BAULKHAM HILLS HIGH SCHOOL
MARKING COVER SHEET

YEAR 11 YEARLY
Extension 1
2011

STUDENT NAME:____________________

TEACHER NAME:____________________

QUESTION	MARK
1	
2	
3	
4	
5	
6	
7	
TOTAL	/ 61

Section 1 - Answer Sheet	
a)	☐☐☐☐
b)	☐
c)	$\theta =$ ☐☐☐☐☐ $b =$ ☐☐☐☐☐
d)	Circle the correct answer i) True / False ii) True / False
e)	☐☐☐☐☐☐
f)	☐
g)	$r =$ ☐☐☐☐☐ $r =$ ☐☐☐☐☐

BAULKHAM HILLS HIGH SCHOOL

2011
YEAR 11 YEARLY

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Marks may be deducted for careless or badly arranged work
- Attempt all questions

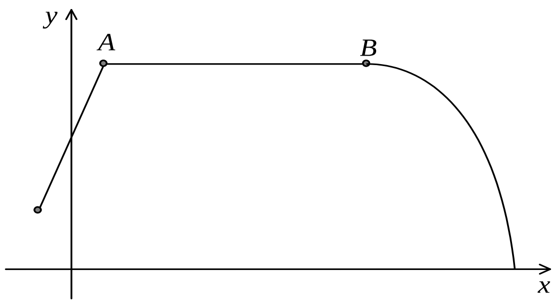
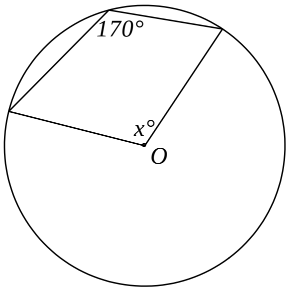
Total marks – 54

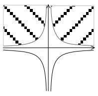
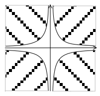
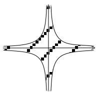
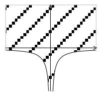
This paper consists of TWO sections.

Section 1 – Short Response
7 marks

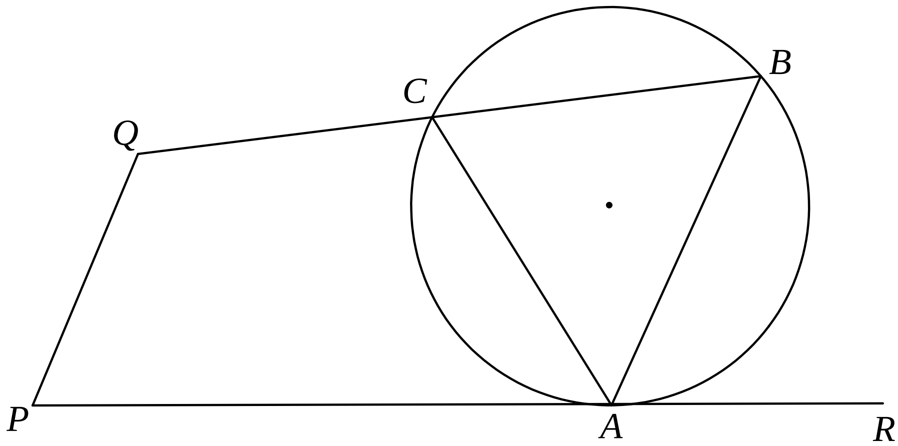
Section 2 – Extended Response
marks
Attempt all questions
Start a new page for each question

Section 1 – Short Response (7 marks)
Attempt all questions. Show all necessary working

