TEACHER'S NAME: ____________________

STUDENT'S NAME: ____________________

BAULKHAM HILLS HIGH SCHOOL

YEAR 11

YEAR 11 YEARLY EXAMINATION

2008

MATHEMATICS
EXTENSION 1

Time allowed - 2 hours
(Plus five minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- The 7 questions are of equal value. Start each question on a new page.
- All necessary working should be shown.
- Write your teacher's name and your name on the cover sheet provided.
- At the end of the exam, staple your answers in order behind the cover sheet provided.

QUESTION 1

(a) Calculate, to the nearest minute, the acute angle between the lines $x-2y=4$ and $3x+y=5$. **3**

(b) Find the exact value of $\cos 15^0$ **2**

(c) Solve $\dfrac{2x+1}{x-2} \le 1$ **3**

(d) Solve $9^x + 6(3^x) - 27 = 0$ **2**

(e) The quadratic equation $x^2 + 16x + k = 0$ has one root three times the other. Find k. **2**

QUESTION 2 (Start a new page)

(a) Prove that $\dfrac{1-\cos 2\theta}{\sin 2\theta} = \tan\theta$. **3**

(b) Solve for $0^0 \le \theta \le 360^0$

(i) $\cos(2\theta - 30^0) = \dfrac{\sqrt{3}}{2}$ **2**

(ii) $7\cos^2\theta - \sin\theta\cos\theta = 3$ **4**

(c) Find $f'(x)$ if $f(x) = (3x^2 - 2x)^{-1}$ **2**

(d) Evaluate $\lim\limits_{x\to\infty}\left(\dfrac{x^2+2x+3}{x^2+2x+1}\right)$ **1**

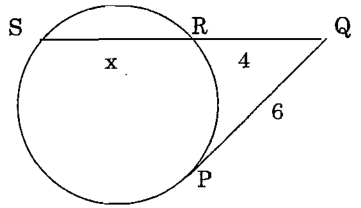
QUESTION 3 (Start a new page)

(a) Simplify $\dfrac{4^n + 2^{n+3}}{2^{n-1}}$ **2**

(b) P(26,-16) divides the interval AB externally in the ratio 3:2. If A is (-4,-1), find the coordinates of B. **2**

(c) Find the number of terms in the series $e^x + e^{-x} + e^{-3x} + e^{-5x} + \ldots + e^{-(2k+1)x}$. **3**

(d) PQ is a tangent to the circle. Find the value of x. **2**



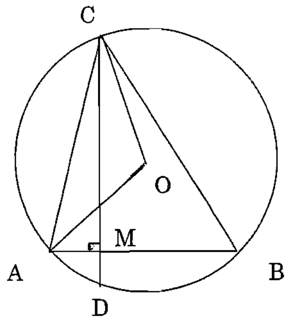
(e) If the equation $x^2 - (k+1)x = 3k - 4$ has real roots, find the range of values of k. **3**

QUESTION 4 (Start a new page)

(a) The limiting sum of a geometric series $1 + 3^x + 3^{2x} + 3^{3x} + \ldots$ is $\dfrac{-1}{80}$. Find x. **2**

(b) Prove by mathematical induction that for $n \geq 1$, $11^n - 5^n$ is divisible by 6. **4**

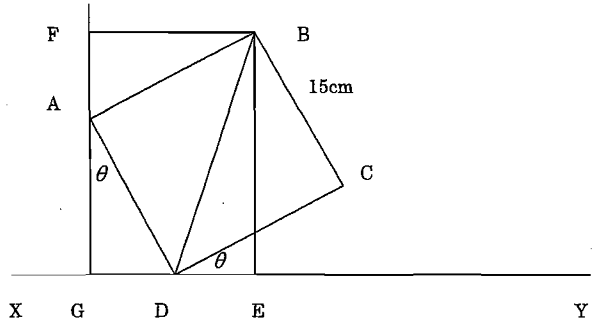
(c) AB and CD are perpendicular chords of a circle, centre O. Prove $\angle DAB = \angle OAC$. **4**



(d) If $y = \dfrac{x-2}{x+2}$, prove that $y + \dfrac{dy}{dx} = \dfrac{x^2}{(x+2)^2}$. **2**

QUESTION 5 (Start a new page)

(a) In the diagram below ABCD is a square of side 15 cm leaning against a wall at an angle θ to the vertical and as well to the ground XY.



