BAULKHAM HILLS HIGH SCHOOL

2010

YEAR 11 YEARLY EXAMINATIONS

Mathematics Extension 1

General Instructions
- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Marks may be deducted for careless or badly arranged work

Total marks – 84
- Attempt Questions 1 – 7
- All questions are of equal value

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Attempt Questions 1 – 7
All questions are of equal value
Answer each question on a SEPARATE piece of paper clearly marked Question 1, Question 2, etc. Each piece of paper must show your name.

Marks

Question 1 *(12 marks)* Use a *separate* piece of paper

a) (i) Expand $\left(x+\frac{1}{x}\right)^2$ *1*

(ii) Suppose that $x+\frac{1}{x}=3$, evaluate $x^2+\frac{1}{x^2}$ without finding the value of x. 2

b) Solve the inequality $\frac{x+1}{x-1}\geq 3$ *3*

c) Find the coordinates of the point which divides the interval joining the points $(-6,2)$ and $(4,7)$ in the ratio 3:2 *2*

d) Find the exact value of $\sin 15^\circ$ by using the expansion for $\sin(\alpha+\beta)$ 2

e) In how many different ways can people between the ages of 20 and 40 (inclusive), be classified by sex, age and political affiliation? 2
(Assume that the political affiliates are Labor, Liberal, Greens and Independent)

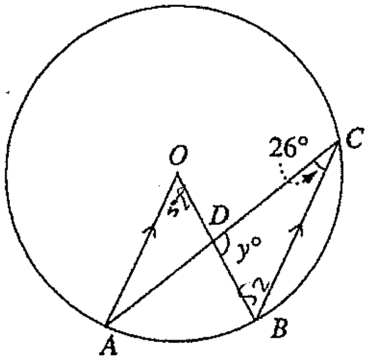
2

Marks

Question 2 *(12 marks)* Use a *separate* piece of paper

a) The variable point $\left(5t, 4t^2\right)$ lies on a parabola. Find the cartesian equation for this parabola. 2

b) The points A, B and C lie on a circle with centre O.
The lines AO and BC are parallel, and OB and AC intersect at D.
Also $\angle ACB = 26^\circ$ and $\angle BDC = y^\circ$, as shown in the diagram.

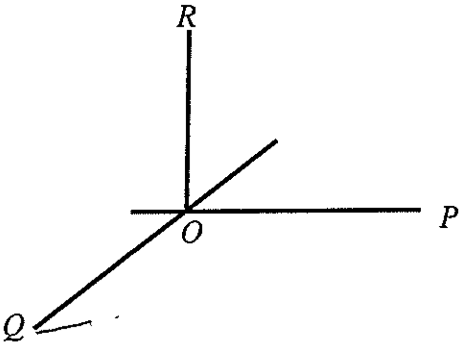


(i) State why $\angle AOB = 52^\circ$ 1

(ii) Find y. Justify your answer. 2

c) A, B and C are the points $(-3,-2)$, $(-4,2)$ and $(2,5)$ respectively. Find the size of $\angle ABC$, correct to the nearest degree. 3

d)



In the diagram, the points O, P and Q are in the same plane. R is a point vertically above O. P and Q are 750 metres apart and $\angle POQ = 120^\circ$, $\angle QRO = 30^\circ$ and $\angle PRO = 60^\circ$.

(i) Redraw the diagram on your answer sheet, labelling all of the given information. 1

(ii) Find the height of R above O, correct to the nearest metre. 3

Marks

Question 3 *(12 marks)* Use a *separate* piece of paper

a) Write down the equation of the vertical asymptote of $y = \dfrac{2x}{x+6}$ *1*

b) If $\sin\theta = \dfrac{3}{7}$ and $90^\circ < \theta < 180^\circ$, find the exact value of;

(i) $\sin 2\theta$ *2*

(ii) $\tan\dfrac{\theta}{2}$ *3*

c) Solve for θ, correct to the nearest degree where necessary, where $0^\circ \le \theta \le 360^\circ$

