BAULKHAM HILLS HIGH SCHOOL

YEARLY 2013
YEAR 11 TASK 4

Mathematics Extension 1

General Instructions
- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 11-13
- Marks may be deducted for careless or badly arranged work

Total marks – 55
Exam consists of 8 pages.

This paper consists of TWO sections.

<u>Section 1</u> – Pages 2-4 (10 marks)
Questions 1-10
- Attempt Questions 1-10
- Allow about 15 minutes for this section

<u>Section II</u> – Pages 5-8 (45 marks)
- Attempt questions 11-14
- Allow about 1 hour and 15 minutes for this section

Section I

10 marks
Attempt questions 1-10
Allow about 15 minutes for this section
Use the multiple choice answer sheet for questions 1-10

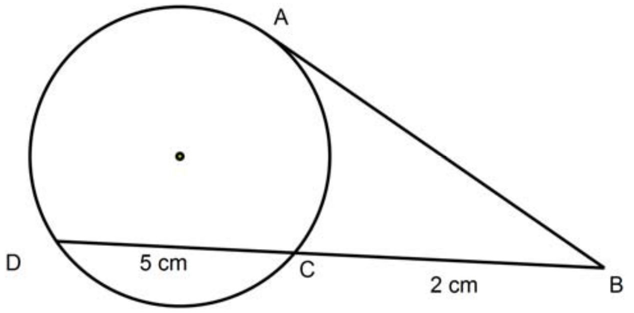
1. Make G the subject of the formula $E = 1 + \sqrt{\frac{G}{R}}$

(A) $G = R(1+E)^2$ (B) $G = R(1+E^2)$ (C) $G = R(1-E^2)$ (D) $G = R(1-E)^2$

2. The number of different arrangements of the letters of the word **S E R V I C E S** which begin and end with the letter S are

(A) $\frac{6!}{(2!)^2}$ (B) $\frac{8!}{(2!)^2}$ (C) $\frac{6!}{2!}$ (D) $\frac{8!}{2!}$

3.



AB is a tangent to the circle, $BC = 2$cm and $CD = 5$cm. The length of AB is

(A) $\sqrt{10}$ (B) $\sqrt{14}$ (C) $\sqrt{20}$ (D) $\sqrt{35}$

4. If $t = \tan\frac{\theta}{2}$ then $4\sin\theta - 3\cos\theta = 4$ results in the equation

(A) $7t^2 - 8t + 7 = 0$
(B) $4t^2 + 3t = 0$
(C) $3t^2 - 8t + 7 = 0$
(D) $t^2 - 8t + 7 = 0$

5. The number of solutions of the equation $\sin 2x = \cos x$ where $0° \le x \le 360°$ is

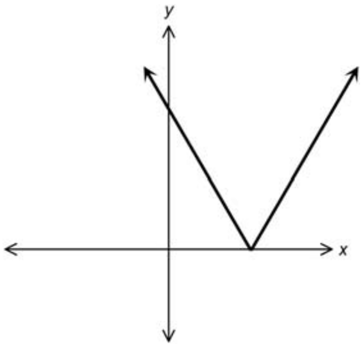
(A) 2 (B) 3 (C) 4 (D) 5

6. What is the size of the acute angle between the lines $x = 1$ and $y = 2x - 1$?

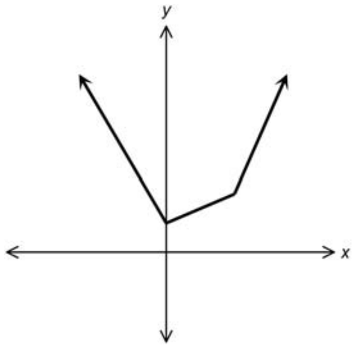
(A) $22\frac{1}{2}°$ (B) $27°$ (C) $45°$ (D) $63°$

7. The graph of $y = |x| + |x - 1|$ is

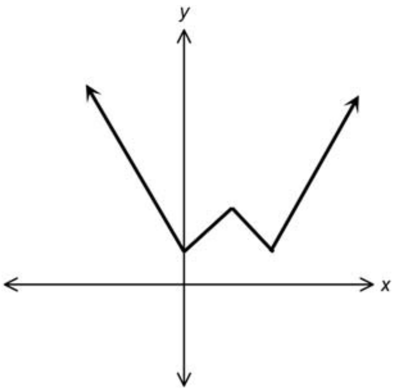
(A)



