general form **2**

(iii) Find the shortest distance of the point D from AB (ie CD) **2**

(iv) Find the area of ΔABD **1**

QUESTION 15: (10 Marks) *Start a new page*

Marks

(a) If $f(x) = 3x^2$, find $\frac{f(x+h)-f(x)}{h}$ **3**

(b) Prove that $\frac{tan^2x}{secx+1} = secx - 1$ **2**

(c) Solve $4sin^2\theta - 3 = 0$ for $0^o \leq \theta \leq 360^o$ **3**

(d) If $f(x) = x^{\frac{3}{2}}$ find the value of $f'(4)$ **2**

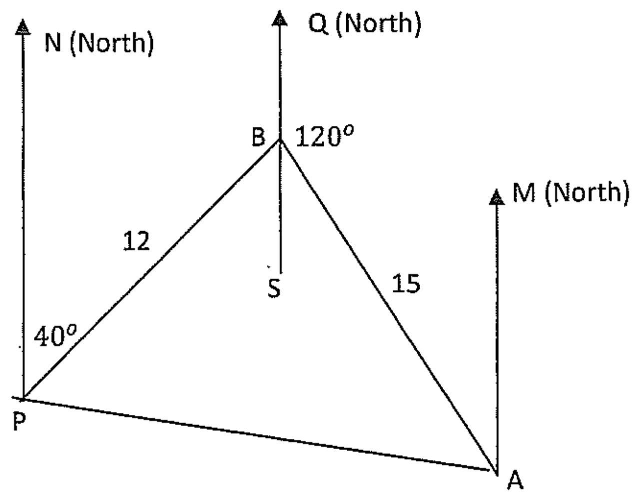
QUESTION 16: (10 Marks) *Start a new page*

(a) Find $\frac{dy}{dx}$ if:

(i) $y = \sqrt{x^3 + 3}$ **2**

(ii) $y = \frac{x}{x+1}$ **2**

(b) The diagram below shows the course of a ship, which sails from a port P on a bearing of 040^0 for 12 km before changing course to a bearing of 120^0 and travelling a further 15 km to a destination A.



(i) Explain why $\angle PBA = 100^0$ **1**

(ii) Find the distance of A from P to the nearest km. **2**

(iii) Find the bearing of P from A to the nearest degree. **3**

YEAR 11 PRELIMINARY FINAL EXAMS 2017

Mathematics Solutions

MULTIPLE CHOICE

Q1. (C)

Q2. $\Delta = 16 - 4(2)(5)$
< 0
$\therefore$ (A)

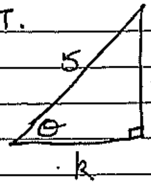
Q3. (D)

Q4. (C)

Q5. $\dfrac{(2\sqrt{3}+3)(\sqrt{3}+2)}{3-4} = \dfrac{6+6+7\sqrt{3}}{-1}$
$= -12 - 7\sqrt{3}$ $\therefore$ (D)

Q6. $\dfrac{d}{dx}(5x-1)^{-2} = -2(5)(5x-1)^{-3}$
$= \dfrac{-10}{(5x-1)^3}$ (A)

Q7.



← By Pythagoras $= \sqrt{25-k^2}$

$\therefore \tan\theta = \dfrac{\sqrt{25-k^2}}{k}$ (A)

Q8. $5^{2x-1} = 5^{-3}$
$\therefore 2x-1 = -3$
$2x = -2$
$\therefore x = -1$. (D)

QUESTION 9:

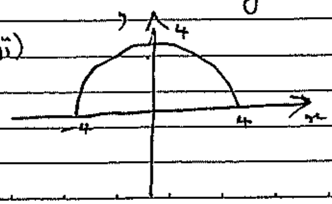
(a) $x^3 + 27$

(b) $3x = 9$ or $3x = -1$
$\therefore x = 3$ or $x = -\frac{1}{3}$

(c) Sum $= 360°$ $\therefore$ Each angle $\frac{360}{5} = 72°$

(d) (i) D: $-4 \le x \le 4$
R: $0 \le y \le 4$

(ii)



(e) $\dfrac{dy}{dx} = \dfrac{3}{4}x^2$
At $(2,-2)$ $m_T = 3$
$\therefore$ Equation is
$y+2 = 3(x-2)$
$y = 3x - 8$

QUESTION 10:

(a) (i) $3x^2 + 3$ (ii) $12(3x-5)^3$
(iii) $-\frac{2}{x^2}$ (iv) $x^{-\frac{1}{2}}$ or $\frac{1}{\sqrt{x}}$.

