STUDENT'S NAME: ______________________

TEACHER'S NAME: ______________________

BAULKHAM HILLS HIGH SCHOOL

YEAR 11

YEARLY EXAMINATION

2007

MATHEMATICS EXTENSION 1

Time allowed - Two hours
(Plus five minutes reading time)

DIRECTIONS TO CANDIDATES:

- Attempt **ALL** questions.
- Start each of the questions on a new page.
- All necessary working should be shown.
- Write your teacher's name and your name on the cover sheet provided.

QUESTION 1 (12 marks)

		Marks
(a)	Solve the inequality $\frac{3x-2}{x} \geq 1$	3
(b)	Find the value of k if $(x-2)$ is a factor of $P(x) = x^4 - 3x^3 + kx^2 - 4$	2
(c)	If P is the point $(-3,5)$ and Q is the point $(1,-2)$ find the coordinates of the point R which divides PQ externally in the ratio 3 : 2.	2
(d)	Find the obtuse angle between the lines $2x - y = 0$ and $x - 2y = 0$ giving your answer correct to the nearest degree.	3
(e)	Differentiate $\left(\sqrt{x}+3\right)\left(\frac{1}{\sqrt{x}} - 2\sqrt{x}\right)$	2

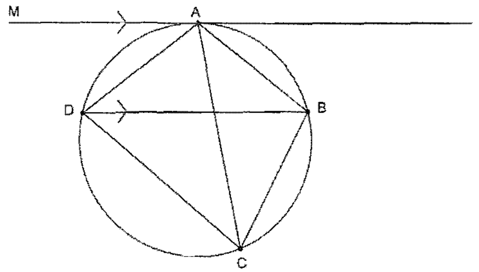
QUESTION 2 (12 marks)

		Marks
(a)	If $\sin 2A = \frac{1}{2}$ find the exact value of $\frac{1}{\sin A\cos A}$	2
(b)	The equation $x^3 - 6x^2 + 4x + 2 = 0$ has three real roots α, β and γ.	
	(i) Write down the values of $\alpha+\beta+\gamma$, $\alpha\beta+\beta\gamma+\alpha\gamma$ and $\alpha\beta\gamma$	1
	(ii) Hence find the value of $(\alpha-2)(\beta-2)(\gamma-2)$	3
(c)	Prove using mathematical induction that $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$	4
(d)	Simplify $\frac{xy^{-1} - yx^{-1}}{x-y}$	2

QUESTION 3 (12 marks)

Marks

(a)



ABCD is a cyclic quadrilateral. The tangent to the circle at A is parallel to DB.

(i) Copy the diagram

(ii) Give a reason why $\angle ACD = \angle MAD$ **1**

(iii) Give a reason why $\angle ACB = \angle ADB$ **1**

(iv) Hence show that AC bisects $\angle BCD$ **2**

(b) (i) Show that $1 - \frac{2}{x-2} = \frac{x-4}{x-2}$ **1**

(ii) Show that the function $f(x) = 1 - \frac{2}{x-2}$ is increasing for all values of x in its domain. **2**

(iii) Sketch the graph showing clearly the coordinates of any points of intersection with the x and y axis and the equations of any asymptotes. **2**

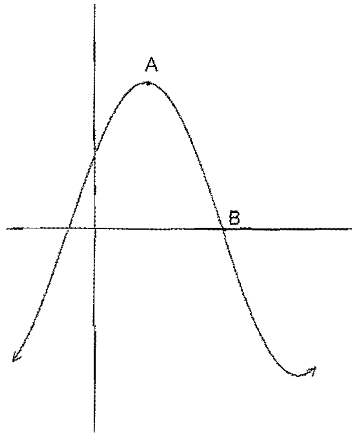
(c) Use the definition $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to find the derivative of $f(x) = \frac{1}{x}$. **3**

QUESTION 4 (12 marks)

Marks

(a) (i) Express $\sqrt{3}\sin x + \cos x$ in the form $R\sin(x + \alpha)$ where $R>0$ and $0 < \alpha < 90°$. **2**

