

Name:

Maths Class:

Year 11
MATHEMATICS

PRELIMINARY EXAMINATION
ASSESSMENT 3

SEPTEMBER, 2018

Time allowed: 120 minutes

General Instructions:

- Write using black or blue pen
- In Questions 11–18, show relevant mathematical reasoning and/ or calculations
- Approved calculators may be used
- Full marks may not be awarded for careless work or illegible writing
- **Begin each question on a new page**
- All answers are to be in the writing booklet provided
- A reference sheet is provided at the back of this paper

Total Marks 74

Section I: Multiple Choice
Questions 1-10
10 Marks

Section II Questions 11-18
64 Marks

Section 1

Multiple Choice (10 marks)

Use the multiple choice answer sheet for Question 1-10

1. What is the value of $\frac{18.81 - 3.47}{2.79 + 7.75}$ correct to 2 significant figures?

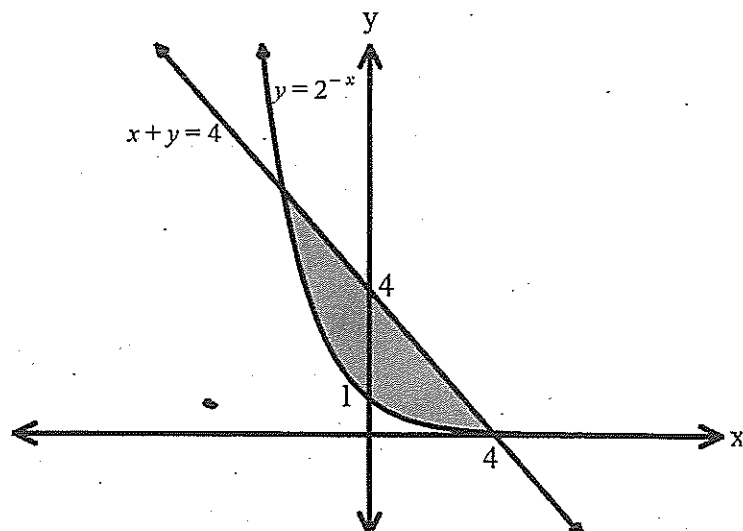
- (A) 1.4
- (B) 1.45
- (C) 1.46
- (D) 1.5

2. What is $\frac{2}{3 - \sqrt{2}}$ as a fraction with a rational denominator?

- (A) $6 - 2\sqrt{2}$
- (B) $6 + 2\sqrt{2}$
- (C) $\frac{6 - 2\sqrt{2}}{7}$
- (D) $\frac{6 + 2\sqrt{2}}{7}$

3. Which pairs of inequalities define the shaded region?

- (A) $\begin{cases} x + y \geq 4 \\ y \leq 2^{-x} \end{cases}$
- (B) $\begin{cases} x + y \leq 4 \\ y \geq 2^{-x} \end{cases}$
- (C) $\begin{cases} x + y \geq 4 \\ y \leq 2^{-x} \end{cases}$
- (D) $\begin{cases} x + y \leq 4 \\ y \leq 2^{-x} \end{cases}$



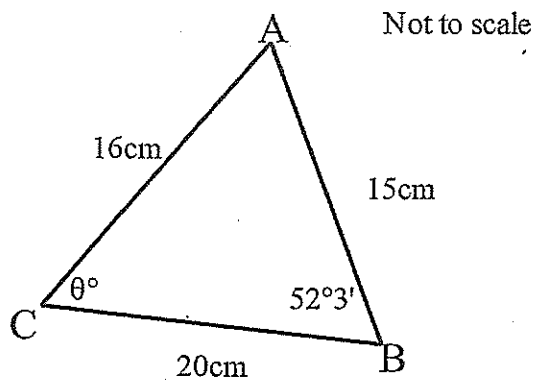
4. Which of the following is a correct expression involving θ in the triangle ABC?