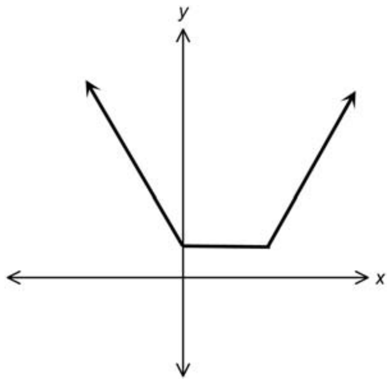
(B)



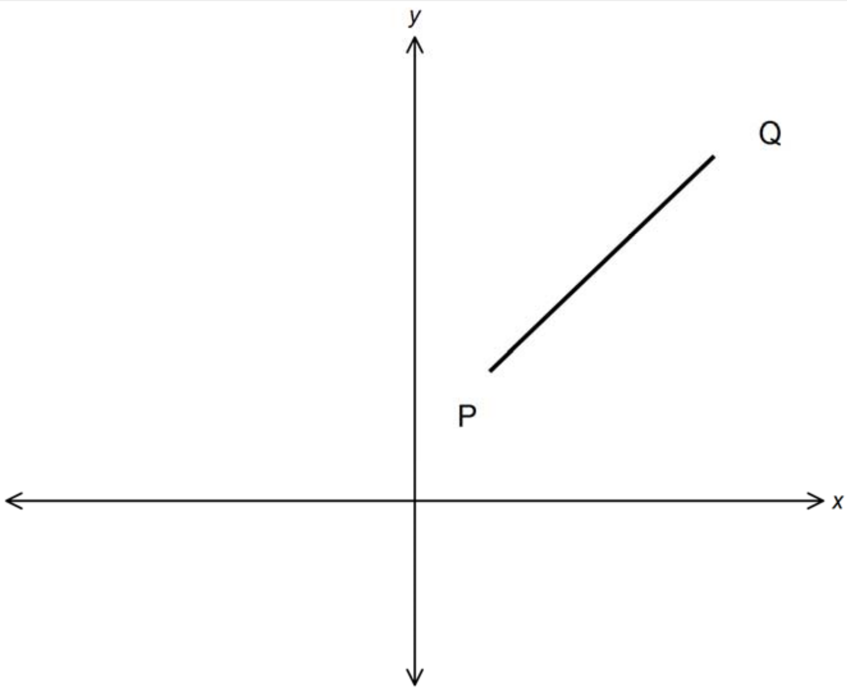
(C)



(D)



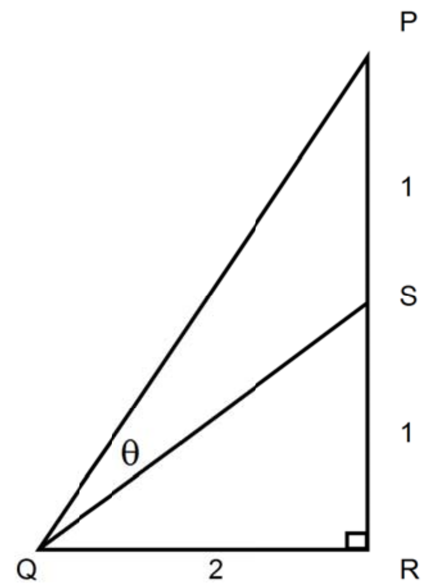
8.



T divides PQ internally in the ratio $2:1$
R divides PQ externally in the ratio $2:1$.
In what ratio does Q divide RT?