(ii) The graph of $y = \sqrt{3}\sin x + \cos x$ is shown below. Hence, or otherwise, find the coordinates of A and B if A is a maximum turning point and B is where the curve cuts the x axis. **3**



(b) (i) Show that $\tan(45° + A) = \frac{\cos A + \sin A}{\cos A - \sin A}$ **2**

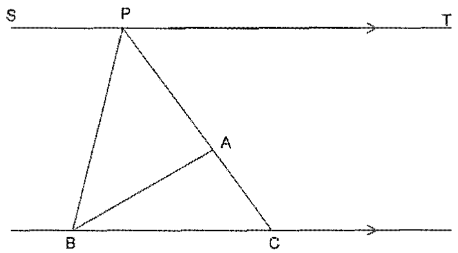
(ii) Hence show that $\tan(45° + A) = \frac{1 + \sin 2A}{\cos 2A}$ **2**

(c) Solve the equation $\cos^2 x + \sin 2x = 0$ for $0 \le x \le 360°$, giving your answer to the nearest degree. **3**

QUESTION 5 (12 marks)

Marks

(a) In the diagram, ST||BC and $\angle SPB = \angle PAB$.



(i) Prove that ΔPBA is similar to ΔPCB **2**

(ii) Deduce that $PB^2 = PA \times PC$ **2**

(b) (i) Sketch the polynomial $p(x) = (x+7)(1-x)(x-3)^2$ **1**

(ii) Hence, for what values of x is $p(x) > 0$ **1**

(c) If the equation $3x^3 - 4x^2 + 2x + 1 = 0$ has roots α, β and γ, find the equation with roots 2α, 2β and 2γ **3**

(d) (i) Show that the sum:

$$a - ap + ap^2 - ap^3 + ap^4 + \ldots + ap^{2k} = a\frac{(1+p^{2k+1})}{1+p}$$ **2**

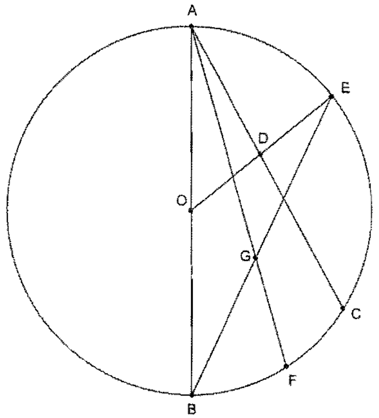
(ii) Hence, or otherwise, find an expression for the sum: **1**

$$3^n - 3^{n+1} + 3^{n+2} - 3^{n+3} + \ldots + 3^{3n}$$

QUESTION 6 (12 marks)

(a) Factorise completely: $x^2 + y^2 + 2xy - z^2$. **1**

(b)



In the figure above, AOB is the diameter of a circle centre O. D is a point on the chord AC such that DA = DO and OD is produced to E. AF is the bisector of $\angle BAC$ and cuts BE in G.

(i) Prove that GA = GB. **3**

(ii) Prove that AOGE is a cyclic quadrilateral. **2**

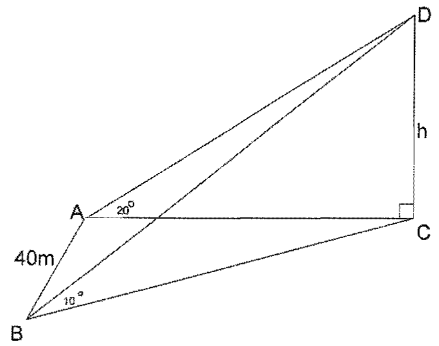
(iii) If CE is joined, prove that CE $\parallel$ AF. **2**

(c) Use mathematical induction to show that $5^n + 2(11^n)$ is a multiple of 3 for all positive integers n. **4**

QUESTION 7

Marks

(a) The new Baulkham Hills High School pole, CD, of height h metres, near the office, stands with base C on horizontal ground. A is a point on the ground due west of C and B is a point on the ground 40 metres due south of A. From A and B the angles of elevation to the top of the pole are 20° and 10° respectively.