(i) $2\cos 2\theta = 4\cos\theta - 3$ *3*

(ii) $\cos\theta - \sqrt{3}\sin\theta = 2$ *3*

Question 4 *(12 marks)* Use a *separate* piece of paper

a) In how many ways can the letters of the word **PARRAMATTA** be arranged if;

(i) all the letters are used? 2

(ii) all the letters are used and the word begins and ends with the letter **A**? 2

b) There are nine people applying for four jobs. How many different ways can the four jobs be allocated if;

(i) the jobs are allocated randomly? 2

(ii) out of the nine people, five have their HSC and three of the jobs require the HSC? 2

c) There are two rows of chairs, with three in the first row and four in the second row. How many ways can seven people be seated if;

(i) Daniel and Joshua must sit in the second row? 2

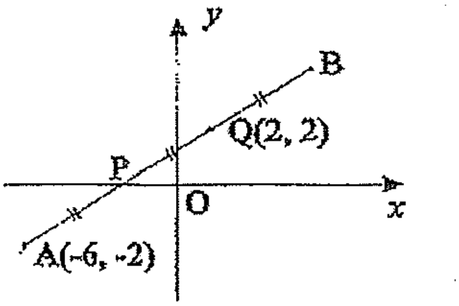
(ii) Bryan will not sit in the same row as Shirley and Harleen must sit in the first row? 2

Marks

Question 5 *(12 marks)* Use a *separate* piece of paper

a) Solve $\tan x \tan 2x = 1$ where $0^\circ \le x \le 360^\circ$ 3

b) The diagram shows the line interval AB **trisected** at P and Q. 3
The coordinates of A are $(-6,-2)$ and of Q are $(2,2)$



Find the coordinates of B using two different methods.

c) $P(2ap, ap^2)$ is a point on the parabola $x^2 = 4ay$, which has focus S.

(i) Derive the equation of the tangent at P. 2

(ii) Hence show that the tangent meets the x-axis at the point $T(ap, 0)$ 1

(iii) Find the coordinates of M, the point that divides ST externally in the ratio 2:1 2

(iv) Explain why $PS = PM$. 1

Marks

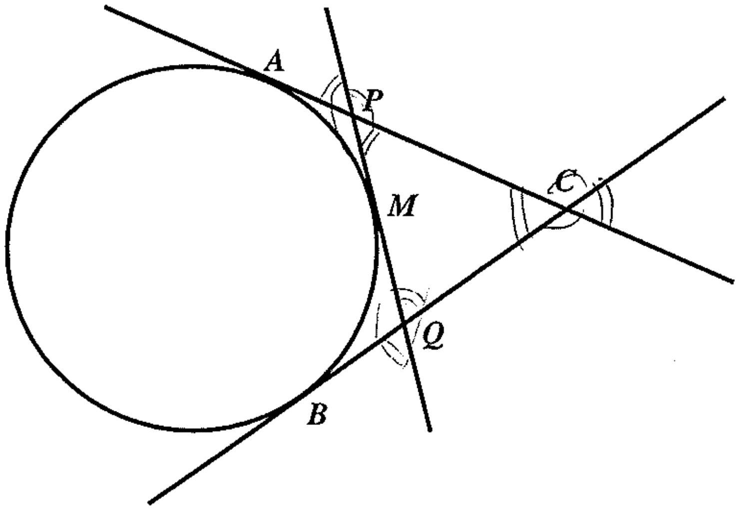
Question 6 *(12 marks)* Use a *separate* piece of paper

a) There are seven points on a plane so that no three points lie on the same straight line. How many quadrilaterals can be formed if the quadrilateral must contain point A? *1*

b) (i) Prove that $\cot\theta - 2\cot 2\theta \equiv \tan\theta$ *1*

(ii) Hence deduce that $\tan x + 2\tan 2x + 4\tan 4x \equiv \cot x - 8\cot 8x$ *2*

c) *3*



A and B are two points on a circle. Tangents at A and B meet at C. A third tangent cuts CA and CB at $P$and Q respectively, as shown in the diagram.

Show that the perimeter of ΔCPQ is independent of PQ.