(A) $3:1$ (B) $1:3$ (C) $3:4$ (D) $4:3$

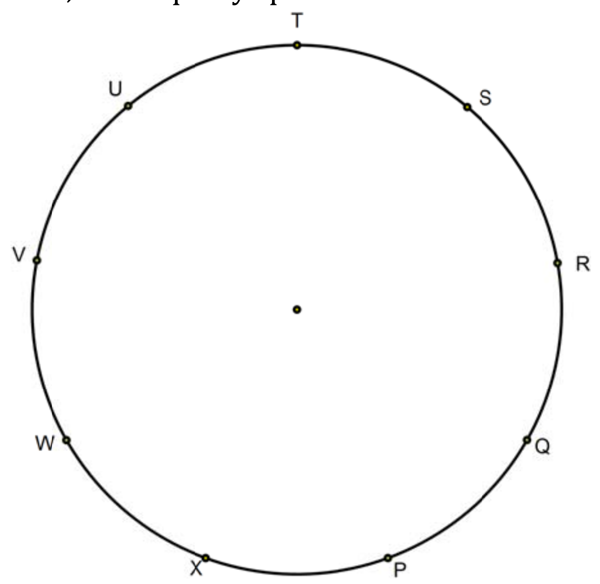
9. In triangle PQR, PS and SR both have a length of 1cm. QR is 2cm.



The value of $\tan\theta$ is

(A) $\frac{1}{3}$ (B) $\frac{1}{\sqrt{5}}$ (C) $\tan\left(22\frac{1}{2}°\right)$ (D) $\frac{1}{2}$

10. The nine points $P, Q, R, S \dots W, X$ lie equally spaced around the circumference of a circle.



The number of distinct triangles that can be formed so that the centre of the circle lies in the interior of each triangle is

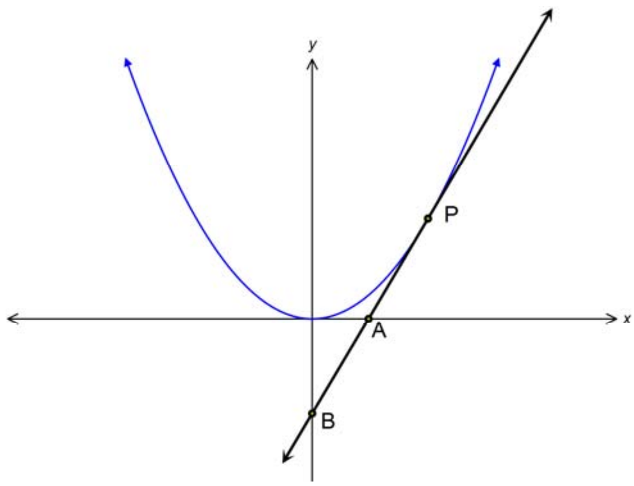
(A) 28 (B) 30 (C) 84 (D) 90

End of Section 1

Section II

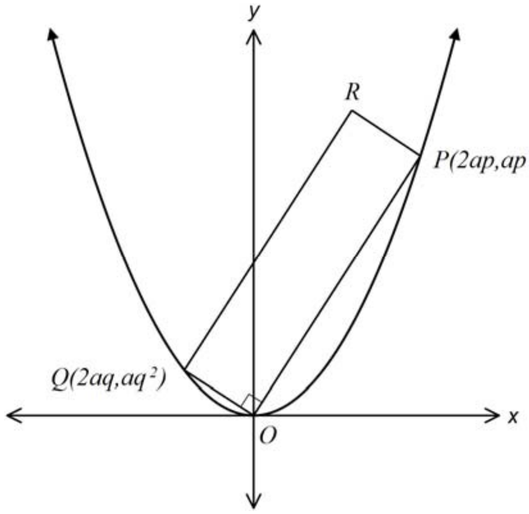
45 marks
Attempt questions 11-13
Answer each question on the appropriate page of your writing booklet. Extra writing paper is available. Each piece of additional writing paper must show your number.
Clearly indicate the question number.
In Questions 11-13, your responses should include relevant mathematical reasoning and/or calculations.

