BAULKHAM HILLS HIGH SCHOOL

2012

YEAR 11 YEARLY

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour and 30 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Marks may be deducted for careless or badly arranged work
- Attempt all questions

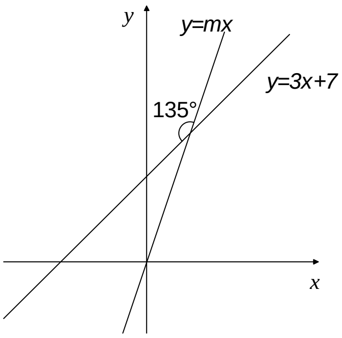
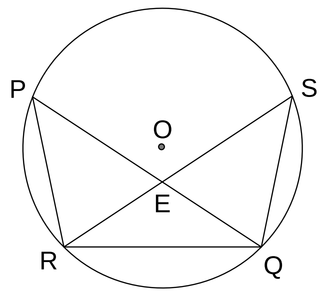
Total marks – 60

This paper consists of TWO sections.

Section 1 – Multiple Choice
6 marks

Section 2 – Extended Response
54 marks
Attempt all questions
Start a new page for each question

Section 1 –Multiple Choice (6 marks)
Attempt all questions.

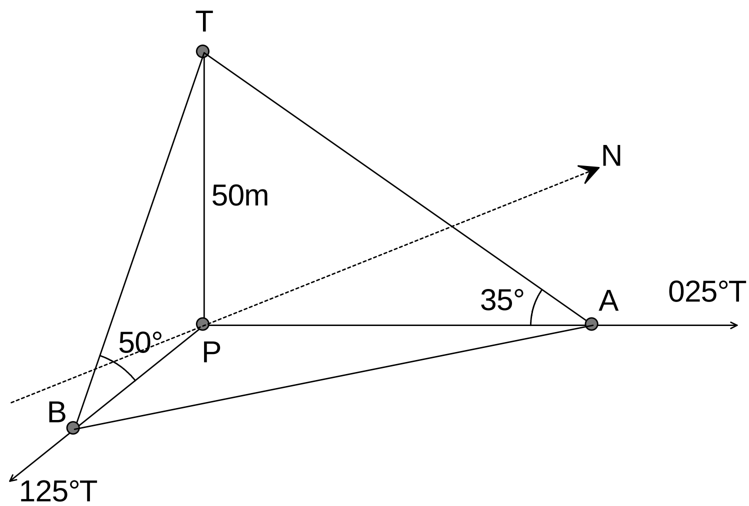
Answer the following on the answer sheet provided.		Marks
1	Let A be the point $(-2,3)$ and B be the point $(3,-4)$. Find the coordinates of the point which divides AB externally in the ratio $3:2$ (A) $\left(0,\frac{1}{5}\right)$ (B) $\left(1,-\frac{6}{5}\right)$ (C) $(-12,17)$ (D) $(13,-18)$	**1**
2	The line $y = 3x + 7$ and $y = mx$ intersect at an angle of $135°$, as shown in the diagram.  A possible value for m is (A) $\frac{1}{3}$ (B) -2 (C) 3 (D) $-\frac{1}{2}$	**1**
3	From six girls and four boys, a committee of 3 girls and 2 boys is to be chosen. How many different committees can be formed? (A) 26 (B) 120 (C) 252 (D) 1440	**1**
4	Let $t = \tan\frac{\theta}{2}$ where $0 < \theta < 180°$. Which of the following gives the correct expression for $\sec\theta + \tan\theta$? (A) $\frac{1-t}{1+t}$ (B) $\frac{1+2t+t^2-t^3}{1-t}$ (C) $\frac{1+t}{1-t}$ (D) $\frac{t^2-2t-1}{1+t^2}$	**1**
5	 O is the centre of the circle. PQ and RS are chords which intersect at E. $PR \neq SQ$. Consider the following ***I*** ***Triangles PER and SEQ are similar*** ***II*** ***Triangles PQR and ERQ are similar*** Which is true? (A) I only (B) II only (C) Both I and II (D) Neither I nor II	**1**
6	The equation of the normal to the parabola $x^2 = 4ay$ at the variable point $P(2ap, ap^2)$ is given by $x + py = 2ap + ap^3$ How many different values of p are there such that the normal passes through the focus of the parabola? (A) 0 (B) 1 (C) 2 (D) 3	**1**
	End of Section I	

Section II – Extended Response
Attempt all questions. Show all necessary working.
Start each question on the appropriate page. Clearly indicate the question number.