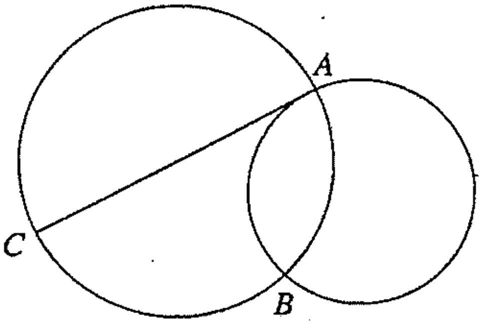
d) The word **EQUATIONS** contains all five vowels. How many seven letter words, including nonsense words, consisting of all five vowels can be formed from the letters of **EQUATIONS**? *2*

e) How many ways can six men and four women be seated around a table, if a particular husband and wife **do not** want to sit next to each other? *3*

Question 7 *(12 marks)* Use a *separate* piece of paper *Marks*

a) If $\sqrt{5} = 2 + \frac{1}{a}$, show that $a = 4 + \frac{1}{a}$ 2

b) The diagram shows two circles intersecting at A and B. The diameter of one circle is AC.



(i) Copy the diagram onto your answer sheet and draw a straight line through A, parallel to CB, meeting the second circle at D. *1*

(ii) Prove that BD is a diameter of the second circle. 2

(iii) Suppose that BD is parallel to CA. Prove that the circles have equal radii. 2

c) The three sides of a triangle have lengths $p - q$, $p + q$ and p, where $p > q > 0$. The largest and smallest angles of the triangle are α and β respectively.

(i) Show, using the cosine rule, that $4(1-\cos\alpha)(1-\cos\beta) = \cos\alpha + \cos\beta$ 3

(ii) In the case that $\alpha = 2\beta$, show that $\cos\beta = \frac{3}{4}$ 2

END OF EXAMINATION

BHHS Extension1 Mathematics Yrly 2010 - Solutions

Question 1 ⑫

a)(i) $(x+\frac{1}{x})^2 = \underline{x^2+2+\frac{1}{x^2}}$ ①

(ii) $x^2+\frac{1}{x^2} = (x+\frac{1}{x})^2 - 2$

$= 3^2 - 2$ -1

$= \underline{7}$ -1 ②

b) $\frac{x+1}{x-1} \geq 3$

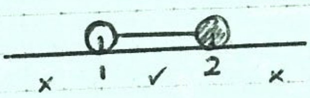
$x-1 \neq 0$

$x \neq 1$

$x+1 = 3x-3$

$2x = 4$

$x = 2$ critical pts -1



evidence of testing -1

$\underline{1 < x \leq 2}$ -1 ③

c) $(-6,2)$ $(4,7)$

3 : 2

$\left(\frac{-12+12}{5}, \frac{4+21}{5}\right)$ evidence of correct method -1

$= \underline{(0,5)}$ -1 ②

d) $\sin 15° = \sin(45-30)$ -1

$= \sin 45 \cos 30 - \cos 45 \sin 30$

$= (\frac{1}{\sqrt{2}})(\frac{\sqrt{3}}{2}) - (\frac{1}{\sqrt{2}})(\frac{1}{2})$

$= \underline{\frac{\sqrt{3}-1}{2\sqrt{2}}}$ -1 ②

e) Ways $= 21 \times 2 \times 4$ evidence of correct method -1

$= \underline{168}$ -1 ②

Question 2 ⑫

a) $x = 5t$

$t = \frac{x}{5}$

$y = 4t^2$

$= 4\left(\frac{x^2}{25}\right)$ attempt to eliminate parameter -1

$\underline{y = \frac{4x^2}{25}}$ -1 ②

b)(i) ∠ at centre twice ∠ at circumference standing on same arc. ①

(ii) $\angle OAC = 26°$ (alternate ∠'s =, AO∥BC)

$\angle ADO = y$ (vertically opposite ∠'s =)

$\angle AOD = 52°$ (proven in (i))