Question 11	Start on the appropriate page in your writing booklet.	**Marks**
a)	Find the exact value of $\sin 15°$	**2**
b)	Find the coordinates of the point which divides the interval AB with $A(1,4)$ and $B(5,2)$ externally in the ratio 1:3.	**2**
c)	Find the Cartesian equation for the parametric equations $$x = \cos 2t \text{ and } y = \cos t$$	**2**
d)	Solve for x: $$|x^2 - 5| = 5x + 9$$	**3**
e)	Solve for x: $\frac{1}{x} < \frac{1}{x+1}$	**3**
f)	Find the acute angle between the tangents to $y = x^2$ at $x = -1$ and $x = 2$. Give your answer to the nearest degree.	**3**
	End of Question 11	

Question 12 Start on the appropriate page in your writing booklet.		**Marks**
a)	(i) Express $\sqrt{3}\cos x - \sin x$ in the form $A\cos(x+\alpha)$ where $A>0$ and α is acute.	**2**
	(ii) Hence, solve $\sqrt{3}\cos x - \sin x = -2$ for $0° \leq x \leq 360°$.	**1**
b)	Solve $\cos 2\theta + \sin^2\theta = \frac{1}{2}$ for $0° \leq \theta \leq 360°$	**3**
c)	A group of tourists waiting to board a flight is comprised of 3 Americans, 4 Germans and 6 New Zealanders.	
	(i) How many ways can they all line up?	**1**
	(ii) How many ways can the Americans and the Germans line up to board the plane if no New Zealanders board?	**1**
	(iii) How many ways can the group line up to board the plane if the New Zealanders are included but no two New Zealanders stand next to each other?	**2**
d)	 P is the point $(2ap, ap^2)$ on the parabola $x^2 = 4ay$ and l is the equation of the tangent at P. The tangent at P intersects the x axis at A and the y axis at B.	
	(i) Prove that the equation of l is $y = px - ap^2$	**1**
	(ii) Find the coordinates of A and B.	**2**
	(iii) In what ratio does P divide AB?	**2**
	End of Question 12	

Question 13 Start on the appropriate page in your writing booklet.	**Marks**

a)



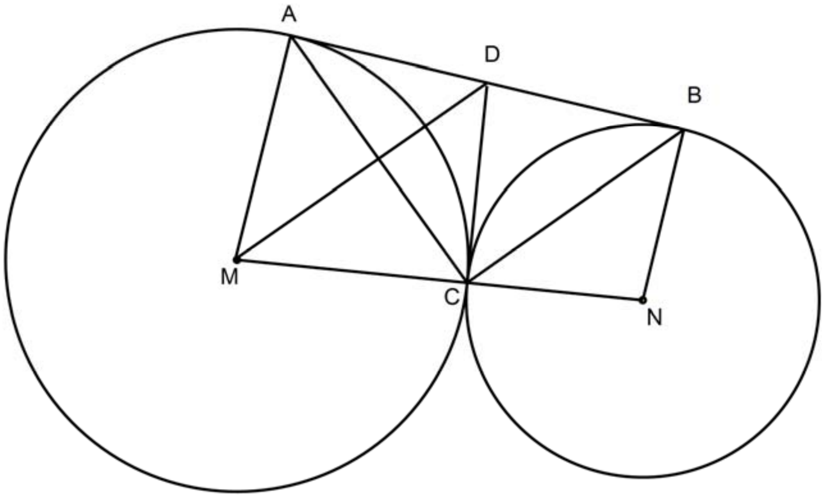
$P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$. The chord PQ subtends a right angle at the origin, O. R is the fourth vertex of the rectangle $POQR$.

(i) Prove $pq = -4$ **2**

(ii) Prove that R has coordinates: $x = 2a(p+q)$, $y = a(p^2+q^2)$ **1**

(iii) Prove that the Cartesian equation of the locus of R is $x^2 = 4a(y - 8a)$ **2**

b) In the diagram, MCN is a straight line. Circles are drawn with centre M, radius MC, and centre N, radius NC. AB is a common tangent to both circles, with the points of contact at A and B respectively. CD is a common tangent and meets AB at D.



(i) Copy the diagram

(ii) It can be proven that $\Delta ADC ||| \Delta BNC$. (Do NOT prove this). **2**

Explain why $AMCD$ is a cyclic quadrilateral.