(i) Find expressions for AC and BC in terms of h. **1**

(ii) Show that $h = \dfrac{40}{\sqrt{\cot^2 10° - \cot^2 20°}}$ **2**

(iii) Hence find the height of the pole. **1**

(b) A car leasing company provides finance to customers. Clients can borrow \$P at r% per month reducible interest calculated monthly. The loan is to be repaid in equal monthly payments of \$M.

Let $R = 1 + \frac{r}{100}$ and let $\$A_n$ be the amount owing after n monthly repayments have been made.

(i) Write an expression for the amount owing after two repayments (i.e. A_2) in terms of P, R and M. **1**

(ii) Show that the amount owing after the n^{th} repayment is given by **2**
$A_n = PR^n - \dfrac{M(R^n - 1)}{R - 1}$

(iii) If the amount owing after the n^{th} repayment is K% of the amount borrowed, show that: **3**

$$R^n = \frac{PK(R-1) - 100M}{100[P(R-1) - M]}$$

(iv) Hence use your calculator to estimate the number of years required for the amount owing to fall to 20% of the amount borrowed if a client borrows \$40,000 and makes monthly repayments of \$800. Interest is charged at 9% per annum compounding monthly. **2**

END OF EXAMINATION

EXTENSION 1

Yr 11 SOLUTIONS YEARLY '07

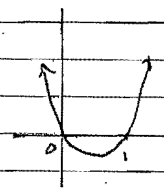
1(a) $\frac{3x-2}{x} \geq 1$ where $x \neq 0$

$x(3x-2) \geq x^2$

$3x^2 - 2x \geq x^2$

$2x^2 - 2x \geq 0$

$2x(x-1) \geq 0$ (1)



$\therefore x \leq 0$ and $x \geq 1$ (1)

(b) $P(2) = 0$ by remainder theorem } (1)

$0 = 2^4 - 3 \times 8 + 4k - 4$

$4k = 12$

$k = 3$ (1)

(c) $P(-3, 5)$ $\quad Q(1, -2)$

$-3 : 2$

Point is $\left(\frac{2 \times -3 + -3 \times 1}{-1}, \frac{2 \times 5 - 3 \times -2}{-1}\right)$

(1) (1) (NB: 1 for correct internal division)

Pt $= (9, -16)$

(d) $2x - y = 0 \qquad x - 2y = 0$

$y = 2x \qquad 2y = x$

$m_1 = 2$ ← both (1) → $y = \frac{x}{2}$, $m_2 = \frac{1}{2}$

acute $\angle$: $\tan\theta = \left|\frac{2 - \frac{1}{2}}{1 + 2 \times \frac{1}{2}}\right|$ (1) subst into formula

$= \left|\frac{1\frac{1}{2}}{2}\right|$

obtuse $\angle$: $\tan\theta = -\frac{3}{4}$

$\therefore \theta = 143°7'48.37''$

$\theta = 143°$ nearest degree (1) (ignore rounding)

1(e) $(\sqrt{x} + 3)\left(\frac{1}{\sqrt{x}} - 2\sqrt{x}\right)$

$= 1 - 2x + \frac{3}{\sqrt{x}} - 6\sqrt{x}$

$= 1 - 2x + 3x^{-\frac{1}{2}} - 6x^{\frac{1}{2}}$ (1)

$\frac{d}{dx}\left(1 - 2x + 3x^{-\frac{1}{2}} - 6x^{\frac{1}{2}}\right)$

$= -2 - \frac{3}{2}x^{-\frac{3}{2}} - 3x^{-\frac{1}{2}}$

$= -2 - \frac{3}{2x\sqrt{x}} - \frac{3}{\sqrt{x}}$ } (1)

2(a) $\sin 2A = \frac{1}{2}$

$2\sin A\cos A = \frac{1}{2}$

$\sin A\cos A = \frac{1}{4}$ (1)