Question 7 (9 marks) - Start a new page		**Marks**
a)	Solve $\tan 2x = 3\tan x$ for $0° \le x \le 360°$	3
b)	i) How many seven digit numbers can be formed using the digits 3, 3, 3, 3, 5, 5, 5 ?	1
	ii) How many five digit numbers can be formed using the digits 3, 3, 3, 3, 5, 5, 5 ?	2
c)	Solve $\dfrac{2x+1}{3x-2} \le 2$	3

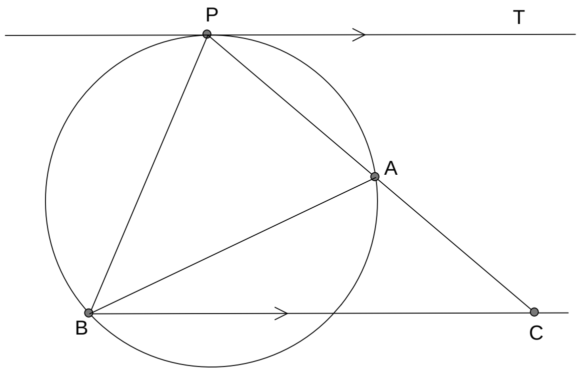
Question 8 (9 marks) - Start a new page		
a)	A committee of four is to be formed from 5 men and 4 women. In how many ways can the committee be chosen if :-	
	(i) there are no restrictions?	1
	(ii) it is formed with 2 men and 2 women?	1
	(iii) it contains at least 2 women?	2

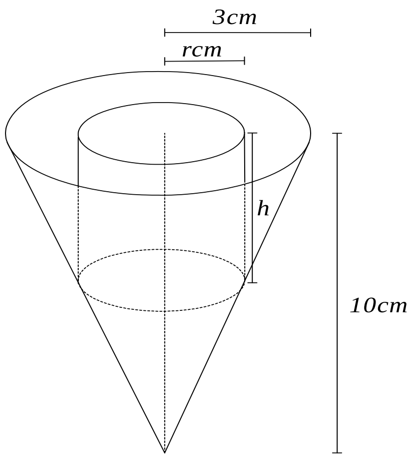
b)



PT is an observation tower $50m$ high. The bearing of two points A and B from P are $025°T$ and $125°T$ respectively. The angles of elevation from the points A and B to the top of the tower are 35° and 50° respectively.

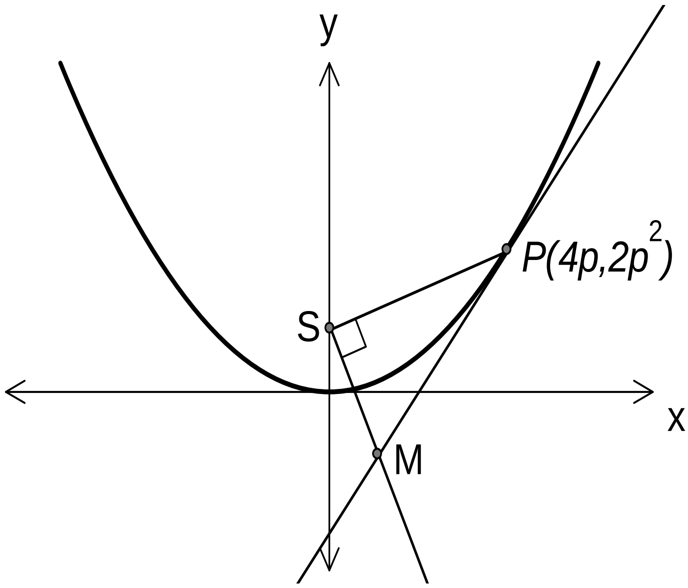
(i)	Find $\angle APB$	1
(ii)	Show that $AB^2 = 50^2(\cot^2 35° + \cot^2 50° - 2\cot 35° \cot 50° \cos 100°)$	3
(iii)	Hence find AB to the nearest metre.	1