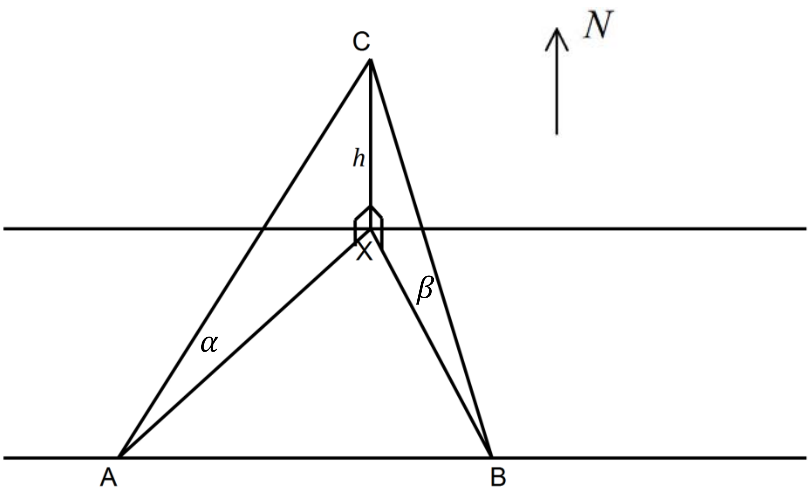
(iii) Prove that $MD \parallel CB$ **2**

Question 13 continues on the following page

Question 13 (continued)

c)	A bag contains 3 red balls, 2 green balls, 2 yellow balls, 2 white balls and 1 black ball. Four balls are chosen from the bag.	
	(i) How many ways are possible in which 3 of the balls are the same colour?	1
	(ii) How many different combinations are possible?	2

d) A vertical mast stands on the north bank of a river with straight parallel banks running east to west.



The angle of elevation of the top of the mast is α when measured from point A on the south bank at a distance of 5 m to the west of the mast and β when measured from point B on the south bank at a distance of 3 m to the east of the mast.

Show that the height, h, of the mast is $h = \frac{4}{\sqrt{cot^2\alpha - cot^2\beta}}$ **3**

End of paper

YEAR 11 PRELIMINARY EXTENSION 1 2013
YEARLY

1. $E-1=\sqrt{\frac{c}{R}}$

$(E-1)^2=\frac{c}{R}$

$c=R(E-1)^2$

$c=R(1-E)^2$

$\therefore D$

2. $\underline{S}$ _ _ _ _ _ _ $\underline{S}$

E R V I C E

No. of ways $=\frac{6!}{2!}$

$\therefore C$

3. $AB^2=BC.BD$

$=2\times7$

$=14$

$AB=\sqrt{14}$

$\therefore B$

4. $4\times\frac{2t}{1+t^2}-3\times\frac{(1-t^2)}{1+t^2}=4$

$\frac{8t-3+3t^2}{1+t^2}=4$

$8t-3+3t^2=4+4t^2$

$0=t^2-8t+7$

$\therefore D$

5. $2\sin x\cos x=\cos x$

$2\sin x\cos x-\cos x=0$

$\cos x(2\sin x-1)=0$

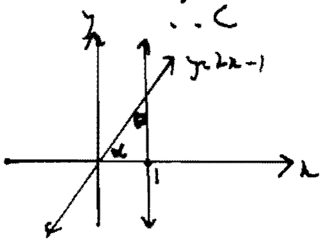
$\cos x=0 \quad \sin x=\frac{1}{2}$

$x=\frac{\pi}{2},\frac{3\pi}{2},\frac{\pi}{6},\frac{5\pi}{6}$

$\therefore$ 4 solutions

$\therefore C$

6.



$\tan\alpha=2$

$\alpha=63^\circ$

$\theta=90^\circ-63^\circ$

$\theta=27^\circ$

$\therefore B$

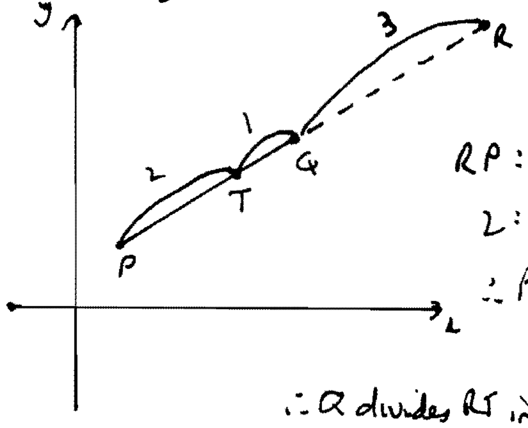
7. Between $x=0$ and 1 ie $0\le x<1$

$y=x+-(x-1)$

$y=1$

$\therefore D$

8.