$\frac{1}{\sin A\cos A} = \frac{1}{1/4}$

$= 4$ (1)

b)(i) $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{6}{1} = 6$ (1) all 3 correct

$\alpha\beta + \beta\gamma + \alpha\gamma = 4$

$\alpha\beta\gamma = -2$

(ii) $(\alpha - 2)(\beta\gamma - 2\beta - 2\gamma + 4)$ (1)

$= \alpha\beta\gamma - 2\alpha\beta - 2\alpha\gamma - 2\beta\gamma + 4\alpha + 4\beta + 4\gamma - 8$

$= \alpha\beta\gamma - 2(\alpha\beta + \alpha\gamma + \beta\gamma) + 4(\alpha + \beta + \gamma) - 8$ (1)

$= -2 - 8 + 24 - 8$

$= 6$ (1)

c) $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots \frac{1}{n(n+1)}$

Testing $n = 1$

LHS $= \frac{1}{1 \times 2}$ $\qquad$ RHS $= \frac{1}{1+1}$

$= \frac{1}{2}$ $\qquad = \frac{1}{2}$ (1)

$= LHS$

$\therefore$ True for $n = 1$

Assume true for $n = k$

ie $\frac{1}{1.2} + \frac{1}{2.3} + \ldots \frac{1}{k(k+1)} = \frac{k}{k+1}$ (1)

For $n = k+1$

$\frac{1}{1.2} + \frac{1}{2.3} + \ldots \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$

$\sum_{r=1}^{k} \frac{1}{r(r+1)} + \frac{1}{(k+1)(k+2)} = \frac{k(k+2)+1}{(k+1)(k+2)}$

$= \frac{k^2 + 2k + 1}{(k+1)(k+2)}$

$= \frac{(k+1)^2}{(k+1)(k+2)}$

$= \frac{k+1}{k+2}$

$= \frac{k+1}{(k+1)+1}$ (1)

which is of the form $\frac{n}{n+1}$ where $n = k+1$.

$\therefore$ If true for $n = k$, proved true for $n = k+1$

But is true for $n = 1$ $\therefore$ true for $n = 2$.

Is true for $n = 2$, $\therefore$ true for $n = 3$ and so on for all $n \geq 1$. (1) conclusion

$\therefore \sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$

(d) $= \dfrac{\frac{x}{y} - \frac{y}{x}}{x - y}$

$= \frac{x^2 - y^2}{xy(x-y)}$ (multiplying numerator and denominator by xy) (1)

$= \frac{(x+y)(x-y)}{xy(x-y)}$

$= \frac{x+y}{xy}$ (1)

3(a) NB -1 if diagram not copied from either (ii)(iii) or (iv)

(ii) Angle between a tangent and a chord drawn to the point of contact is equal to the angle in the alternate segment. (1)

(iii) Angles at the circumference standing on the same arc are equal (1)

iv) $\angle MAD = \angle ADB$ (alternate angles $MA \parallel BD$)

$\therefore \angle ACD = \angle ADB$ from (i) (1)

and since $\angle ACB = \angle ADB$ (from (ii))

$\therefore \angle ACD = \angle ACB$

$\therefore AC$ bisects $\angle BCD$ } (1)

b(i) LHS $= 1 - \frac{2}{x-2}$

$= \frac{x-2-2}{x-2}$

$= \frac{x-4}{x-2}$ (1)

$= RHS$

$\therefore 1 - \frac{2}{x-2} = \frac{x-4}{x-2}$

(ii) $f'(x) = \frac{(x-2).1 - (x-4).1}{(x-2)^2}$

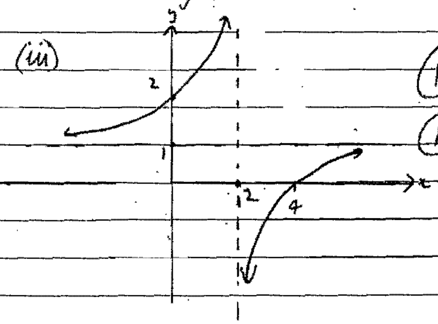
$= \frac{x - 2 - x + 4}{(x-2)^2}$

$f'(x) = \frac{2}{(x-2)^2}$ (1)