(i) Show that $BD = 15\sqrt{2}$ cm **1**

(ii) Hence by using triangle DBE prove that the perpendicular distance from B to the line XY is $15\sqrt{2}\sin(45^0 + \theta)$ **2**

(iii) By using triangles DAG and BFA find an expression for the length of FG. **2**

(iv) Hence prove that $\sin\theta + \cos\theta = \sqrt{2}\sin(45^0 + \theta)$ **2**

(b) Find the equation of the parabola with focus (2,3) and directrix $y = -1$ **2**

(c) The elevation of a hill at a point P due east of it is 35⁰. At a point Q due south of P the elevation is 25⁰. If PQ = 300m, find the height of the hill to the nearest metre. **3**

QUESTION 6 (Start a new page)

(a) Using first principles find $\dfrac{dy}{dx}$ if $y = 2\sqrt{x}$. **3**

(b) Consider the function $y = \dfrac{6}{3 + x^2}$.

(i) Show that the function is even. **1**

(ii) Find the coordinates of any stationary points and determine their nature. **3**

(iii) Sketch the curve. **2**

(iv) State the range of the function. **1**

(c) If $\dfrac{\sin(A+B)}{\sin(A-B)} = \dfrac{5}{3}$ show that $\tan A = 4\tan B$. **2**

QUESTION 7 (Start a new page)

(a) A parabola is represented by the equations $x = 2at,\ y = at^2$.

(i) Find $\frac{dy}{dt}, \frac{dx}{dt}$ and hence find $\frac{dy}{dx}$. 1

(ii) Hence find the equation of the normal to the parabola at $P(2ap, ap^2)$ 2

(b) (i) If $t = \tan\frac{\alpha}{2}$, find expressions for $\sin\alpha$ and $\cos\alpha$ in terms of t. 2

(ii) Using part (i) solve $\sqrt{3}\sin\alpha - \cos\alpha = 1$ for $0^0 \le \alpha \le 360^0$ 3

(c) Sam and Bob set out from two towns. They travel on roads that meet at right angles and they walk towards the intersection. Sam is initially 15 km from the intersection and walks at 3 km/h. Bob is initially 10 km from the intersection and walks at 4 km/h.

(i) Show that their distance apart after t hours is given by $D^2 = 25t^2 - 170t + 325$. 1

(ii) Hence find how long it takes them to reach their minimum distance apart. 3

END OF PAPER

BHHS Yr 11 EXT 1 YEARLY SOLUTIONS /82

Q1 (a) [12]

$y = \frac{1}{2}x - 2 \therefore m_1 = \frac{1}{2}$

$y = -3x + 5 \therefore m_2 = -3$ } —(1)

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$ —(1)

$= \left|\frac{\frac{1}{2} - -3}{1 + \frac{1}{2}\times -3}\right|$

$= \left|\frac{\frac{7}{2}}{-\frac{1}{2}}\right|$

$= 7$

$\therefore \theta = 81°52'$ —(1)

(b) $\cos 15° = \cos(45° - 30°)$ —(1)

$= \cos 45° \cos 30° + \sin 45° \sin 30°$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$

or $= \frac{\sqrt{6} + \sqrt{2}}{4}$ } —(1)

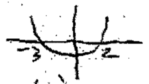
(c) $\frac{2x+1}{x-2} \times (x-2)^2 \le 1 \times (x-2)^2$ —(1)

$(2x+1)(x-2) - (x-2)^2 \le 0$

$(x-2)\{2x+1 - (x-2)\} \le 0$

$(x-2)(x+3) \le 0$ —(1)

$-3 \le x \le 2$



But $x \ne 2$

$\therefore -3 \le x < 2$ —(1)

(d) $(3^x)^2 + 6 \cdot 3^x - 27 = 0$

$m^2 + 6m - 27 = 0$

$(m+9)(m-3) = 0$

$\therefore m = -9$ or $m = 3$

$3^x = -9$ $3^x = 3$

No soln $x = 1$ —(2)

(e) let roots be $\alpha, 3\alpha$

$\therefore \alpha + 3\alpha = -\frac{b}{a} = -16$ —(1)

$\therefore \alpha = -4$

and $\alpha \times 3\alpha = \frac{c}{a} = k$

$3 \times 16 = k$

$k = 48$ —(1)

Q2 (a) [12]

LHS $= \frac{1 - (1 - 2\sin^2\theta)}{2\sin\theta\cos\theta}$ —(1)

$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$

$= \frac{\sin\theta}{\cos\theta}$ —(1)

$= \tan\theta =$ RHS (1)