(A) $15^2 = 16^2 + 20^2 - 16 \times 20 \cos \theta$

(B) $\cos \theta = \frac{16^2 + 20^2 - 15^2}{2 \times 20 \times 15}$

(C) $\frac{15}{\sin \theta} = \frac{16}{\sin 52^\circ 3'}$

(D) $\frac{\sin \theta}{16} = \frac{\sin 52^\circ 3'}{15}$



5. Which parabola has a vertex at (2,-1) and directrix $y = 1$?

(A) $(x - 2)^2 = 4(y + 1)$

(B) $(x + 2)^2 = 4(y - 1)$

(C) $(x - 2)^2 = 8(y + 1)$

(D) $(x + 1)^2 = 8(y - 1)$

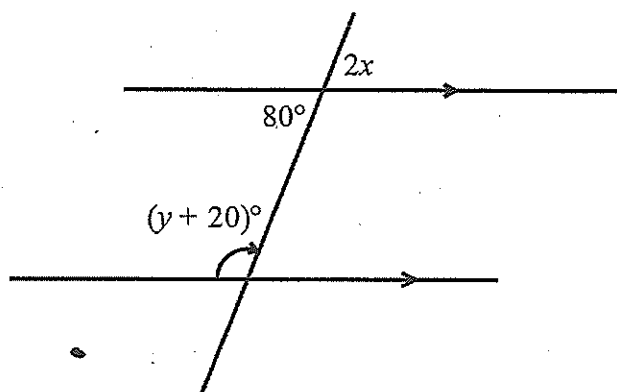
6. What are the values of x and y ?

(A) $x = 40^\circ$ $y = 60^\circ$

(B) $x = 40^\circ$ $y = 80^\circ$

(C) $x = 80^\circ$ $y = 80^\circ$

(D) $x = 80^\circ$ $y = 80^\circ$



7. What is the value of $f(-1)$ if $f(x) = x^3 - 4x$?

(A) $f(-1) = -3$

(B) $f(-1) = -5$

(C) $f(-1) = 3$

(D) $f(-1) = 5$

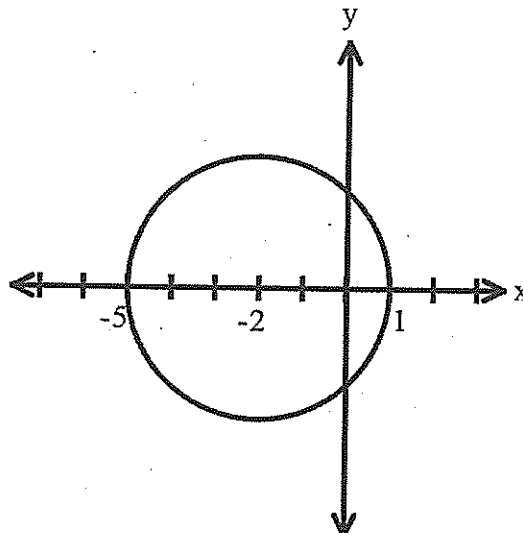
8. What is the equation of the circle shown in the diagram below?

(A) $(x - 2)^2 + y^2 = 9$

(B) $(x - 2)^2 + y^2 = 3$

(C) $(x + 2)^2 + y^2 = 9$

(D) $(x + 2)^2 + y^2 = 3$



9. What is the derivative of $(x^3 + 4)^3$?

(A) $3(x^3 + 4)^2$

(B) $3(3x^2 + 4)^2$

(C) $9x^2(x^3 + 4)^2$

(D) $9x^2(3x^2 + 4)^2$

10. For what value of k does the quadratic equation $x^2 + (k + 2)x + 2k = 0$ have equal roots?

(A) -2

(B) 0

(C) 2

(D) 4

Section II

Total Marks (60)

Attempt Questions 11 – 18.

Answer each question in your writing booklet.