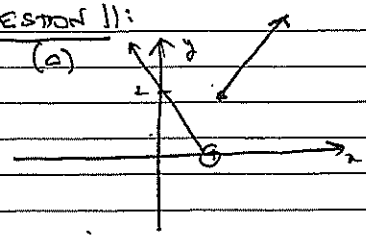
(b) $\theta = 60°$

(c) (i) A is $(0,3)$ C is $(-2,0)$ $\therefore$ B is $(-2,3)$.

(ii) $m_{AC} = \frac{3}{2}$ $\therefore$ m of perp $= -\frac{2}{3}$
$\therefore$ Equation is
$y - 3 = -\frac{2}{3}(x+2)$
$3y - 9 = -2x - 4$
$2x + 3y - 5 = 0$

QUESTION 11:

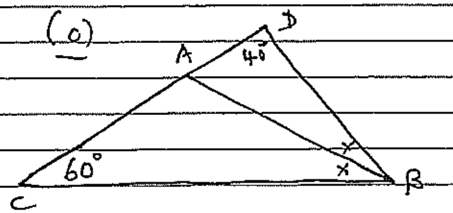
(a)



(ii) $f(-1) + f(1) + f(3)$
$= 4 + 2 + 6$
$= 12$

(b) $8x - 2y = 38$ (1)
$x + 2y + 2 = 0$ (2)
(1)+(2) $9x + 2 = 38$
$\therefore \begin{cases} x = 4 \\ y = -3 \end{cases}$

(c)



In $\triangle CDB$, $2x = 80°$ (angle sum)
$\therefore x = 40°$

In $\triangle CBA$ and $\triangle CDB$
$\angle ACB = \angle BCD$ (same angle)
$\angle CBA = \angle CDB = 40°$.
$\therefore \triangle CBA \;|||\; \triangle CDB$ (equiangular)

QUESTION 12:

(a) $\frac{dy}{dx} = 6x^2 - 8x$

at $(1,-2)$ $\quad m_T = -2$

$m_N = \frac{1}{2}$

EQUATION:

$y + 2 = \frac{1}{2}(x-1)$

$2y + 4 = x - 1$

$2y = x - 5$

(b) (i) $\alpha + \beta = \frac{3}{2}$ $\quad$ (ii) $\alpha\beta = \frac{5}{2}$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$= \frac{3/2}{5/2}$

$= \frac{3}{5}$

(iv) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \frac{9}{4} - 5$

$= -\frac{11}{4}$

(v) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= \frac{-11/4}{5/2}$

$= -\frac{11}{10}$

QUESTION 13:

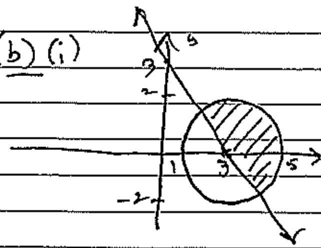
(a) By sine rule, $\frac{\sin\theta}{\sqrt{12}} = \frac{\sin 45^\circ}{\sqrt{8}}$

$\therefore \sin\theta = \frac{1/\sqrt{2} \cdot \sqrt{12}}{\sqrt{8}}$

$= \frac{\sqrt{3}}{2}$

$\therefore \theta = 60^\circ$

(b) (i)



(ii) P is (5,0)

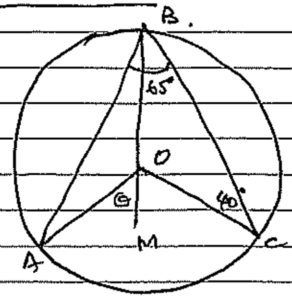
(c) Let the roots be α and $\frac{1}{\alpha}$.

PRODUCT $= 1 = \frac{2k+1}{k}$

$\therefore k = -1$

QUESTION 14:

(a)



(i) In $\triangle BOC$, OB and OC are radii.

$\therefore \angle MBC = 40^\circ$ (isosceles $\triangle BMC$)

$\therefore \angle ABM = 25^\circ$ (angle sum)

(ii) $\triangle AOB$ is isosceles (equal radii)

$\therefore \angle BAO = 25^\circ$ (base angles are equal)

$\therefore \theta = 50^\circ$ (exterior angle of $\triangle ABO$)

(b)

(i) $AB = \sqrt{(6-2)^2 + 3^2}$

$= 5$

(ii) $m_{AB} = \frac{3}{4}$ $\quad$ EQUATION AB: $y = \frac{3}{4}(x-2)$

$4y = 3x - 6$

ie $3x - 4y - 6 = 0$

(iii) $p = \left| \frac{3(4) - 4(5) - 6}{5} \right|$

$= \frac{14}{5}$

(iv) Area (ABD) $= \frac{1}{2}(AB)(CD)$

$= \frac{1}{2} \times \frac{14}{5} \times 5$

$= 7$ units.