(b)(i) $2\theta - 30° = -30°, 30°, 330°, 390°, 690°$ (1)

$2\theta = 0°, 60°, 360°, 420°, 720°$

$\theta = 0°, 30°, 180°, 210°, 360°$ (1)

(ii) Divide by $\cos^2\theta$

$7 - \frac{\sin\theta}{\cos\theta} = \frac{3}{\cos^2\theta}$

$7 - \tan\theta = 3\sec^2\theta$

$7 - \tan\theta = 3(1 + \tan^2\theta)$ —(1)

$3\tan^2\theta + \tan\theta - 4 = 0$

$3\tan^2\theta - 3\tan\theta + 4\tan\theta - 4 = 0$

$3\tan\theta(\tan\theta - 1) + 4(\tan\theta - 1) = 0$

$(\tan\theta - 1)(3\tan\theta + 4) = 0$ —(1)

$\therefore \tan\theta = 1$ or $\tan\theta = -\frac{4}{3}$

$\theta = 45°, 225°, 126°52', 306°52'$ —(2)

(c) $f(x) = (3x^2 - 2x)^{-1}$

$f'(x) = -(3x^2 - 2x)^{-2} \cdot (6x - 2)$ —(2)

or $= \frac{2 - 6x}{(3x^2 - 2x)^2}$

(d) Exp $= \lim_{x \to \infty} \left(\frac{1 + \frac{2}{x} + \frac{3}{x^2}}{1 + \frac{2}{x} + \frac{1}{x^2}}\right)$ —(1)

$= \frac{1 + 0 + 0}{1 + 0 + 0}$

$= 1$

Q3 (a) [12]

$\frac{2^{2n} + 2^{n+3}}{2^{n-1}} = \frac{2^{n-1}(2^{n+1} + 2^4)}{2^{n-1}}$ (1)

$= 2^{n+1} + 2^4 = 2^{n+1} + 16$ —(1)

(b) $x_p = \frac{mx_2 + nx_1}{m+n}$ (1) $y_p = \frac{my_2 + ny_1}{m+n}$

$26 = \frac{3x_2 + -2 \times 4}{3 + -2}$ $-16 = \frac{3y_2 + -2 \times -1}{3 + -2}$

$26 = 3x_2 + 8$ $-16 = 3y_2 + 2$

$3x_2 = 18$ $3y_2 = -18$

$x_2 = 6$ $y_2 = -6$

$\therefore B = (6, -6)$ —(1)

(If internal (1) mark)

(c) G.S where $a = e^x$, $r = e^{-2x}$ (1)

$T_n = ar^{n-1}$

$e^{-(2k+1)x} = e^x \cdot (e^{-2x})^{n-1}$ —(1)

$= e^x \cdot e^{-2nx + 2x}$

$= e^{(-2n+3)x}$

$\therefore -2n + 3 = -(2k+1)$

$2n = 2k + 4$

$n = k + 2$

$\therefore$ No. of terms $= k + 2$ —(1)

(d) $(x+4) \cdot 4 = 6^2$ —(1)

$4x = 20$

$x = 5$ —(1)

(e) $x^2 - (k+1)x + 4 - 3k = 0$

If real roots $\Delta \ge 0$ —(1)

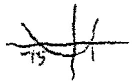
$[-(k+1)]^2 - 4 \cdot 1 \cdot (4 - 3k) \ge 0$

$k^2 + 2k + 1 - 16 + 12k \ge 0$

$k^2 + 14k - 15 \ge 0$ —(1)

$(k+15)(k-1) \ge 0$

$\therefore k \le -15$ or $k \ge 1$ —(1)



Q4 (a) [10]

$a = 1$, $r = 3^x$

$S_\infty = \frac{a}{1-r}$ —(1)

$-\frac{1}{80} = \frac{1}{1 - 3^x}$

$1 - 3^x = -80$

$3^x = 81$

$\therefore x = 4$ —(1)

(b) (i) Prove true for $n = 1$

$11^1 - 5^1 = 6 = 6 \times 1$

$\therefore$ True for $n = 1$ —(1)

(ii) Assume true for $n = k$

i.e. $11^k - 5^k = 6P$ where $P \in J$ —(1)

(iii) Prove true for $n = k+1$

i.e. $11^{k+1} - 5^{k+1} = 6Q$ where $Q \in J$

LHS $= 11 \cdot 11^k - 5^{k+1}$

$= 11(6P + 5^k) - 5^{k+1}$ by assum.