$f'(x) > 0$ since $(x-2)^2 > 0$ (1)

$\therefore$ Increasing for all x in its domain

(iii)



(1) shape

(1) asymp & intercept

when $x = 0$, $y = 2$

$y = 0$, $\quad 1 = \frac{2}{x-2}$

$x - 2 = 2$

$x = 4$

c) $f'(x) = \lim_{h\to 0} \dfrac{\frac{1}{x+h} - \frac{1}{x}}{h}$ (1)

$= \lim_{h\to 0} \dfrac{\frac{x-(x+h)}{x(x+h)}}{h}$

$= \lim_{h\to 0} \dfrac{-h}{hx(x+h)}$

$= \lim_{h\to 0} \dfrac{-1}{x(x+h)}$ (1)

$f'(x) = \dfrac{-1}{x^2}$ (1)

4 a(i) $\sqrt{3}\sin x + \cos x \rightarrow R = \sqrt{3+1} = 2$ (1)

$2\left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right)$

$= 2(\sin x\cos\alpha + \cos x\sin\alpha)$

$\therefore \cos\alpha = \frac{\sqrt{3}}{2}$ & $\sin\alpha = \frac{1}{2}$

$\alpha = 30°$ (1)

$\therefore \sqrt{3}\sin x + \cos x = 2\sin(x+30°)$

(ii) Max value of $2\sin(x+30°) = 2\times 1 = 2$

$\sqrt{3}\sin x + \cos x = 0$ — For A, $2\sin(x+30) = 2$

$2\sin(x+30°) = 0$ — $\sin(x+30) = 1$

$\sin(x+30°) = 0$ — $x + 30 = 90$

$x + 30° = 0, 180, 360°$ — $x = 60$

$x = -30, 150°, 330°$ $\therefore A$ is $(60°, 2)$ (1)

First intercept is $150°$ (1)

$\therefore B$ is $(150°, 0)$ (1)

b(i) $\tan(45°+A) = \dfrac{\sin(45°+A)}{\cos(45°+A)}$

$= \dfrac{\sin 45°\cos A + \cos 45°\sin A}{\cos 45°\cos A - \sin 45°\sin A}$ (1)

$= \dfrac{\frac{\cos A}{\sqrt{2}} + \frac{\sin A}{\sqrt{2}}}{\frac{\cos A}{\sqrt{2}} - \frac{\sin A}{\sqrt{2}}}$

$= \dfrac{\cos A + \sin A}{\cos A - \sin A}$ (1)

(ii) $\tan(45°+A) = \dfrac{(\cos A + \sin A)(\cos A + \sin A)}{(\cos A - \sin A)(\cos A + \sin A)}$ (1)

$= \dfrac{\cos^2 A + \sin^2 A + 2\sin A\cos A}{\cos^2 A - \sin^2 A}$

$\tan(45°+A) = \dfrac{1 + \sin 2A}{\cos 2A}$ — since $\sin^2 A + \cos^2 A = 1$ and $\sin 2A = 2\sin A\cos A$ and $\cos 2A = \cos^2 A - \sin^2 A$ (1)

(c) $\cos^2 x + \sin 2x = 0$

$\cos^2 x + 2\sin x\cos x = 0$

$\cos x(\cos x + 2\sin x) = 0$

$\cos x = 0$ — $2\sin x + \cos x = 0$ (1)

$2\sin x = -\cos x$

$\cos x = 0$ — $\tan x = -\frac{1}{2}$

$x = 90°, 270°$ (1) — $x = 180 - 27°, 360 - 27°$

$x = 153°, 333°$

$\therefore x = 90°, 153°, 270°, 333°$ (1)

5 (a) In ΔPAB and ΔPCB

$\angle BPA = \angle BPC$ (common)