Question 9 (9 marks) - Start a new page		
a)	(i) Prove that $\sin(A+B)-\sin(A-B)=2\cos A\sin B$	1
	(ii) Hence or otherwise show that $$2\sin x\,(\cos 3x+\cos 5x+\cos 7x)=\sin 8x-\sin 2x$$	2
b)	 In the diagram A, P and B are points on the circle. The line PT is tangent to the circle at P, and PA is produced to C so that BC is parallel to PT.	
	(i) Show that $\angle PBA=\angle PCB$	2
	(ii) Prove that triangles ABP and BCP are similar	2
	(iii) Hence deduce that $PB^2=PA\times PC$	2

Question 10 (9 marks) - Start a new page		**Marks**
a)	Solve $12\cos\theta-5\sin\theta+6=0$, for $0°\le\theta\le 360°$, correct to the nearest degree.	3
b)	 A cylinder of radius $r$cm and height $h$cm is inscribed in a cone with base radius 3cm and height 10cm as in the diagram.	
	(i) Show that the volume V of the cylinder is given by $V=\frac{10\pi r^2(3-r)}{3}$	2
	(ii) Hence find the values of r for the cylinder which has the maximum volume.	2
	(iii) What is the maximum volume of the cylinder ?	2

Question 11 (9 marks) - Start a new page			
a)	(i) By considering the difference of two cubes, show that $$(2+x)^{\frac{1}{3}} - 2^{\frac{1}{3}} = \frac{x}{(2+x)^{\frac{2}{3}} + 2^{\frac{1}{3}}(2+x)^{\frac{1}{3}} + 2^{\frac{2}{3}}}$$		2
	(ii) Hence evaluate $\lim_{x\to 0} \frac{(2+x)^{\frac{1}{3}} - 2^{\frac{1}{3}}}{x}$		2
b)	(i) For the curve $y = \frac{4}{\sqrt{16+x^2}}$ find any stationary points and determine their nature.		2
	(ii) Find any points of inflexion and hence sketch the curve showing any significant features.		3

Question 12 (9 marks) - Start a new page		**Marks**
$P(4p, 2p^2)$ is a point on the parabola $x^2 = 8y$, with focus S, as in the diagram.		
(i)	Show that the equation of the tangent at P is given by $y = px - 2p^2$	2
(ii)	Show that the equation of the line through S and perpendicular to SP is $$2px + (p^2-1)y = 2(p^2-1)$$	2
(iii)	The tangent and this line meet at M. Prove that the co-ordinates of M are $\left(\frac{2(p^2-1)}{p}, -2\right)$.	3
(iv)	Show that the area of $\Delta PSM = \frac{2(p^2-1)^2}{\lvert p \rvert}$	2



End of Examination

YEAR 11 EXT 1 YEARLY 2012

MC: 1 - D, 2 - B, 3 - B, 4 - C, 5 - A, 6 - B

7a) $\tan 2x = 3\tan x$

$\frac{2\tan x}{1-\tan^2 x} = 3\tan x$ — 1

$2\tan x = 3\tan x - 3\tan^3 x$

$\therefore \tan x - 3\tan^3 x = 0$ — LOSS OF TAN x

$\tan x(1 - 3\tan^2 x) = 0$ — 1 — OR LOSS OF 0

$\tan x = 0$ or $\tan x = \mp\frac{1}{\sqrt{3}}$ — 1 only.