$= 66P + 11 \cdot 5^k - 5 \cdot 5^k$

$= 66P + 6 \cdot 5^k$

$= 6(11P + 5^k)$ —(1)

$= 6Q$ since $11P + 5^k$ is an integer

$\therefore$ True for $n = k+1$ if true for $n = k$.

(iv) Since true for $n = 1$ and since true for $n = k+1$ if true for $n = k$ then it is true for $n = 2, 3, 4$, etc

$\therefore$ True for $n \ge 1$. —(1)

(c) Draw AD let $\angle DAB = x°$

$\therefore \angle DCB = x°$ ($\angle$'s on circ. stan. same arc) —(1)

Let $\angle CBA = y°$

$\therefore \angle AOC = 2y°$ ($\angle$ at centre $= 2 \times \angle$ at circ) —(1)

$\triangle AOC$ is isos (2 eq. radii)

$\therefore \angle OAC = \frac{1}{2}(180° - 2y°)$ ($\angle$ sum of isos $\triangle$)

$= 90° - y°$ —(1)

But $x + y = 90$ ($\angle$ sum of right $\triangle BCM$)

$\therefore \angle OAC = x°$ —(1)

(d) $\frac{dy}{dx} = \frac{(x+2) \cdot 1 - (x-2) \cdot 1}{(x+2)^2} = \frac{4}{(x+2)^2}$ —(1)

$\therefore y + \frac{dy}{dx} = \frac{x-2}{x+2} + \frac{4}{(x+2)^2}$

$= \frac{x^2 - 4 + 4}{(x+2)^2}$ —(1)

$= \frac{x^2}{(x+2)^2} =$ RHS

5(a)(i) $BD^2 = BC^2 + DC^2 = 15^2 + 15^2 = 2 \times 15^2$ (1)

$\therefore BD = 15\sqrt{2}$ cm

$\angle BDC = 45^\circ$ (Diags of square bisect ∠)

$\therefore \angle BDE = 45^\circ + \theta$

In ΔDBE, $\frac{BE}{DB} = \sin(45^\circ + \theta)$ – (1)

$BE = 15\sqrt{2}\sin(45^\circ + \theta)$ – (1)

In ΔFAB, $\angle FAB = 90^\circ - \theta$ (∠ sum of str. ∠)

$\frac{FA}{AB} = \cos(90^\circ - \theta)$ – (1)

$FA = 15\cos(90^\circ - \theta) = 15\sin\theta$

In ΔAGD $\frac{AG}{AD} = \cos\theta$ $\therefore AG = 15\cos\theta$ (1)

$\therefore FG = FA + AG = 15(\sin\theta + \cos\theta)$

$FG = BE = 15\sqrt{2}\sin(45^\circ + \theta)$ – (1)

$\therefore 15(\sin\theta + \cos\theta) = 15\sqrt{2}\sin(45^\circ + \theta)$ – (1)

$\therefore \sin\theta + \cos\theta = \sqrt{2}\sin(45^\circ + \theta)$

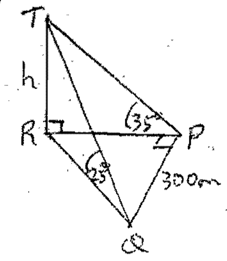
$2a = 3 - 1 = 4$ $\therefore a = 2$

$\therefore V = (2, 1)$ – (1)

Using $(x-h)^2 = 4a(y-k)$

$(x-2)^2 = 8(y-1)$ – (1)

)



In ΔPRT, $\frac{h}{RP} = \tan 35^\circ$

$RP = \frac{h}{\tan 35^\circ}$

In ΔQRT, $\frac{h}{RQ} = \tan 25^\circ$ – (1)

$RQ = \frac{h}{\tan 25^\circ}$

$\therefore$ In ΔPQR

$RQ^2 = PQ^2 + PR^2$

$\frac{h^2}{\tan^2 25^\circ} = 300^2 + \frac{h^2}{\tan^2 35^\circ}$ – (1)

$h^2 = \frac{300^2}{\frac{1}{\tan^2 25^\circ} - \frac{1}{\tan^2 35^\circ}}$

$= 35165.82166$

$h = 187.5 \doteqdot 188$ m (1)

Q6(a) [12] let $f(x) = 2\sqrt{x}$

$\therefore f(x+h) = 2\sqrt{x+h}$

$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ – (1)