Question 1 (7 marks) - Answer the following on the answer sheet provided.		**Marks**
a)	The letters of the word M O U S E are to be rearranged. How many arrangements start with M and end with E?	**1**
b)	Which of the following can be a cyclic quadrilateral? (A) kite (B) parallelogram (C) trapezium (D) all of the above	**1**
c)	For what value of θ and *b* does cos θ = *b* have only one solution in the domain $0° \leq \theta \leq 360°$	**1**
d)	 At which point/s is the curve differentiable. Are the following True or False. i) Differentiable at A ii) Differentiable at B	**1**
e)	 In the diagram above, what is the value of *x* ? NOT TO SCALE	**1**

f)	Which diagram below is the correct graph of $\lvert xy \rvert \geq 1$ (A)  (B)  (C)  (D) 	**1**
g)	For what two values of r does ${}^{18}P_r = {}^{18}C_r$ $r =$ ☐☐☐ and $r =$ ☐☐☐	**1**
	End of Section 1	

Section II – Extended Response
Attempt all questions. Show all necessary working.
Start each question on a new page. Clearly indicate question number.
Write your name and teacher's name at the top of each new page.

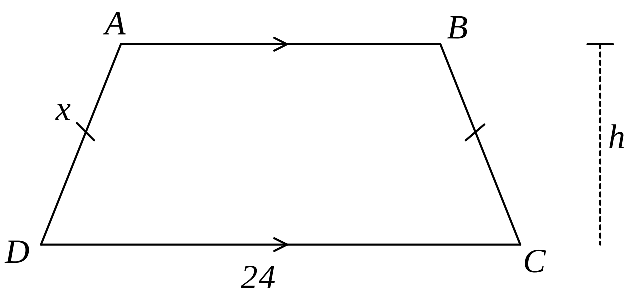
Question 2 (9 marks) - Start a new page		**Marks**
a)	Solve $\frac{x}{x-3} \geq 2$	3
b)	Solve $\sin 2x - \sin x = 0$ for $0° \leq x \leq 360°$	3
c)	*ABC* is a triangle inscribed in a circle. *PA* is a tangent to the circle and is produced to *R*. *PQ* is drawn parallel to *AB* and meets *BC* produced at *Q*. i) Copy the diagram onto your own paper ii) Prove *APQC* is a cyclic quadrilateral 	3

Question 3 (9 marks) - Start a new page		
a)	i) On the same diagram sketch the graphs of $y = \lvert x-2 \rvert$ and $y = \frac{3}{x}$ ii) For what values of *x* is $\lvert x-2 \rvert < \frac{3}{x}$?	2 2
b)	i) Show that the acute angle between the lines $y = x + 2$ and $y = mx + b$ is given by $\tan \alpha = \left\lvert \frac{m-1}{m+1} \right\rvert$	1
	ii) Write down a similar result for the angle β between the lines $y = 3x - 1$ and $y = mx + b$	1
	iii) Hence find the gradients of the lines bisecting the angles between the lines $y = x + 2$ and $y = 3x - 1$	3

Question 4 (9 marks) - Start a new page		**Marks**
a)	Prove $\dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}=\tan\dfrac{\theta}{2}$	**3**
b)	Consider the curve $y=\dfrac{x}{x^2+1}$	
	i) Explain why there are no vertical asymptotes	**1**
	ii) Find $\lim\limits_{x\to\infty}\dfrac{x}{x^2+1}$.	**1**
	iii) Find the stationary point/s and determine their nature	**3**
	iv) Sketch the curve showing the above information, given there are points of inflection when $x=\pm\dfrac{1}{\sqrt{3}}$.	**1**

Question 5 (9 marks) - Start a new page		
a)	There are 8 green cards, 8 red card and 8 yellow cards in a pack. Four cards are chosen at random without replacement. Find the probability that	
	i) 4 green cards are chosen	**2**
	ii) At least 2 green cards are chosen	**2**
b)	i) Prove that $\cot x + \tan x = 2\operatorname{cosec} 2x$	**2**
	ii) Hence prove that $\cot 15^\circ = 2+\sqrt{3}$	**3**