$RP:RQ$

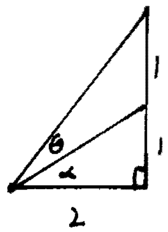
$2:1$

$\therefore PQ=QR$

$\therefore Q$ divides RT in ratio $3:1$

$\therefore A$

9.



$\tan(\theta+\alpha)=\frac{2}{2}$

$\frac{\tan\theta+\tan\alpha}{1-\tan\theta\tan\alpha}=1$

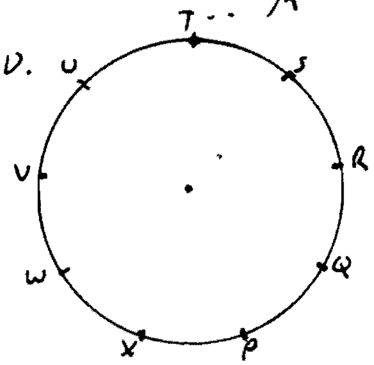
$\tan\theta+\frac{1}{2}=1-\tan\theta\times\frac{1}{2}$

$\frac{3}{2}\tan\theta=\frac{1}{2}$

$\tan\theta=\frac{1}{3}$

$\therefore A$

10.



Total possible $\Delta={}^9C_3$

$=84$

Determine triangles not containing centre.

From T, choose T and 2 others from U, V, W, X ie ${}^4C_2=6$

or choose T and 2 others from S, R, P, Q ie ${}^4C_2=6$

or choose T and 2 others from V, U, S, R ie ${}^4C_2=6$

But there are 9 vertices $\therefore(6+6+6)\times9=162$

But T(and SR), S(and TR), R and(TS) are all the same $\therefore\frac{162}{3}=54$.

$84-54=30 \therefore B$

11 a) $\sin 15° = \sin(60-45)°$

$= \sin 60° \cos 45° - \cos 60° \sin 45°$ ✓

$= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$ ✓

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

b) $A(1,4) \quad B(5,2)$
$\quad -1 : 3$ } ✓

$x = \frac{3-5}{3-1} \quad y = \frac{12-2}{3-1}$ ① for a single error

$x = \frac{-2}{2}, \; y = \frac{10}{2}$ ① if correct internal coordinates

$\therefore P$ is $(-1, 5)$ ✓

c) $x = \cos 2t$

$y = \cos t$

$x = 2\cos^2 t - 1$ ✓

$x = 2y^2 - 1$ ✓ must be in this (ISE) form (no $\cos^{-1}$)

d) $|x^2 - 5| = 5x + 9$ NB $5x + 9 \geq 0$

$5x \geq -9$

$x \geq -\frac{9}{5}$

$x^2 - 5 = 5x + 9 \qquad x^2 - 5 = -5x - 9$

$x^2 - 5x - 14 = 0 \qquad x^2 + 5x + 4 = 0$

$(x-7)(x+2) = 0 \qquad (x+4)(x+1) = 0$

$x = \cancel{-2}, 7$ ✓ $\qquad x = \cancel{-4}, -1$ ✓

$\therefore x = 7 \qquad x = -1$

$\therefore x = -1$ and 7

② if $x = 7$ only or $x = -1$ only

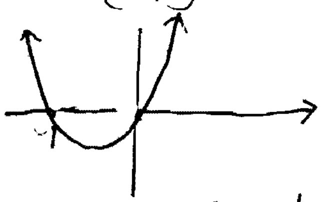
① if $x = -2, 7$ or $x = -4, -1$

e) $\frac{1}{x} < \frac{1}{x+1} \quad (x \neq 0, -1)$

$x(x+1)^2 < x^2(x+1)$ ✓

$x(x+1)[x+1-x] < 0$

$x(x+1) < 0$ ✓



(NB squaring both sides max 1 mark)