$= \lim_{h \to 0} \frac{2\sqrt{x+h} - 2\sqrt{x}}{h}$

$= \lim_{h \to 0} \frac{2\sqrt{x+h} - 2\sqrt{x}}{h} \cdot \frac{2\sqrt{x+h} + 2\sqrt{x}}{2\sqrt{x+h} + 2\sqrt{x}}$

$= \lim_{h \to 0} \frac{4(x+h) - 4x}{h(2\sqrt{x+h} + 2\sqrt{x})}$ (1)

$= \lim_{h \to 0} \frac{4}{2\sqrt{x+h} + 2\sqrt{x}}$

$= \frac{4}{4\sqrt{x}} = \frac{1}{\sqrt{x}}$ – (1)

(b)(i) $f(a) = \frac{6}{3+a^2}$

$f(-a) = \frac{6}{3+(-a)^2} = \frac{6}{3+a^2} = f(a)$ (1)

$\therefore$ Even

(ii) $\frac{dy}{dx} = \frac{0 - 6 \cdot 2x}{(3+x^2)^2} = \frac{-12x}{(3+x^2)^2}$ (1)

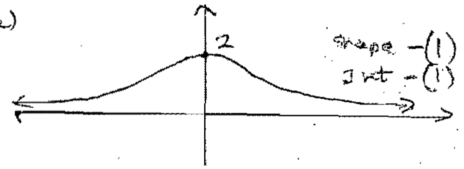
St pts when $\frac{dy}{dx} = 0$

$\therefore x = 0$, $y = 2$ – (1)

when $x < 0$, $\frac{dy}{dx} > 0$ when $x > 0$, $\frac{dy}{dx} < 0$

$\therefore$ Max turn. pt at $(0, 2)$ (1)

(iii)



shape – (1)

y int – (1)

(iv) Range: $0 < y \leq 2$ – (1)

(c) $3\sin A\cos B + 3\cos A\sin B$ – (1)

$= 5\sin A\cos B - 5\cos A\sin B$

$2\sin A\cos B = 8\cos A\sin B$

$\frac{\sin A}{\cos A} = 4\frac{\sin B}{\cos B}$ – (1)

$\tan A = 4\tan B$

Q7(a)(i) $\frac{dy}{dt} = 2at$ $\frac{dx}{dt} = 2a$

$\therefore \frac{dy}{dx} = 2at \times \frac{1}{2a} = t$ (1)

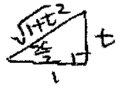
(ii) $m_T = p$ $\therefore m_N = -\frac{1}{p}$ – (1) (1)

$\therefore$ Eqn is $y - ap^2 = -\frac{1}{p}(x - 2ap)$

$py - ap^3 = -x + 2ap$

or $x + py = 2ap + ap^3$

(b)(i) $\tan\frac{\alpha}{2} = \frac{t}{1}$



$\therefore \sin\frac{\alpha}{2} = \frac{t}{\sqrt{1+t^2}}$

$\cos\frac{\alpha}{2} = \frac{1}{\sqrt{1+t^2}}$

$\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$

$= 2\frac{t}{\sqrt{1+t^2}} \cdot \frac{1}{\sqrt{1+t^2}}$

$\therefore \sin\alpha = \frac{2t}{1+t^2}$ – (1)

$\cos\alpha = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}$

$= \frac{1}{1+t^2} - \frac{t^2}{1+t^2}$

$\therefore \cos\alpha = \frac{1-t^2}{1+t^2}$ – (1)

(ii) $\sqrt{3} \cdot \frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2} = 1$

$2\sqrt{3}t - 1 + t^2 = 1 + t^2$

$2\sqrt{3}t = 2$

$\tan\frac{\alpha}{2} = \frac{1}{\sqrt{3}}$ – (1)

$\therefore \frac{\alpha}{2} = 30^\circ$

$\alpha = 60^\circ$ – (1)

Check 180°: LHS $= \sqrt{3}.0 - -1 = 1 =$ RHS (1)

$\therefore \alpha = 60^\circ, 180^\circ$

c)(i) $S = 15 - 3t$, $B = 10 - 4t$ (1)

$D^2 = S^2 + B^2 = (15-3t)^2 + (10-4t)^2$

$= 225 - 90t + 9t^2 + 100 - 80t + 16t^2$

$= 25t^2 - 170t + 325$

$\frac{d(D^2)}{dt} = 50t - 170$. – (1)

$= 0$ when $t = 3.4$ (1)

$\frac{d^2(D^2)}{dt^2} = 50 > 0$ (shows) $\therefore$ Min distance (1)

apart when $t = 3.4$ h or 3h 24m.