Question 6 (9 marks) - Start a new page		**Marks**
a)	Consider the parabola $y = x^2$	
	i) Find the equation of the tangent, T, to the parabola at the point $P(t, t^2)$	1
	ii) Show that the line passing through the focus of the parabola and perpendicular to T has the equation $y = \dfrac{t-2x}{4t}$	3
	iii) M is the foot of the perpendicular drawn from the focus to the tangent at T. Find the locus of M.	2
b)	A radius of a circle divides a chord in the ratio 3 : 2 and is bisected by the chord. Show that $\cos A = \dfrac{\sqrt{2}}{4}$ where A is the acute angle between the radius and the chord.	3

Question 7 (9 marks) - Start a new page		
a)	Seven girls including Kara and Lara form a queue at the canteen. Kara does not like Lara and will refuse to stand next to her. If they happen to be together, Kara will go back to the end of the queue. If she can't avoid Lara in this way she will leave and go to the library. What is the probability that Kara will go to the library?	2
b)	An isosceles trapezium is drawn with side CD 24cm. Sides AD and BC are equal and $AD + AB + BC = 42$cm. 	
	i) Show that the perpendicular height h of the trapezium is $h = 3\sqrt{2x-9}$	3
	ii) Hence find the dimensions of the trapezium with greatest area.	4

End of Examination

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a) 3! or 6

b) 0

c) $\theta = 180^\circ$, $b = -1$

d) i) False

ii) True

e) 20

f) B

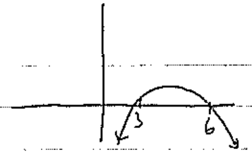
g) 0 and 1

Q2 a) $\frac{x}{x-3} \geq 2$ $\quad x \neq 3$

$x(x-3) \geq 2(x-3)^2$ ✓

$(x-3)(x-2x+6) \geq 0$

$(x-3)(6-x) \geq 0$ ✓



$3 < x \leq 6$ ✓

b) $2\sin x\cos x - \sin x = 0$

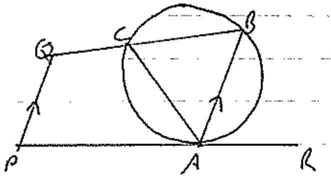
$\sin x(2\cos x - 1) = 0$

$\sin x = 0$ ✓ $\quad \cos x = \frac{1}{2}$ ✓

$x = 0^\circ, 180^\circ, 360^\circ \quad x = 60^\circ, 300^\circ$

$\therefore x = 0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ$ ✓

2 c)



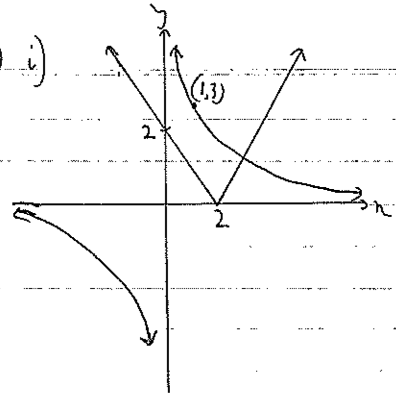
let $\angle BAR = \alpha$ ✓

$\angle ACB = \alpha$ (angle between a tangent and a chord at point of contact is equal to the angle in the alternate segment)

$\angle QPA = \alpha$ (corresponding $\angle$'s, $QP \parallel BA$) ✓

$\therefore APQC$ is cyclic quad (exterior angle of a cyclic quadrilateral equals interior opposite $\angle$) ✓

3 a) i)



① each graph

ii) Pts of intersection

When $x \geq 2$: $x - 2 = \frac{3}{x}$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = -1, 3$ ✓

$\therefore x = 3, y = 1$ is pt of intersection

$x - 2 = -\frac{3}{x}$

$x^2 - 2x + 3 = 0$

$\Delta = 4 - 4 \times 1 \times 3$

$= -8$

< 0

$\therefore$ no solution

$\therefore$ Solution is $0 < x < 3$ ✓

b i) let $m_1 = m \quad m_2 = 1$

$\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{m-1}{1+m\times 1}\right|$ ← ① showing substitution