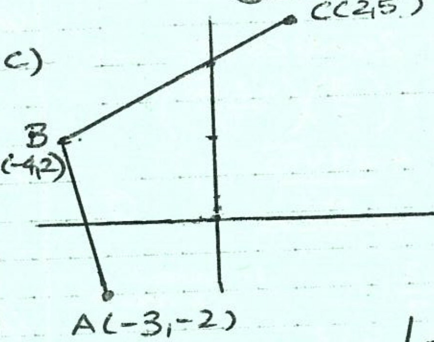
$\angle OAC + \angle ADO + \angle AOD = 180$ (∠ sum △ADO)

$26 + y + 52 = 180$

1 - solution
1 - reasoning

$\underline{y = 102}$ ②

c)



$m_{AB} = \frac{2+2}{-4+3}$

$= -4$

$m_{BC} = \frac{5-2}{2+4}$

$= \frac{1}{2}$

$\tan \angle ABC = \left|\frac{-4-\frac{1}{2}}{1+(-4)(\frac{1}{2})}\right|$

$= \left|\frac{-\frac{9}{2}}{-1}\right|$ 1 - subbing correct slopes into correct formula

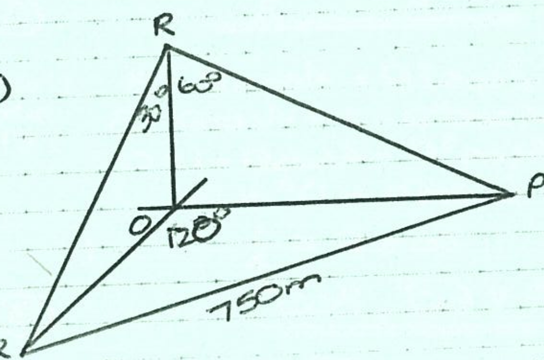
$= \frac{9}{2}$

$\angle ABC = 77°$ -1

however ∠ABC is obtuse

$\therefore \underline{\angle ABC = 103°}$ -1 ③

d)
(i)



①

(ii) $\frac{QO}{OR} = \tan 30°$

$QO = \frac{OR}{\sqrt{3}}$

$\frac{PO}{OR} = \tan 60°$

$PO = \sqrt{3}\, OR$

1 - expressing QO, PO in terms of OR.

$PQ^2 = OQ^2 + OP^2 - 2.OR.OP \cos 120°$

$750^2 = \frac{1}{3}OR^2 + 3OR^2 - 2OR^2(-\frac{1}{2})$

$750^2 = \frac{13}{3}OR^2$ 1 - attempting to use Cosine Rule.

$OR^2 = \frac{3 \times 750^2}{13}$

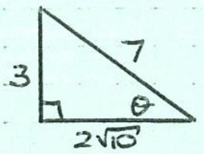
$OR = 360.2883461...$ -1

$\underline{OR = 360\text{ m}}$ (to nearest m) ③

Question 3 ⑫

a) $\underline{x = -6}$ ①

b)



(i) $\sin 2\theta = 2\sin\theta\cos\theta$ −1

$= 2\left(\frac{3}{7}\right)\left(-\frac{2\sqrt{10}}{7}\right)$

$= \underline{-\frac{12\sqrt{10}}{49}}$ −1 ②

(ii) $\frac{2t}{1+t^2} = \frac{3}{7}$ 1 – correct t result

$14t = 3 + 3t^2$

$3t^2 - 14t + 3 = 0$

$t = \frac{14 \pm \sqrt{160}}{6}$ −1

$= \frac{14 \pm 4\sqrt{10}}{6}$

$= \frac{7 \pm 2\sqrt{10}}{3}$ 1 – recognising $t > 0$ and explaining why.

however $\frac{\theta}{2}$ is acute

$\therefore \underline{\tan\frac{\theta}{2} = \frac{7+2\sqrt{10}}{3}}$ ③

c) $2\cos 2\theta = 4\cos\theta - 3$ 1 – using cos2θ result

$4\cos^2\theta - 2 = 4\cos\theta - 3$

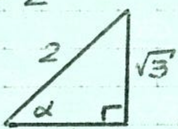
$4\cos^2\theta - 4\cos\theta + 1 = 0$

$(2\cos\theta - 1)^2 = 0$

$\cos\theta = \frac{1}{2}$ −1

$\underline{\theta = 60^\circ, 300^\circ}$ −1 ③

(ii) $\cos\theta - \sqrt{3}\sin\theta = 2$



$2\cos(\theta + 60) = 2$ 1 – attempt to transform into single trig function

$\cos(\theta + 60) = 1$

$\theta + 60 = 0, 360$ −1 (both required)