QUESTION 15:

(a) $\frac{f(x+h) - f(x)}{h} = \frac{3(x+h)^2 - 3x^2}{h}$

$= \frac{6xh + 3h^2}{h}$

$= 6x + 3h$

(b) $\frac{\tan^2 x(\sec x - 1)}{\sec^2 x - 1} = \frac{\tan^2 x(\sec x - 1)}{\tan^2 x}$

$= \sec x - 1$

(c) $\sin\theta = \pm\frac{\sqrt{3}}{2}$

$\therefore \theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ$

(d) $f'(x) = \frac{3}{2}x^{1/2} \Rightarrow f'(4) = \frac{3}{2}(2)$

$= 3$

QUESTION 16:

(a) (i) $dy/dx = \frac{1}{2}(x^3+3)^{-1/2} \cdot 3x^2$

$= \dfrac{3x^2}{2\sqrt{x^3+3}}$

(ii) $dy/dx = \dfrac{(x+1).1 - x.1}{(x+1)^2}$

$= 1/(x+1)^2$

(b) (i) $\angle PBS = 40°$ (alternate angles NP ∥ QS)

and $\angle SBA = 60°$ (straight $\angle QBS$)

$\therefore \angle PBA = 100°$

(ii) In $\triangle PBA$,

$PA^2 = 12^2 + 15^2 - 2 \times 12 \times 15 \cos 100°$

$PA \doteqdot 21$ km

(iii) $\angle BAm = 60°$ (co-interior angles QS ∥ mA)

Let $\angle BAP = \theta$

$\dfrac{\sin\theta}{12} = \dfrac{\sin 100°}{21}$

$\therefore \theta = 34.24°$

$\therefore \angle PAm = 94.24°$

$\therefore$ Bearing is $(360 - 94.24)°$

$= 265.76°$

$= 266°$

EXAMINERS' COMMENTS – YEAR 11 ASSESSMENT 3 MATHEMATICS 2017

QUESTION 9:

d) (i) The Domain was between -4 and 4, not just $x \leq 4$. In most cases, to do the range you really have to know what the graph looks like, and the half-circle gives y as between 0 and 4, not just $y \geq 0$.

QUESTION 10:

a) Make sure you use the correct notation when differentiating and for each step of working. Students need to be aware of setting out – E.g. $y = x^2 \neq 2x$
For parts (iii) and (iv), those who did not change the original equations to a negative/fractional index first had less success differentiating.

c) (ii) Some students used the incorrect gradient. Needed to find the gradient perpendicular to line AC. Some found the perpendicular distance and lost 3 marks – Read the question!

QUESTION 11:

c) If the question asks you to copy the diagram, please do so. Then
   a. A formal proof requires you to start with "In ΔCDB and ……" not just waffle on.
   b. The only similarity proof involving sides is that two sides are in ratio, about a common angle. SAS is NOT a similarity proof. AA is NOT a reason for similarity. We would accept AAA, but better to say "equiangular"
   c. For similarity, you only need to prove 2 angles are the same (by angle sum, the third HAS to be the same)
   d. In this example, x had to be found before progressing.
   e. Many people used ΔCDB and ΔCAB (ok), and then quoted angle ABD which is in neither triangle.

QUESTION 12:

This question was generally well done by all candidates.

QUESTION 13:

a) You need to mention why Θ=60 is not an answer. The question is worth 3 marks!

b) ii) Find the point furthest from the origin (0,0) not the centre of the circle.

c) k is not the root! Since the roots are reciprocal to each other, start with $\propto$ and $\frac{1}{\propto}$ then use product of the roots formula.

QUESTION 14:

a) Students need to give a correct reason for geometry (e.g. the triangle was isosceles as the radii were equal resulting in equal sides)

b) General form is $ax + by + c = 0$

QUESTION 15:

a) answer the given question – asked to find an expression not a limit, or a substitution of x=0 or x=h

b) convoluted methods used, subject to silly mistakes

c) forgot the plus/minus sign when solving the equations

QUESTION 16:

a) (ii) Use the quotient rule when you have a quotient! Do not use the product rule as this is often messy and does not give the simplest form.

b) (i) "Explain" means "set out with reasons"
(iii) Use AP = 21 (from (ii)). You do not need to use more accurate (exact) value for AP since it was found in part (ii)