$\therefore -1 < x < 0$ ✓

f) $y = x^2$

$\frac{dy}{dx} = 2x$

① correct gradients

At $x = -1, \; m_1 = -2$ ✓

At $x = 2, \; m_2 = 4$

$\tan\theta = \left| \frac{4-(-2)}{1+4\times(-2)} \right|$ ① correctly substitutes

$= \left| \frac{6}{-7} \right|$

$= 40° 36' 4.6...''$ ✓

$\doteqdot 41°$ nearest degree

12 a (i) $A = \sqrt{(\sqrt{3})^2 + 1^2}$

$= 2$ ✓

$\sqrt{3}\cos x - \sin x \equiv 2\cos(x+\alpha)$

$\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = \cos x \cos\alpha - \sin x \sin\alpha$

$\therefore \cos\alpha = \frac{\sqrt{3}}{2}$

$\sin\alpha = \frac{1}{2}$

$\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = 30°$ ✓

$\therefore \sqrt{3}\cos x - \sin x = 2\cos(x + 30°)$

12 a ii)

$2\cos(x+30^\circ) = -2$

$\cos(x+30^\circ) = -1 \quad 0 \le x \le 360^\circ$

$x+30^\circ = 180^\circ \quad 30^\circ \le (x+30^\circ) \le 390^\circ$

$x = 150^\circ$ ✓

b)

$\cos 2\theta + \sin^2\theta = \frac{1}{2}$

$\cos^2\theta - \sin^2\theta + \sin^2\theta = \frac{1}{2}$ ✓ (Double ∠ for $\cos 2\theta$)

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\frac{1}{\sqrt{2}}$

$\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$ ✓

c) i) $13!$ ✓

ii) $7!$ $(= 5040)$ ✓

iii) Americans & Germans 7! ways

_ A _ G _ A _ G _ A _ G _ G _

insert 6 New Zealanders in $^8P_6$ ways. ✓

∴ Total ways $= 7! \times {}^8P_6$

$= 5040 \times 20160$

$= 101\,606\,400$ ways ✓

12 d i)

$y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a} = \frac{x}{2a}$

At $x = 2ap$, $\frac{dy}{dx} = \frac{2ap}{2a}$

$= p$ ✓

$y - y_1 = m(x - x_1)$

$y - ap^2 = p(x - 2ap)$

$y - ap^2 = px - 2ap^2$

$y = px - ap^2$ ✓

d ii) For A: $0 = px - ap^2$

$px = ap^2$

$x = ap$ ∴ $A(ap, 0)$ $p \ne 0$. ✓

For B: $y = -ap^2$

∴ $B(0, -ap^2)$ ✓

d iii) let ratio be $k:1$ $\quad A(ap, 0)\ B(0, -ap^2)$ $\quad k:1$

∴ $2ap = \frac{ap + 0}{k+1}$

$k + 1 = \frac{1ap}{2ap}$ ✓

$k = -\frac{1}{2}$

∴ P divides AB externally in ratio $1:2$ ✓

or

P divides AB in ratio $-1:2$.

13 a) i) $m_{OP} = \frac{ap^2}{2ap}$ $\quad m_{OQ} = \frac{aq^2}{2aq}$

$= \frac{p}{2}$ ✓ $\quad = \frac{q}{2}$ $\quad$ ① finds one of the gradients.