$\tan\alpha = \left|\frac{m-1}{m+1}\right|$

ii) $\tan\beta = \left|\frac{m-3}{3m+1}\right|$ ✓

iii) Gradients of lines bisecting is given when

$\tan\alpha = \tan\beta$

$\left|\frac{m-1}{m+1}\right| = \left|\frac{m-3}{3m+1}\right|$

$\frac{m-1}{m+1} = \pm \frac{m-3}{3m+1}$ ✓

$(m-1)(3m+1) = (m+1)(m-3)$ or $(m-1)(3m+1) = -(m+1)(m-3)$

$3m^2 - 2m - 1 = m^2 - 2m - 3$; $3m^2 - 2m - 1 = -m^2 + 2m + 3$

$2m^2 = -2$; $4m^2 - 4m - 4 = 0$

$m^2 = -1$; $m^2 - m - 1 = 0$

no real soln. ✓ ; $m = \frac{1 \pm \sqrt{1 - 4 \times 1 \times -1}}{2}$ ✓

$\therefore m = \frac{1 \pm \sqrt{5}}{2}$ are the gradients

Q4. a) let $t = \tan\frac{\theta}{2}$, $\sin\theta = \frac{2t}{1+t^2}$, $\cos\theta = \frac{1-t^2}{1+t^2}$

$LHS = \frac{1 + \frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}}{1 + \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}$ ✓

$= \frac{1+t^2+2t-1+t^2}{1+t^2+2t+1-t^2}$

$= \frac{2t^2+2t}{2t+2}$

$= \frac{2t(t+1)}{2(t+1)}$ ✓

$= t$

$= \tan\frac{\theta}{2}$ ✓

$= RHS.$

$\therefore \frac{1+\sin\theta - \cos\theta}{1+\sin\theta+\cos\theta} = \tan\frac{\theta}{2}$

b) i) For vertical asymptotes, denominator must equal 0.
But $x^2+1=0$ has no real soln since $x^2 = -1$ ✓
$\therefore$ No vertical asymptotes

ii) $\lim_{x\to\infty} \frac{(x) \div x^2}{(x^2+1) \div x^2}$

$= \lim_{x\to\infty} \frac{\frac{1}{x}}{1 + \frac{1}{x^2}}$ as $\lim_{x\to\infty} \frac{1}{x} \to 0$ and $\lim_{x\to\infty} \frac{1}{x^2} \to 0$ ✓

$= \frac{0}{..} = 0$

4 b iii) $y = \frac{x}{x^2+1}$

$\frac{dy}{dx} = \frac{(x^2+1).1 - x(2x)}{(x^2+1)^2}$

$\frac{dy}{dx} = \frac{-x^2+1}{(x^2+1)^2}$ ✓

Stat points occur when $\frac{dy}{dx} = 0$

ie $0 = -x^2+1$

$x^2 = 1$

$x = \pm 1$ ✓

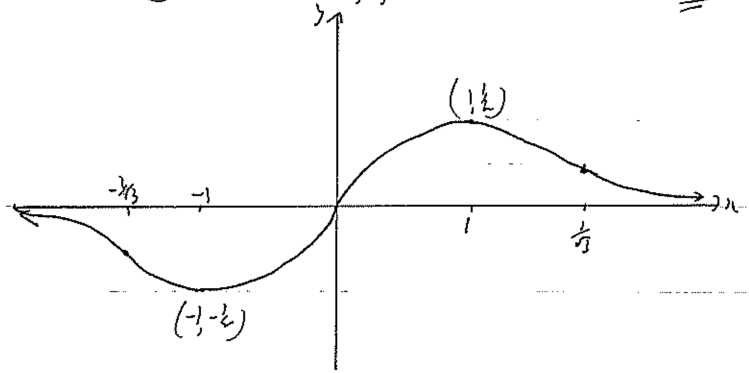
Testing

x	-2	-1	0	1	2
$\frac{dy}{dx}$	$\frac{-3}{25}$	0	1	0	$\frac{-3}{25}$

$\therefore$ local min at $(-1, -\frac{1}{2})$; $\therefore$ local max t.p. at $(1, \frac{1}{2})$ ✓

b iv) when $x=0$, $y=0$

NB function is odd as $f(-x) = -f(x)$



✓ correct shape with turning points labelled (NB not needed to include inflection