In Questions 11-18, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (8 Marks)

- (a) Factorise $2x^2 + 7x - 4$ 2
- (b) Solve $|x - 4| = 1 - 2x$ 2
- (c) Evaluate $\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x - 1}$ 2
- (d) Sketch on $\frac{1}{3}$ of your page $f(x) = |x| - 4$ showing all main features
(Hint use a ruler) 2

End of Question 11

Question 12 (8 Marks)

Use a Separate Sheet of paper

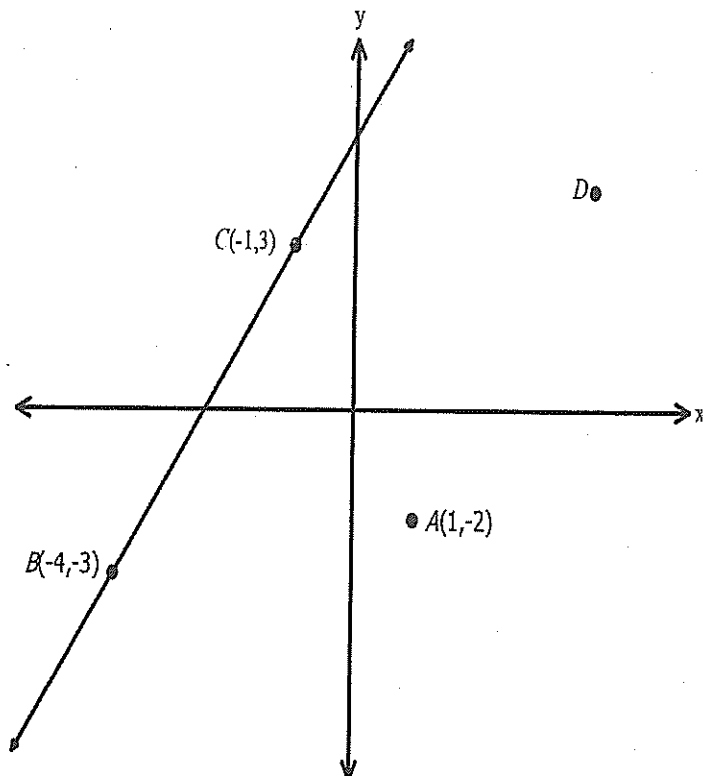
- (a) Solve $2\cos x + 1 = 0$ for $0^\circ \leq x \leq 360^\circ$ 2
- (b) Simplify $\cot \theta - \cot \theta \cos^2 \theta$ 2
- (c) The quadratic equation $2x^2 - 5x - 3$ has roots α and β . Find:
- i) $\alpha + \beta$ 1
- ii) $\alpha \beta$ 1
- iii) $\alpha^2 + \beta^2$ 2

End of Question 12

Question 13 (8 Marks)

Use a Separate Sheet of paper

In the diagram below the points A, B and C have coordinates (1,-2), (-4,-3) and (-1,3) respectively.



Not drawn to scale

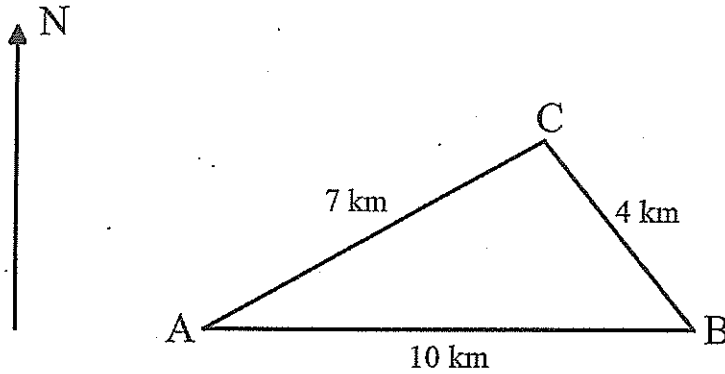
- a) Calculate the exact length of interval BC 2
- b) Find the gradient of BC 1
- c) Hence, show the equation of BC is $y = 2x + 5$ 1
- d) Find the perpendicular distance between A and the line BC 2
- e) Find the co-ordinates of D, in the first quadrant so that ABCD is a parallelogram. 1
- f) Find the exact area of the parallelogram ABCD 1

End of Question 13

Question 14 (8 Marks)

Use a Separate Sheet of paper

- (a) In the diagram building B is 10km due east of building A. Building C is 7 km from building A and 4 km from building B.