$\therefore x = 0, 30, 150, 180, 210, 330, 360$ — 1 all ans.

b) i) $\frac{7!}{4!3!} = 35$ — 1

ii) 33335 = 5

33355 = 10 — 1

33555 = 10 — TOTAL = 25 — 1

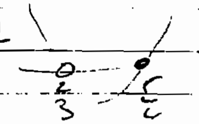
c) $\frac{2x+1}{3x-2} \le 2$ — $x \ne \frac{2}{3}$

$(2x+1)(3x-2) \le 2(3x-2)^2$ — 1

$2(3x-2)^2 - (3x-2)(2x+1) \geq 0$

$(3x-2)(6x-4-2x-1) \geq 0$ — 1

$(3x-2)(4x-5) \geq 0$



$x < \frac{2}{3}$ or $x \geq \frac{5}{4}$ — 1

(8) a) i) ${}^9C_4 = 126$ — 1

ii) ${}^5C_2 \cdot {}^4C_2 = 60$ — 1

iii) 2W ⇒ 60

3W → ${}^5C_1 \cdot {}^4C_3 = 5 \cdot 4 = 20$ — 1

4W → 1

TOTAL = 60 + 20 + 1 = 81 — 1

b) i) $\angle APB = 125 - 25 = 100^\circ$ — 1

ii) $\tan 50^\circ = \frac{50}{BP}$

$BP = 50\cot 50^\circ$ — Similarly $AP = 50\cot 35^\circ$ — 1

in $\triangle APB$ — $AB^2 = BP^2 + AP^2 - 2\,BP.AP.\cos\angle APB$ — 1

$= 50^2\cot^2 50^\circ + 50^2\cot^2 35^\circ - 2\cdot 50^2\cot 50^\circ\cot 35^\circ\cos 100^\circ$

$= 50^2(\cot^2 50^\circ + \cot^2 35^\circ - 2\cot 50^\circ\cot 35\cos 100^\circ$ — 1

$= 89$ m — 1

9 a) i) $\sin(A+B) - \sin(A-B)$

$= \sin A\cos B + \sin B\cos A - \sin A\cos B + \sin B\cos A$

$= 2\cos A\sin B = RHS$ 1

ii) $2\sin x\cos 3x + 2\sin x\cos 5x + 2\sin x\cos 7x$

$\sin(4x) - \sin(2x) + \sin(6x) - \sin(4x) + \sin 8x - \sin 6x$ 1 + 1

$= \sin 8x - \sin 2x$

b) i) $\angle TPA = \angle PBA$ (ANGLE FORMED BY TANGENT AND CHORD = ANGLE IN ALT. SEGMENT) 1

$\angle TPA = \angle PCB$ (ALTERNATE ∠s ON || LINES PT, BC)

$\therefore \angle PBA = \angle PCB$ (1)

ii) IN Δs APB AND BCP

$\angle PBA = \angle PCB$ (PROVED IN i) 1

$\angle BPC$ IS COMMON TO BOTH TRIANGLES

$\therefore \triangle ABP \;|||\; \triangle BCP$ (AA, AAA, equiangular, all matching ∠s =) (1)

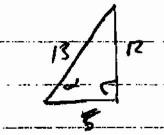
iii) (1) $\dfrac{PB}{AP} = \dfrac{PC}{PB}$ (ratio of similar Δs)

$\therefore PB^2 = PA \cdot PC$ (1)

16 a) $12\cos\theta - 5\sin\theta + 6 = 0$

$\therefore 5\sin\theta - 12\cos\theta = 6$ $\left(\sqrt{5^2+12^2} = 13\right)$

$\frac{5}{13}\sin\theta - \frac{12}{13}\cos\theta = \frac{6}{13}$



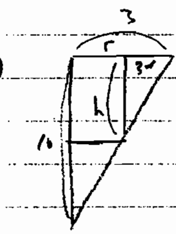
$\therefore \sin\theta\cos\alpha - \cos\theta\sin\alpha = \frac{6}{13}$ 1

$\sin(\theta - 67.38) = \frac{6}{13}$ $\tan\alpha = \frac{12}{5}$

$\theta - 67.38 = 27.49$ or 152.51 $\alpha = 67.38$ 1

$\therefore \theta = 95^\circ$ or 220 1

b)