Q5 a) i) $P(\text{4 green cards}) = \dfrac{{}^8C_4}{{}^{24}C_4}$ (1) for ${}^8C_4$

$= \dfrac{5}{759}$ (1) for denominator of ${}^{24}C_4$ and evaluated.

ii) $P(\text{at least 2 green cards}) = 1 - P(0 \text{ greens}) - P(1 \text{ green})$

$= 1 - \dfrac{{}^{16}C_4}{{}^{24}C_4} - \dfrac{{}^4C_1 \times {}^{16}C_3}{{}^{24}C_4}$ ✓

$= 1 - \dfrac{130}{759} - \dfrac{160}{759}$

$= \dfrac{469}{759}$ ✓

5 b) i) $LHS = \cot x + \tan x$

$= \dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x}$

$= \dfrac{\cos^2 x + \sin^2 x}{\frac{1}{2} \times 2\sin x \cos x}$ ✓ ie (1) finds common denominator

$= \dfrac{1}{\frac{1}{2}\sin 2x}$ ✓ (1) uses $\sin 2x = 2\sin x\cos x$

$= 2\operatorname{cosec} 2x$

$= RHS$ as reqd.

ii) $\cot 15° + \tan 15° = \dfrac{2}{\sin 30°}$

$\cot 15° + \dfrac{1}{\cot 15°} = 4$

$\cot^2 15° - 4\cot 15° + 1 = 0$ (✓)

$\cot 15° = \dfrac{4 \pm \sqrt{16-4}}{2}$

$\cot 15° = \dfrac{4 \pm 2\sqrt{3}}{2}$ (✓)

$\cot 15° = 2 \pm \sqrt{3}$

but $\cot 15° = \tan 75°$ and $\tan 75° > 1$ (✓)

$\therefore \cot 15° = 2 + \sqrt{3}$

or $\cot 15° + \tan 15° = \dfrac{2}{\frac{1}{2}}$

$\cot 15° + \tan 15° = 4$

but $\tan 15° = \tan(60° - 45°)$

$= \dfrac{\tan 60° - \tan 45°}{1 + \tan 60° \tan 45°}$ ✓

$= \dfrac{\sqrt{3}-1}{1+\sqrt{3}} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \dfrac{4 - 2\sqrt{3}}{2}$ ✓

$= 2 - \sqrt{3}$

$\cot 15° = 4 - (2 - \sqrt{3})$

$= 2 + \sqrt{3}$ ✓

6 a) i) $y = x^2$

$\dfrac{dy}{dx} = 2x$

when $x = t$, $m = 2t$ ✓

$\therefore y - t^2 = 2t(x - t)$

$y - t^2 = 2tx - 2t^2$

$y = 2tx - t^2$ is eqn of tangent

ii) $x^2 = 4ay \quad \therefore a = \frac{1}{4}$

focus is $(0, \frac{1}{4})$ ✓

$M_{\perp} = -\dfrac{1}{2t}$

$\therefore$ line is $y - \frac{1}{4} = -\dfrac{1}{2t}(x - 0)$ ✓

$y = \dfrac{-x}{2t} + \dfrac{1}{4}$

$y = \dfrac{-2x}{4t} + \dfrac{t}{4t}$

$y = \dfrac{t - 2x}{4t}$ ✓

(iii) M is point of int of (ii) and (i)