- i) Find the size of $\angle ABC$ correct to the nearest degree. 2
- ii) Hence find the bearing of building B from building C correct to the nearest degree. 2
- (b) For the function $f(x) = 3 + \sqrt{4 - x^2}$ find the
- i) the domain 1
- ii) the range 1
- (c) If $y = \frac{2x+3}{(x-1)^2}$ find $\frac{dy}{dx}$ in its simplest form. 2

End of Question 14

Question 15 (8 Marks)

Use a Separate Sheet of paper

- (a) Find the equation of the line through the point of intersection of the lines $6x - 5y = 3$ and $4x + y = -11$ and also through the point $(2,1)$ 3
- (b) A right-angled triangle has sides of length x cm, $(x + 1)$ cm and $(x + 3)$ cm
- i) Show that $x^2 - 4x - 8 = 0$ 2
- ii) Hence find the length of the hypotenuse in simplest exact form. 1
- (c) Find values for A and B if $3(x - 2)^2 \equiv A(x^2 + 4) + Bx$. 2

End of Question 15

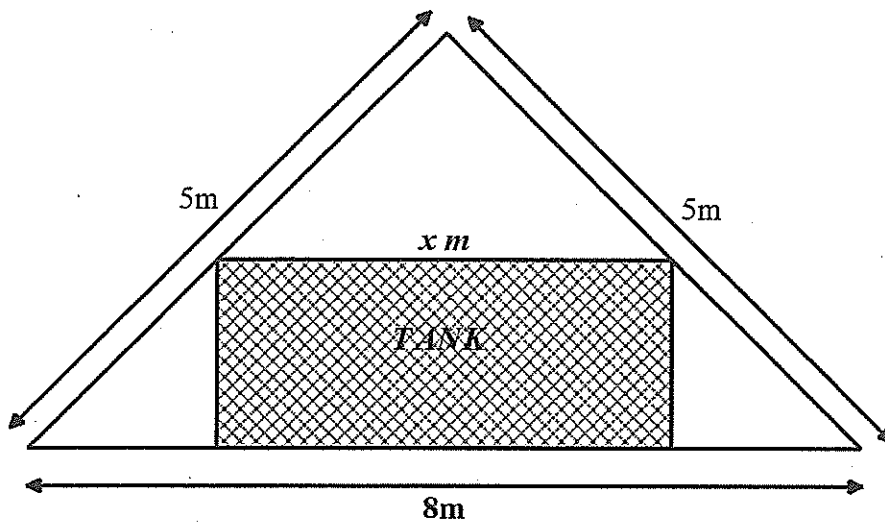
Question 16 (8 Marks)

Use a Separate Sheet of paper

(a) For what values of m is $-2x^2 + 3x + m$ negative definite.

1

(b) A rectangular hot water tank x metres wide, y metres high and 1.5 metres long, Which fits exactly into the roof of the house. The cross-section of the roof is an isosceles triangle with base 8 metres and equal sides 5 metres in length, as shown below.



i) Explain why the roof of the house is 3 metres high.

1

ii) Show that $y = \frac{3}{8}(8 - x)$.

2

iii) Show that the volume of the tank is given by $V = \frac{9x}{16}(8 - x)$

1

iv) Calculate the maximum volume of the tank. (without calculus)

3

End of Examination



Year 11 Mathematics

Assessment 3 September 2018.

Multiple Choice:

- | | |
|------|-------|
| 1. D | 6. B |
| 2. D | 7. C |
| 3. B | 8. C |
| 4. C | 9. C |
| 5. B | 10. C |

Question 11

a) $2x^2 + 7x - 4 = (2x - 1)(x + 4)$

b) $|x - 4| = 1 - 2x$

$x - 4 = 1 - 2x$

$3x = 5$

$x = \frac{5}{3}$

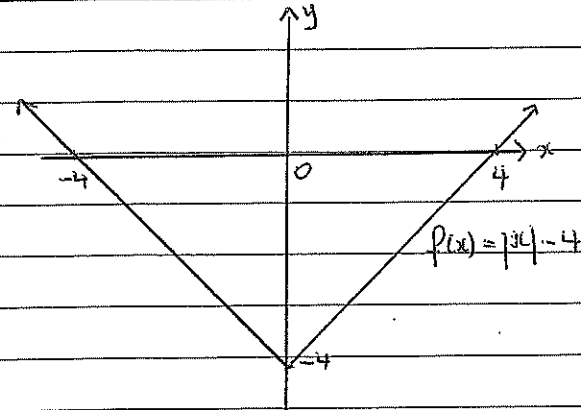
$-x + 4 = 1 - 2x$

$x = -3$

c) $\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x - 1} = \lim_{x \rightarrow 1} \frac{(x - 3)(x - 1)}{x - 1}$

$= \lim_{x \rightarrow 1} x - 3$

$= -2$



Question 12

a) $2 \cos x + 1 = 0$

$2 \cos x = -1$

$\cos x = -\frac{1}{2}$

$x = 120^\circ, 240^\circ$

b) $\cot \theta - \cot \theta \cos^2 \theta$
 $= \cot \theta (1 - \cos^2 \theta)$

$= \cot \theta \times \sin^2 \theta$

$= \frac{\cos \theta}{\sin \theta} \times \sin^2 \theta$

$\sin \theta$

$= \sin \theta \times \cos \theta$

c) $2x^2 - 5x - 3$

$a = 2$

$b = -5$

$c = -3$

i) $\alpha + \beta = \frac{-b}{a}$
 $= \frac{-(-5)}{2} = \frac{5}{2}$

Student Name: _____ Teacher Name: _____

$$c) \text{ ii) } \alpha\beta = \frac{c}{a} = \frac{-3}{2}$$

$$\text{iii) } \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \left(\frac{5}{2}\right)^2 - 2 \times \frac{-3}{2}$$

$$= \frac{37}{4} \text{ or } 9\frac{1}{4}$$

Question 13

$$a) \quad B(-4, -3) \quad C(-1, 3)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-4 - (-1))^2 + (-3 - 3)^2}$$

$$d = \sqrt{(-3)^2 + (-6)^2}$$

$$d = \sqrt{45}$$

$$b) \quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 3}{-4 - (-1)}$$

$$= \frac{-6}{-3}$$

$$= 2$$

$$c) \quad y - 3 = 2(x + 1)$$

$$y - 3 = 2x + 2$$

$$y = 2x + 5$$

Student Name: _____ Teacher Name: _____

$$\text{ik) } A(1, -2) \quad 2x - y + 5 = 0$$

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$d = \frac{|2 \times 1 + (-1)(-2) + 5|}{\sqrt{2^2 + (-1)^2}}$$