$\theta = -60, 300$

$\therefore \underline{\theta = 300^\circ}$ −1 ③

OR $\frac{1-t^2}{1+t^2} - \frac{2\sqrt{3}t}{1+t^2} = 2$ 1 – correct t results

$1 - t^2 - 2\sqrt{3}t = 2 + 2t^2$

$3t^2 + 2\sqrt{3}t + 1 = 0$

$(\sqrt{3}t + 1)^2 = 0$

$t = -\frac{1}{\sqrt{3}}$ −1

$\frac{\theta}{2} = 150^\circ$

$\underline{\theta = 300^\circ}$ −1

Question 4 ⑫

Note: final answer not required for mark.

a)(i) Ways $= \frac{10!}{4!2!2!}$ 1 – account for some repetition of letters.

$= \underline{37\,800}$ ②

(ii) Ways $= 1 \times 1 \times \frac{8!}{2!2!2!}$ 1 – correct handling of restriction

$= \underline{5040}$ ②

b)(i) Ways $= {}^9C_4$ 1 – evidence of acknowledging order unimportant.

$= \underline{126}$ ②

(ii) Ways $= {}^5C_3 \times {}^6C_1$ 1 – ${}^5C_3$.

$= \underline{60}$ ②

c)(i) Ways $= {}^4P_2 \times 5!$ 1 – ${}^4P_2$

$= \underline{1440}$ ②

(ii) Ways $= \overset{\textcircled{H}}{{}^3P_1} \times \overset{\textcircled{B+S}}{2 \times 4} \times 2 \times 4!$ 1 – progress towards correct solution

$= \underline{1152}$ ②

NOTE: If student assumes (i) conditions.

Ways $= {}^4P_2 \times {}^3P_1 \times 2 \times 2 \times 2 \times 2!$

$= 576$

Question 5 ⑫

a) $\tan x \tan 2x = 1$ let $t = \tan x$

$t\left(\frac{2t}{1-t^2}\right) = 1$ 1 – using t result correctly.

$2t^2 = 1 - t^2$

$3t^2 = 1$

$t^2 = \frac{1}{3}$ −1

$t = \pm\frac{1}{\sqrt{3}}$

$\tan x = \pm\frac{1}{\sqrt{3}}$

$\underline{x = 30^\circ, 150^\circ, 210^\circ, 330^\circ}$ −1 ③

b) B divides AQ externally
in the ratio 3 : 1

A(−6, −2) $\times$ Q(2, 2)
− 3 : 1

$$B\left(\frac{-6-6}{-2}, \frac{-2-6}{-2}\right)$$
$$= \underline{\underline{(6, 4)}}$$

Q divides AB internally
in the ratio 2 : 1

A(−6, −2) $\times$ B(x, y)
2 : 1

$$Q\left(\frac{-6+2x}{3}, \frac{-2+2y}{3}\right)$$

$$\frac{-6+2x}{3} = 2 \qquad \frac{-2+2y}{3} = 2$$
$$-6+2x = 6 \qquad -2+2y = 6$$
$$2x = 12 \qquad 2y = 8$$
$$x = 6 \qquad y = 4$$
$$\therefore \underline{\underline{B(6,4)}}$$

P is midpoint of AQ
$$P\left(\frac{-6+2}{2}, \frac{-2+2}{2}\right) = (-2, 0)$$

Q is midpoint of PB
$$Q\left(\frac{-2+x}{2}, \frac{0+y}{2}\right)$$
$$\frac{-2+x}{2} = 2 \qquad \frac{y}{2} = 2$$
$$-2+x = 4 \qquad y = 4$$
$$x = 6$$
$$\therefore \underline{\underline{B(6,4)}}$$