$\dfrac{t - 2x}{4t} = 2tx - t^2$

$t - 2x = 8t^2x - 4t^3$

$8t^2x + 2x = 4t^3 + t$

$2x(4t^2+1) = t(4t^2+1)$ ✓

$x = \dfrac{t}{2}$

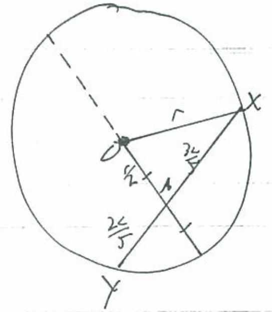
$y = 2t\left(\dfrac{t}{2}\right) - t^2$

$y = 0$

$\therefore M$ is $\left(\dfrac{t}{2}, 0\right)$

locus of M is $y = 0$ ✓

6(b)



let $XY = c$

$AX = \frac{3c}{5}$

$AY = \frac{2c}{5}$

r be radius

$\frac{3r}{2} \times \frac{r}{2} = \frac{3c}{5} \times \frac{2c}{5}$ (products of intercepts of chords)

$\frac{3r^2}{4} = \frac{6c^2}{25} \rightarrow \frac{c^2}{25} = \frac{r^2}{8}$

$\cos A = \dfrac{\left(\frac{r}{2}\right)^2 + \left(\frac{3c}{5}\right)^2 - r^2}{2\left(\frac{r}{2}\right)\left(\frac{3c}{5}\right)}$

$= \dfrac{\left(\frac{r}{2}\right)^2 + \frac{9c^2}{25} - r^2}{r \cdot \frac{3r}{2\sqrt{2}}}$

$= \dfrac{\frac{r^2}{4} + \frac{9r^2}{8} - r^2}{\frac{3r^2}{2\sqrt{2}}}$

$= \dfrac{\frac{3}{8}}{\frac{3}{\sqrt{8}}}$

$= \frac{\sqrt{8}}{8}$

$= \frac{2\sqrt{2}}{8}$

$= \frac{\sqrt{2}}{4}$

Q7 a) _ _ _ _ _ _ _

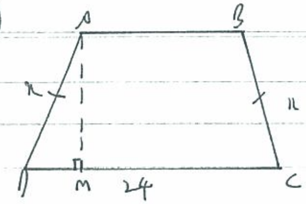
Girls being in last 2 places means Kara will go to library.

$P(\text{library}) = \frac{2! \times 5!}{7!}$

$= \frac{2}{42}$

$= 1$

7b) i)



$AB + 2x = 42$

$AB = 42 - 2x$

$DM = \frac{24 - (42 - 2x)}{2}$

$= \frac{2x - 18}{2}$

$DM = x - 9$

In ΔAMD $h^2 = x^2 - (x-9)^2$

$= x^2 - (x^2 - 18x + 81)$

$= 18x - 81$

$h^2 = 9(2x - 9)$

$h = 3\sqrt{2x-9}$ $(h > 0)$

b ii) $A = \frac{1}{2} 3\sqrt{2x-9}\,(42 - 2x + 24)$

$= \frac{3}{2}\sqrt{2x-9}\,(66 - 2x)$

$= 3\sqrt{2x-9}\,(33 - x)$

$\frac{dA}{dx} = 3\sqrt{2x-9} \times (-1) + (33 - x) \cdot \frac{3}{2}(2x-9)^{-\frac{1}{2}} \times 2$

$\frac{dA}{dx} = -3\sqrt{2x-9} + \frac{3(33-x)}{\sqrt{2x-9}}$

max or min occur when $\frac{dA}{dx} = 0$

$3\sqrt{2x-9} = \frac{3(33-x)}{\sqrt{2x-9}}$

$2x - 9 = 33 - x$

$3x = 42$

$x = 14$

Test

x	13	14	15
$\frac{dA}{dx}$	2.18	0	-1.96

$\therefore$ max area when $x = 14$

Dimension 24cm, 14cm, 14cm, 14cm