Since $OP \perp OQ$ $\quad \frac{p}{2} \cdot \frac{q}{2} = -1$

$pq = -4$ ✓

a ii) Since POQR is a rectangle and opposite sides are equal and parallel then R is $2aq$ units left of P and aq^2 units above P ✓

ie $x = 2ap + 2aq \quad y = ap^2 + aq^2$

$x = 2a(p+q) \quad y = a(p^2+q^2)$

OR Midpoint PQ $= \frac{2ap+2aq}{2}, \frac{ap^2+aq^2}{2}$ ✓

∴ R is $\frac{2ap+2aq}{2} = \frac{0+x_R}{2} \quad \frac{ap^2+aq^2}{2} = \frac{0+y_R}{2}$

∴ $R = 2a(p+q), a(p^2+q^2)$

13a iii) $x = 2a(p+q)$ $y = a(p^2+q^2)$

$x^2 = 4a^2(p^2+q^2+2pq)$ ✓

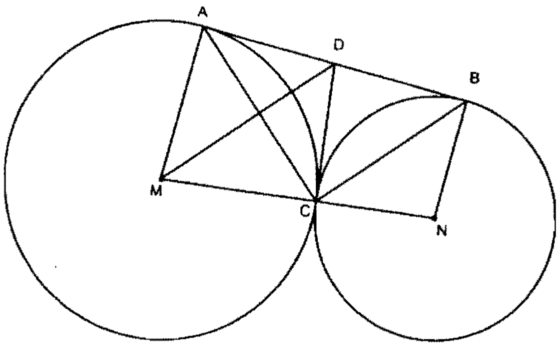
$x^2 = 4a^2\left(\frac{y}{a} + 2 \times -4\right)$ ✓

$x^2 = 4a^2\left(\frac{y}{a} - 8\right)$

$x^2 = 4a(y - 8a)$

(1) attempts to use $(p+q)^2 = p^2+q^2+2pq$

13b) i)



13b ii) $AM \perp AD$ (radius $\perp$ tangent)

$\therefore \angle MAD = 90°$ ✓

Also $CM \perp CD$ (radius $\perp$ tangent)

$\therefore \angle MCD = 90°$] either

In $AMCD$, $\angle MAD + \angle MCD = 90° + 90°$

$= 180°$ ✓

$\therefore$ AMCD is a cyclic quadrilateral

(opposite interior $\angle$s are supplementary)

iii) Similarly BNCD is a cyclic quadrilateral.

✓ $\angle DAC = \angle NCB$ (matching $\angle$'s in similar Δs ADC and CNB)

✓ $\angle DMC = \angle DAC$ ($\angle$'s at circumference standing on same chord of cyclic quad. ADCM)

✓ $\therefore \angle DMC = \angle NCB$

$\therefore MD \parallel CB$ (corresponding angles are equal)

All 3 correct **2** marks

some progress (relevant) 1 mark

13c i) All 3 red and 1 from G, Y, W, B

ie. ${}^4C_1 = 4$ ways. ✓

ii) 4 cases to consider

3 red 1 other $= 4$ ways

2 pairs $= {}^4C_2 = 6$ ways

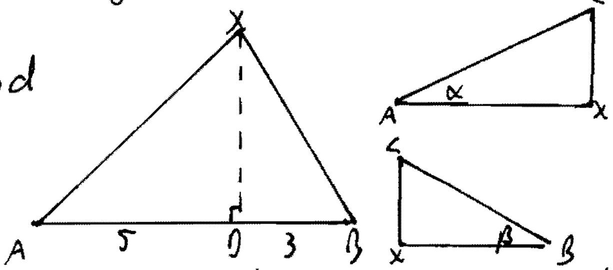
1 pair, 2 different $= {}^4C_1 \times {}^4C_2 = 4 \times 6 = 24$ ways

all different $= {}^5C_4 = 5$ ways

Total $= 39$ ways ✓

(1) for any of these cases

13d



$\tan\alpha = \frac{CX}{AX} = \frac{h}{AX}$ $\tan\beta = \frac{CX}{BX} = \frac{h}{BX}$

$AX = h\cot\alpha$ & $BX = h\cot\beta$ (1) for either

$XO^2 = AX^2 - 5^2$ & $XO^2 = BX^2 - 3^2$

$\therefore AX^2 - 25 = BX^2 - 9$

$h^2\cot^2\alpha - h^2\cot^2\beta = 16$ (1) for equating the expressions

$h^2(\cot^2\alpha - \cot^2\beta) = 16$

$h^2 = \frac{16}{\cot^2\alpha - \cot^2\beta}$ (1) final solution

$\therefore h = \frac{4}{\sqrt{\cot^2\alpha - \cot^2\beta}}$