A to Q = 2 parts
x: 2 + 6 = 8 ∴ 1 part = 4
y: 2 + 2 = 4 ∴ 1 part = 2

A to B = 3 parts
$x = -6 + 3 \times 4 = 6$
$y = -2 + 3 \times 2 = 4$

$$\therefore \underline{\underline{B(6,4)}}$$

③

1 − progress towards one correct solution

1 − one correct solution

1 − another different correct solution
must use different method

c)(i) $y = \frac{x^2}{4a}$
$$\frac{dy}{dx} = \frac{2x}{4a}$$
$$= \frac{x}{2a}$$

when $x = 2ap$, $\frac{dy}{dx} = p$ −1
∴ slope of tangent is p

$$y - ap^2 = p(x - 2ap)$$
$$y - ap^2 = px - 2ap^2$$
$$\underline{\underline{y = px - ap^2}}$$ −1 ②

(ii) x intercept occurs when y = 0
i.e. $0 = px - ap^2$
$px = ap^2$
$x = ap$
$\therefore \underline{\underline{T(ap, 0)}}$

Note! if tangent in (i) incorrect T must be correct for their equation! ①

(iii)
S(0, a) $\times$ T(ap, 0)
−2 : 1

1 − progress towards solution

$$M\left(\frac{-2ap}{-\cancel{3}}, \frac{a}{-\cancel{3}}\right)$$ ②

$$= \underline{\underline{(2ap, -a)}}$$ −1

(iv) M lies on the directrix, by the definition of a parabola any point on the parabola is equidistant to the focus and directrix. ①

Question 6 ⑫

a) Quadrilaterals $= {}^6C_3$ ①
$$= \underline{20}$$

b)(i) $\cot\theta - 2\cot 2\theta$
$$= \frac{1}{\tan\theta} - \frac{2(1-\tan^2\theta)}{2\tan\theta}$$
$$= \frac{1 - 1 + \tan^2\theta}{\tan\theta}$$
$$= \frac{\tan^2\theta}{\tan\theta}$$
$$= \underline{\underline{\tan\theta}}$$ ①

(iii) $\tan x + 2\tan 2x + 4\tan 4x$

$= \cot x - 2\cot 2x + 2\cot 2x - 4\tan 4x + 4\tan 4x - 8\tan 8x$

$= \underline{\cot x - 8\tan 8x}$ ②

1- for showing use of (ii)

c) Perimeter = CP + CQ + PQ

AP = PM (tangents from external pt =)

BQ = QM (")

PQ = PM + MQ (common side)

∴ PQ = AP + BQ

Perimeter = CP + AP + CQ + BQ

= AC + BC ③

which is independent of PQ

2- solution 1- reasoning

d) Words $= 1 \times {}^4C_2 \times 7!$

1- selecting letters
1- arranging letters

$= \underline{\underline{30\,240}}$ ②

e) Total Ways

1- placing H and W in circle

$= 1 \times 7 \times 8!$

1- arranging other 8 ie 8!

$= \underline{\underline{282\,240}}$

1- final answer ③

ie deduct 1 if 7! or similar used.

OR

Total, no restrictions = 9!

1- demonstrate circle (n−1)!

Total, HW together

$= 2! \times 8!$

1- ways together

Total, HW no together

$= 9! - 2!8!$ −1

$= \underline{\underline{282\,240}}$

Question 7 ⑫

a) $\sqrt{5} = 2 + \frac{1}{a}$

$\frac{1}{a} = \sqrt{5} - 2$

$a = \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$

$= \sqrt{5} + 2$

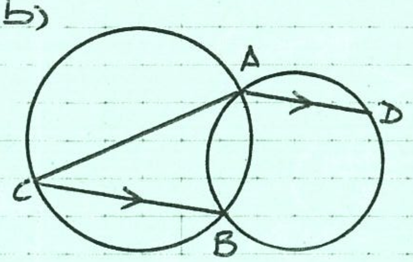
1- progress towards solution

$4 + \frac{1}{a} = 4 + \sqrt{5} - 2$

$= \sqrt{5} + 2$

$= \underline{a}$ ②

b)