$$d = \frac{|2 + 2 + 5|}{\sqrt{5}}$$

$$d = \frac{9}{\sqrt{5}} \text{ or } \frac{9\sqrt{5}}{5}$$

$$e) \quad (4, 4)$$

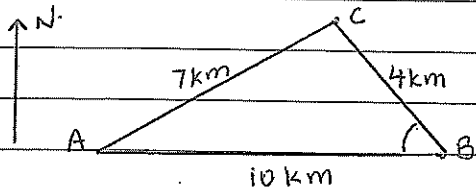
$$f) \quad A = b \times h$$

$$= \sqrt{45} \times \frac{9}{\sqrt{5}}$$

$$= \sqrt{9} \times \sqrt{5} \times \frac{9}{\sqrt{5}}$$

$$= 27 \text{ units}^2$$

Question 14



ai) $\cos B = \frac{4^2 + 10^2 - 7^2}{2 \times 4 \times 10}$

$\cos B = \frac{67}{80}$

$B = \cos^{-1}\left(\frac{67}{80}\right)$

$B = 33^\circ$

aii) Bearing of B from C is $90^\circ + 33^\circ = 123^\circ T$

b i) $f(x) = 3 + \sqrt{4-x^2}$
Domain $-2 \leq x \leq 2$

ii) Range $3 \leq y \leq 5$

e $y = \frac{2x+3}{(x-1)^2}$ $u = 2x+3$ $v = (x-1)^2$
 $du = 2$ $dv = 2(x-1)$

$\frac{dy}{dx} = \frac{(x-1)^2 \times 2 - 2(x-1)(2x+3)}{(x-1)^4}$

$= \frac{2(x-1)[(x-1) - (2x+3)]}{(x-1)^4}$

$\frac{dy}{dx} = \frac{2(x-1)[x-1 - (2x+3)]}{(x-1)^4}$

$= \frac{2(x-1)[x-1 - 2x-3]}{(x-1)^4}$

$= \frac{-2(x+4)}{(x-1)^3}$

Question 15

a) $6x - 5y - 3 + k(4x + y + 11) = 0$
 $6(2) - 5(1) - 3 + 4[4 \times 2 + 1 + 11] = 0$
 $12 - 5 - 3 + 20k = 0$
 $20k = -4$
 $k = -\frac{1}{5}$

$6x - 5y - 3 - \frac{1}{5}(4x + y + 11) = 0$
 $5(6x - 5y - 3) - 1(4x + y + 11) = 0$
 $30x - 25y - 15 - 4x - y - 11 = 0$
 $26x - 26y - 26 = 0$
 $x - y - 1 = 0$

b i) $x^2 + (x+1)^2 = (x+3)^2$
 $x^2 + x^2 + 2x + 1 = x^2 + 6x + 9$
 $x^2 - 4x - 8 = 0$

bii) $x^2 - 4x + 4 = 12$ $x > 0$
 $(x-2)^2 = 12$
 $(x-2) = \pm 2\sqrt{3}$
 $x = 2\sqrt{3} + 2$ since $x > 0$
 $x + 3 = 5 + 2\sqrt{3}$

Hypotenuse has length $(5 + 2\sqrt{3})$ cm.

c) $3(x-2)^2 = A(x^2+4) + Bx$

$A = 3$ $B = -12$

$3(x^2 - 4x + 4) = Ax^2 + 4A + Bx$

$3x^2 - 12x + 12 = Ax^2 + 4A + Bx$

$3x^2 = Ax^2$ $-12x = Bx$

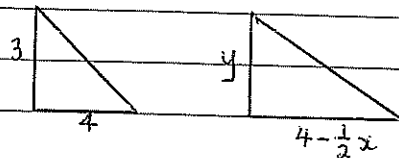
$A = 3$ $B = -12$

Question 16

a) $\Delta < 0$ $-2x^2 + 3x + m$
 $a < 0$ $\Delta = 3^2 - 4 \times -2 \times m$
 $= 9 + 8m < 0$
 $9 < -8m$
 $m < -\frac{9}{8}$

b i) $h^2 = 5^2 - 4^2$
 $h^2 = 9$
 $h = 3$ m

bii)



ratio of sides of similar triangles.

$\frac{y}{3} = \frac{4 - \frac{1}{2}x}{4}$

$4y = 12 - \frac{3}{2}x$

$y = 3 - \frac{3}{8}x$

$y = \frac{3}{8}(8 - x)$

$$16 \text{ ciii) } V = 1.5 \times x \times y$$

$$V = \frac{3}{2} \times x \times \frac{3}{8} (8-x)$$

$$V = \frac{9}{16} x(8-x)$$

16 c iv) Concave down parabola.
maximum at the vertex.

$$\text{Axis of symmetry} = \frac{-b}{2a} = \frac{-\frac{9}{2}}{2 \times \frac{9}{16}}$$

$$= \frac{9 \times 8}{2 \times 9}$$

$$= 4$$

maximum value

$$V = \frac{9}{2} (4) - \frac{9}{16} (4)^2$$

$$= 9 \text{ m}^3$$

$$\text{OR } \max V = \frac{-\Delta}{4a}$$

$$= \frac{-\left(\frac{9}{2}\right)^2 - 0}{4 \times \frac{9}{16}}$$

$$= 9 \text{ m}^3$$