$\angle PBC = \angle SPB$ (alt angles, $ST \parallel BC$) (1)

$\therefore \angle PBC = \angle PAB$ (since $\angle SPB = \angle PAB$ given)

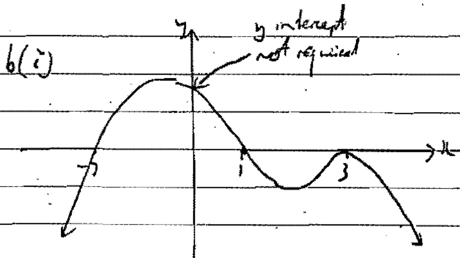
$\therefore \angle PBA = \angle PCB$ (angle sum of Δ's PAB, PCB)

$\therefore \Delta PBA \parallel\!\!\mid \Delta PCB$ (three pairs of matching angles are equal) (1)

(ii) $\dfrac{PA}{PB} = \dfrac{PB}{PC}$ (matching sides in similar Δ's PBA & PCB are in same ratio) (1)

$PB^2 = PA.PC$ (1)

b(i)



(1)

As $x \to \infty$, $P(x) \to -\infty$

When $x = 0$, $P(0) = 7\times 1\times 9 = 63$

(ii) From graph

$-7 < x < 1$ (1)

5(c) $\alpha + \beta + \delta = \frac{4}{3}$

$\alpha\beta + \alpha\delta + \beta\delta = \frac{2}{3}$ — (1) all 3

$\alpha\beta\delta = -\frac{1}{3}$

$2\alpha + 2\beta + 2\delta = 2(\alpha+\beta+\delta)$

$= \frac{8}{3}$

$4\alpha\beta + 4\alpha\delta + 4\beta\delta = 4(\alpha\beta + \beta\delta + \alpha\delta)$ — (1) all 3

$= \frac{8}{3}$

$2\alpha\beta . 2\alpha\delta . 2\beta\delta = 8\alpha\beta\delta$

$= -\frac{8}{3}$

$\therefore x^3 - \frac{8}{3}x^2 + \frac{8}{3}x + \frac{8}{3} = 0$ — (1) other

ie $3x^3 - 8x^2 + 8x + 8 = 0$

d(i) GP with $a = a$, $r = \frac{-ap}{a} = -p$ — (1) for use of sum GP

$S_{2k+1} = \dfrac{a(1-(-p)^{2k+1})}{1-(-p)}$

$= \dfrac{a(1-(-1)^{2k+1}p^{2k+1})}{1+p}$ (1)

$= \dfrac{a(1+p^{2k+1})}{1+p}$ — since $(-1)^{2k+1} = -1$ since $2k+1$ is odd

(ii) $3^n - 3^{n+1} + \ldots + 3^{3n} = 3^n - 3^n 3^1 + \ldots + 3^n 3^{2n}$

Here $a = 3^n$, $p = 3$ and $n = k$

$\therefore S_{2k+1} = \dfrac{3^n(1+3^{2n+1})}{1+3}$

$= \dfrac{3^n(1+3^{2n+1})}{4}$ (1)

6(a) $x^2 + y^2 + 2xy - z^2$

$= (x+y)^2 - z^2$

$= (x+y+z)(x+y-z)$ (1)

b) Let $\angle OAG = y$

$\angle OAG = \angle DAG = y$ (given AF bisects $\angle BAC$)

$\angle DOA = 2y$ (base angles of isosceles ΔDOA, $OA = DO$) (1)

$\angle OBG = y$ (angle at circumference is half angle at centre standing on same arc) (1)

$\therefore \angle OBG = \angle OAG = y$

$\therefore \Delta GAB$ is isosceles (base angles equal)

$\therefore GA = GB$ (equal sides opposite equal angles in isosceles ΔGAB) (1)

(ii) Join AE

$\angle AGE = 2y$ (exterior $\angle$ of ΔAGB) (1)