Note: BC does not have to be drawn in. ①

(ii) ∠ABC = 90° (∠ in semicircle)

∠ABC = ∠BAD (alternate ∠'s =, AD//CB)

∴ ∠BAD = 90°

Thus BD is diameter (∠ subtended at circumference = 90°)

1- solution

1- reasoning ②

(iii) AD//CB (given)

BD//CA (given)

∴ ADBC is a //gram (opposite sides //)

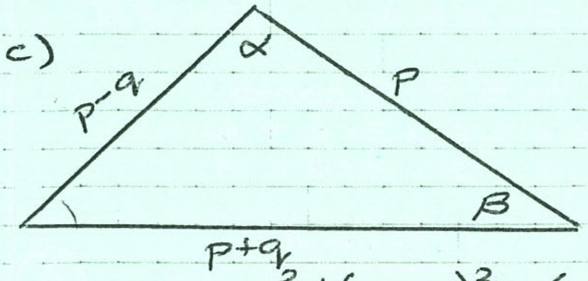
AC = BD (opposite sides in //gram =)

<u>If diameters are equal, circles must have equal radii.</u> ②

1- solution, 1 reasoning

<u>Note</u>: must prove //gram, not just assume.

c)



$\cos\alpha = \frac{p^2 + (p-q)^2 - (p+q)^2}{2p(p-q)}$

$= \frac{p^2 - 4pq}{2p(p-q)}$

$= \frac{p - 4q}{2(p-q)}$

1- using cosine rule to find cosα or cosβ

Similarly $\cos\beta = \frac{p + 4q}{2(p+q)}$

$$4(1-\cos\alpha)(1-\cos\beta)$$

$$= 4\left(1-\frac{p-4q}{2(p-q)}\right)\left(1-\frac{p+4q}{2(p+q)}\right)$$

$$= 4 \times \frac{p+2q}{2(p-q)} \times \frac{p-2q}{2(p+q)}$$

$$= \frac{p^2-4q^2}{p^2-q^2}$$

$$\cos\alpha + \cos\beta$$

$$= \frac{p-4q}{2(p-q)} + \frac{p+4q}{2(p+q)}$$

$$= \frac{p^2+pq-4pq-4q^2+p^2-pq+4pq-4q^2}{2(p-q)(p+q)}$$

$$= \frac{2p^2-8q^2}{2(p-q)(p+q)}$$

$$= \frac{p^2-4q^2}{p^2-q^2}$$

$$= \underline{\underline{4(1-\cos\alpha)(1-\cos\beta)}}$$ ③

1 − substituting for $\cos\alpha$ and $\cos\beta$

1 − $\frac{p^2-4q^2}{p^2-q^2}$ in either LHS or RHS.

1 − $\frac{p^2-4q^2}{p^2-q^2}$ in both.

(ii) $\alpha = 2\beta$

$$4(1-\cos2\beta)(1-\cos\beta) = \cos2\beta + \cos\beta$$

$$4(2-2\cos^2\beta)(1-\cos\beta) = 2\cos^2\beta - 1 + \cos\beta$$

$$8(1-\cos^2\beta)(1-\cos\beta) = 2\cos^2\beta + \cos\beta - 1$$

$$8(1-\cos\beta)^2(1+\cos\beta) = (2\cos\beta - 1)(\cos\beta + 1)$$

$$8(1-\cos\beta)^2 = 2\cos\beta - 1$$

(<u>Note</u>: $\cos\beta \neq -1$)

$$8 - 16\cos\beta + 8\cos^2\beta = 2\cos\beta - 1$$

$$8\cos^2\beta - 18\cos\beta + 9 = 0$$

$$(4\cos\beta - 3)(2\cos\beta + 3) = 0$$

$$\cos\beta = \frac{3}{4} \text{ or } \cos\beta = -\frac{3}{2}$$

however β is acute

$$\therefore \underline{\underline{\cos\beta = \frac{3}{4}}}$$ ②

1 − progress towards correct solution including at least correct use of $\cos 2\alpha$ substitution

1 − correct solution.