$V = \pi r^2 h$

$\dfrac{h}{3-r} = \dfrac{10}{3}$

$\therefore h = \dfrac{10(3-r)}{3}$ 1

Sub → V $\therefore V = \pi r^2 \cdot \dfrac{10(3-r)}{3}$

$V = \dfrac{10\pi r^2(3-r)}{3}$

$= \frac{10\pi}{3}(3r^2 - r^3)$ 1

$V' = \frac{10\pi}{3}(6r - 3r^2)$

$\frac{10}{3}\pi\, 3r(2-r)$

$= 0$ when $r = 0$ or 2

$r=0$ $V=0$

$r=2$ ∃ 1

r	1.9	2	2.1
V'	+ve	0	−ve

1

∴ max

$V = 10\pi \cdot 2^2 \dfrac{(3-2)}{3}$ 1

$= \dfrac{40\pi}{3}$ cm³ 1

(11) a) i) $(A^3 - B^3) = (A - B)(A^2 + AB + B^2)$

$\therefore A - B = \dfrac{A^3 - B^3}{A^2 + AB + B^2}$

let $A = (2+x)^{\frac{1}{3}}$ $\quad B = 2^{\frac{1}{3}}$ 1

$\therefore (2+x)^{\frac{1}{3}} - 2^{\frac{1}{3}} = \dfrac{2+x-2}{(2+x)^{\frac{2}{3}} + 2^{\frac{1}{3}}(2+x)^{\frac{1}{3}} + 2^{\frac{2}{3}}}$ ($2 + x - 2 = x$) $= \dfrac{x}{(2+x)^{\frac{2}{3}} + 2^{\frac{1}{3}}(2+x)^{\frac{1}{3}} + 2^{\frac{2}{3}}}$ 1

ii) $\lim\limits_{x \to 0} \dfrac{(2+x)^{\frac{1}{3}} - 2^{\frac{1}{3}}}{x}$

$= \lim\limits_{x \to 0} \dfrac{1}{(2+x)^{\frac{2}{3}} + 2^{\frac{1}{3}}(2+x)^{\frac{1}{3}} + 2^{\frac{2}{3}}}$ 1

$= \dfrac{1}{2^{\frac{2}{3}} + 2^{\frac{2}{3}} + 2^{\frac{2}{3}}}$

$= \dfrac{1}{3 \cdot 2^{\frac{2}{3}}}$. 1

11 b) i) $y = \dfrac{4}{\sqrt{16+x^2}}$

$y = 4(16+x^2)^{-\frac{1}{2}}$

$y' = -2(16+x^2)^{-\frac{3}{2}} \cdot 2x$ — 1 for $\frac{dy}{dx}$

$y' = \dfrac{-4x}{(16+x^2)^{\frac{3}{2}}}$

Stationary points occur when $y' = 0$

$0 = \dfrac{-4x}{(16+x^2)^{\frac{3}{2}}}$

$x = 0$

$\therefore$ Stat. pt at $(0,1)$

Testing $x = 0$

x	-1	0	1
y'	$\frac{4}{17^{3/2}}$	0	$\frac{-4}{17^{3/2}}$

$\therefore$ Max turning point at $(0,1)$ — ① for classifying stat pt.

ii) $y'' = -4(16+x^2)^{-\frac{3}{2}} - 4x \cdot -\frac{3}{2} \cdot 2x\,(16+x^2)^{-\frac{5}{2}}$ — ① for y''

$= \dfrac{12x^2}{(16+x^2)^{\frac{5}{2}}} - \dfrac{4(16+x^2)}{(16+x^2)^{\frac{5}{2}}}$

$y'' = \dfrac{8x^2 - 64}{(16+x^2)^{\frac{5}{2}}} = \dfrac{8(x^2-8)}{(16+x^2)^{\frac{5}{2}}}$

Points of inflexion may occur when $y'' = 0$

ie $8x^2 - 64 = 0$

$x^2 = 8$

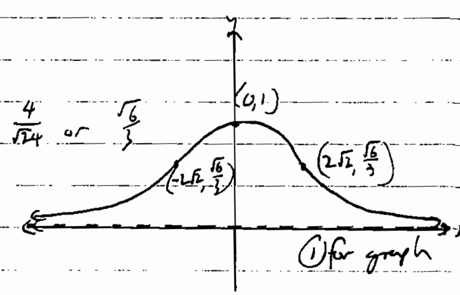
$x = \pm 2\sqrt{2}$ — ① for P.O.I.