$\therefore \angle DOA = \angle AGE = 2y$

$\therefore ADGE$ is a cyclic quadrilateral (angles at the circumference standing on the same arc are equal) (1)

(iii) Join CE

$\angle ACE = \angle ABE$ (angles at circumference standing on the same arc)

$\angle ACE = \angle FAC$ ($\angle FAC = y$ proved above given AF bisects $\angle BAC$)

$\therefore CE \parallel AF$ (alternate angles are equal) (1)

(c) Test $n = 1$

$5^1 + 2\times 11^1 = 27$

$= 3\times 9$

$\therefore$ Divisible by 3

$\therefore$ True for $n = 1$ (1)

Assume true for $n=k$

$5^k + 2(11^k) = 3P$ where P is an integer (1)

For $n = k+1$

$5^{k+1} + 2(11)^{k+1} = 5.5^k + 2.11.11^k$

$= 5(3P - 2(11)^k) + 22.11^k$ (1)

$= 15P - 10.11^k + 22.11^k$

$= 15P + 12.11^k$

$= 3(5P + 4.11^k)$

$= 3 \times Q$ where Q is an integer (1)

(since $5P$ and 11^k are integers)

which is divisible by 3.

$\therefore$ If true for $n=k$, proved true for $n=k+1$

But true for $n=1$, $\therefore$ true for $n=2$

But true for $n=2$, $\therefore$ true for $n=3$ and so on

for all $n \geq 1$

$\therefore 5^n + 2(11^n)$ is divisible by 3

7.(i) $\tan A = \frac{h}{AC}$

$\tan 20° = \frac{h}{AC}$

$AC = \frac{h}{\tan 20°}$

Similarly $BC = \frac{h}{\tan 10°}$

(1) for both

(ii) In $\triangle ABC$

$BC^2 = AB^2 + AC^2$

$AB^2 = BC^2 - AC^2$

$40^2 = h^2 \cot^2 10° - h^2 \cot^2 20°$ (1)

$1600 = h^2(\cot^2 10° - \cot^2 20°)$

$h^2 = \frac{1600}{\cot^2 10° - \cot^2 20°}$

$\therefore h = \frac{40}{\sqrt{\cot^2 10° - \cot^2 20°}}$ (1)

(iii) $h = \frac{40}{\sqrt{24.16}}$

$\therefore$ Height $= 8.06$ m (1)

7(b)(i) $A_1 = PR - M$

$A_2 = (PR - M)R - M$

$A_2 = PR^2 - MR - M$

(1) either

(ii) $A_3 = A_2 R - M$

$= PR^3 - MR^2 - MR - M$

$= PR^3 - M(1 + R + R^2)$

A pattern is emerging

$A_n = PR^n - M(1 + R + R^2 + \ldots + R^{n-1})$ (1)

↑ GP with $a=1$, $r=R$, n terms

$= PR^n - M\left(\frac{1(R^n - 1)}{R-1}\right)$

$A_n = PR^n - \frac{M(R^n - 1)}{R-1}$ (1)

(iii) If $A_n = \frac{K}{100} \times P$

$\frac{PK}{100} = PR^n - \frac{M(R^n - 1)}{R-1}$ (1)

$PK(R-1) = 100(R-1)PR^n - 100M(R^n - 1)$ (1)

$R^n(100P(R-1) - 100M) = PK(R-1) - 100M$

$R^n(100(P(R-1) - M)) = PK(R-1) - 100M$

$\therefore R^n = \frac{PK(R-1) - 100M}{100[P(R-1) - M]}$ (1)

(iv) From calculator,

$R^n = \frac{40000 \times 20\left(\frac{3}{400}\right) - 100 \times 800}{100\left[40000 \times \frac{3}{400} - 800\right]}$

$R^n = 1.48$ (1)

$1.0075^n = 1.48$ (then (1) for 4 or 5)

when $n=48$ $1.0075^{48} = 1.43$ when $n=60$ $1.0075^{60} = 1.565\ldots$

$\therefore$ Between 4 and 5 yrs.