Testing

x	-3	$-2\sqrt{2}$	0	$2\sqrt{2}$	3
y''	$\frac{8}{125}$ ∪	0	-1 ∩	0	$\frac{+8}{125}$ ∪

$\therefore$ concavity changes at $x = \pm 2\sqrt{2}$, $y = \frac{4}{\sqrt{24}}$ or $\frac{\sqrt{6}}{3}$

$\therefore$ Points of infle at $\left(\pm 2\sqrt{2}, \frac{\sqrt{6}}{3}\right)$



as $x \to \infty$, $y \to 0^+$, as $x \to -\infty$, $y \to 0^+$ — ① for graph

12

(ii)(2)

$x^2 = 8y$ or $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

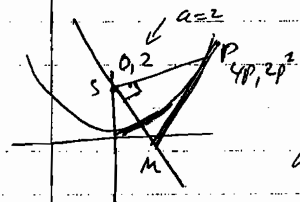
$y = \frac{x^2}{8}$ $= 4t \div 4t$

$y' = \frac{2x}{8} = \frac{x}{4}$ $= t$

$= p$ at $x = 4p$

$= p$ at $x = 4p$ 1

∴ equation of tangent is $y - 2p^2 = p(x - 4p)$



$y = px - 4p^2 + 2p^2$

$y = px - 2p^2$ — * 1

~~gradient of line ⊥ SP = $-\frac{1}{p}$~~

~~∴ line is y … $= -\frac{1}{p}$~~

(9)

gradient of SP $= -\left(\frac{4p}{2p^2-2}\right)$

$= \frac{-2p}{p^2-1}$ — 1

∴ equation of SP is $y - 2 = \frac{-2p}{p^2-1}(x - 0)$

$(p^2-1)y - 2(p^2-1) = -2px$ ~~+4p~~ — 1

∴ $2px + (p^2-1)y = 2(p^2-1)$ — *

$y = px - 2p^2$ — ①

$2px + (p^2-1)y = 2(p^2-1)$ — ②

from ① substitute $px = y + 2p^2 \Rightarrow$ ② 1

~~$y + 2(y + p^2 …) …$ — 2~~

$2y + 4p^2 + p^2y - y = 2p^2 - 2$

$(p^2+1)y = -2 - 2p^2$

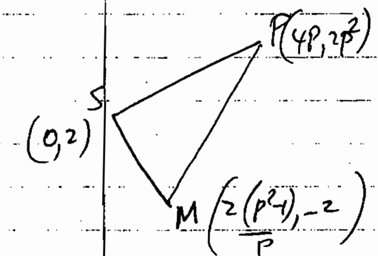
$y = \frac{-2(1+p^2)}{(p^2+1)} = -2$ 1

substitute $y = -2$

∴ $px = -2 + 2p^2$

$x = \frac{2(p^2-1)}{p}$ — 1

∴ M has co-ord $\left(\frac{2(p^2-1)}{p}, -2\right)$ — *



Area of Δ $= \frac{1}{2}$ SP . SM.

~~$= \frac{1}{2}\sqrt{(4p)^2}$~~ — 1

$= \frac{1}{2}\sqrt{(4p)^2 + (2p^2-2)^2} \cdot \sqrt{\left(\frac{2(p^2-1)}{p}\right)^2 + 16}$

$= \frac{1}{2}\sqrt{16p^2 + 4p^4 - 8p^2 + 4} \cdot \sqrt{\frac{4p^4 - 8p^2 + 4 + 16p^2}{p^2}}$

$= \frac{1}{2}\sqrt{4p^4 + 8p^2 + 4} \cdot \sqrt{\frac{4p^4 + 8p^2 + 4}{p^2}}$

$= \frac{1}{2}\sqrt{4(p^4+2p^2+1)} \cdot \sqrt{\frac{4(p^2+1)^2}{p^2}}$

$= \frac{1}{2} \cdot \sqrt{4(p^2+1)^2} \cdot \frac{\sqrt{4(p^2+1)^2}}{|p|}$

$= \frac{1}{2} \cdot \frac{4(p^2+1)^2}{|p|}$

$= \frac{2(p^2+1)^2}{|p|}$ 1