BAULKHAM HILLS HIGH SCHOOL

2014

YEAR 11 YEARLY EXAMINATIONS

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- Show all necessary working in Questions 11 – 14
- Marks may be deducted for careless or badly arranged work

Total marks – 70

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 6 – 10

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

1 If the exact value of $\cos x$ is $\frac{1}{\sqrt{5}}$, what is the exact value of $\cos 2x$?

(A) $-\frac{3}{5}$

(B) $-\frac{2}{\sqrt{5}}$

(C) $\frac{3}{5}$

(D) $\frac{2}{\sqrt{5}}$

2 A curve has parametric equations $x = t^2$, $y = t^4$.
What is the cartesian equation of this curve?

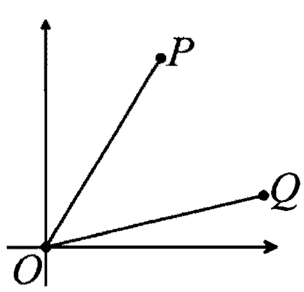
(A) $y = \sqrt{x}$, for all real x

(B) $y = \sqrt{x}$, for $x \geq 0$

(C) $y = x^2$, for all real x

(D) $y = x^2$, for $x \geq 0$

3 If OP has gradient 4 and OQ has gradient $\frac{1}{4}$, then the value of $\cos \angle POQ$ is ?



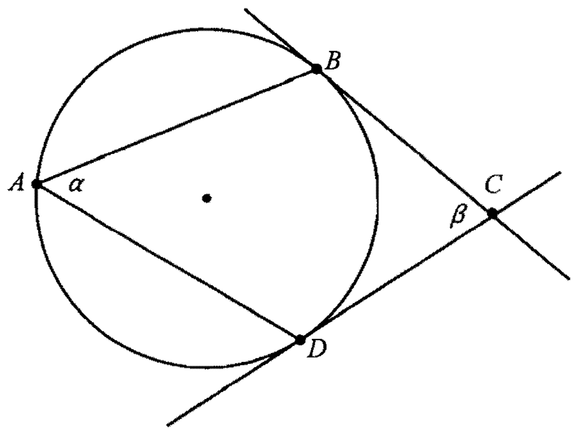
(A) $\frac{8}{\sqrt{17}}$

(B) $\frac{15}{17}$

(C) $\frac{8}{17}$

(D) $\frac{15}{\sqrt{17}}$

4 In the diagram below, BC and DC are tangents.



Which of the following statements is true?

(A) $\alpha + \beta = 180°$

(B) $\alpha + 2\beta = 180°$

(C) $2\alpha + \beta = 180°$

(D) $2\alpha - \beta = 90°$

5 Out of 7 different consonants and 4 different vowels, how many words made up of 3 different consonants and 2 different vowels can be formed?

(A) 210

(B) 10 080

(C) 25 200

(D) 55 440

6 Which is the correct condition for $y = mx + b$ to be a tangent to $x^2 = 4ay$?

(A) $am + b = 0$

(B) $am^2 + b = 0$

(C) $am - b = 0$

(D) $am^2 - b = 0$

7 Given θ in the range $0° \leq \theta \leq 180°$, the equation

$$x^2 + y^2 + 4x\cos\theta + 8y\sin\theta + 10 = 0$$

represents a circle for

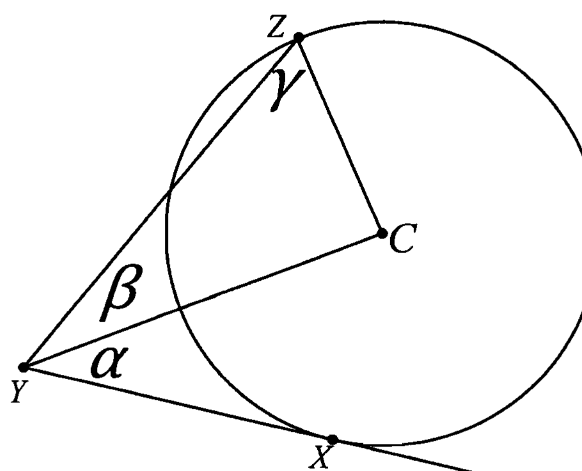
(A) $0° < \theta < 60°$

(B) $45° < \theta < 135°$

(C) $0° < \theta < 90°$

(D) all values of θ

8 The circle in the diagram has centre C. XY is a tangent to the circle at X. The angles α, β and γ are also indicated.



The angles α, β and γ are related by the equation

(A) $\sin\beta = \sin\alpha\sin\gamma$

(B) $\cos\alpha = \sin(\beta + \gamma)$

(C) $\sin\beta(1 - \cos\alpha) = \sin\gamma$

(D) $\sin(\alpha + \beta) = \cos\gamma\sin\alpha$

9 There are *positive* real numbers x and y which satisfy the equations

$$2x + ky = 4$$
$$x + y = k$$

for;

(A) only $k = 2$

(B) only $k > -2$

(C) all values of k

(D) no values of k

10 Let $N = 2^k \times 4^m \times 8^n$ where k, m and n are positive whole numbers. Then N will definitely be a square number whenever;

(A) k is even

(B) $k + n$ is odd

(C) k is odd but $m + n$ is even

(D) $k + n$ is even

END OF SECTION I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour 45 minutes for this section

Answer each question on the appropriate answer sheet. Each answer sheet must show your name. Extra paper is available.

All necessary working should be shown in every question.

Marks

Question 11 ***(15 marks)*** Use a *separate* answer sheet

a) The point P divides the interval joining $A(2,-3)$ and $B(-1,7)$ externally in the ratio 2:1.

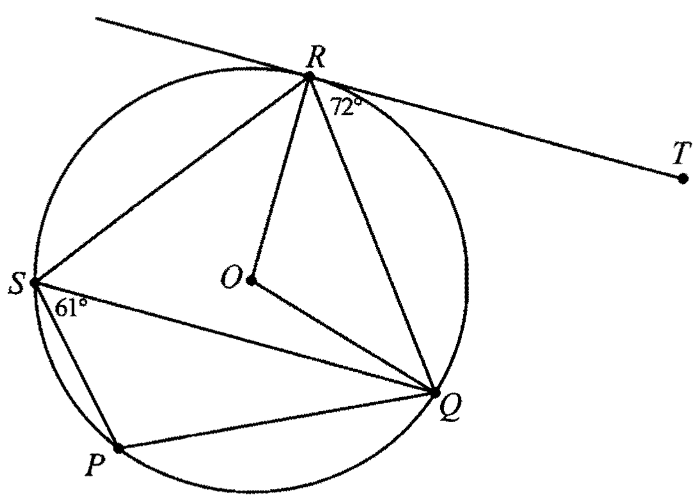
(i) Find the coordinates of P. *2*

(ii) Find the ratio in which B divides AP. *1*

b) Solve $\dfrac{3}{x-1} \leq 4$ *3*

c) Find the acute angle between $y = 2x - 1$ and $3x + 2y - 4 = 0$, correct to the nearest degree. *2*

d) In the diagram $\angle QRT = 72°$ and $\angle PSQ = 61°$.



Find the value of each of the following angles, giving a brief reason for your answer.

(i) $\angle RSQ$ *1*

(ii) $\angle PQR$ *1*

(iii) $\angle ROQ$ *1*

Question 11 continues on page 7

Marks

Question 11...*continued.*

e) (i) Prove that $\cos 105° = \dfrac{\sqrt{2}-\sqrt{6}}{4}$ 2

(ii) Hence, or otherwise, find the exact value of $\cos 52\frac{1}{2}°$ 2

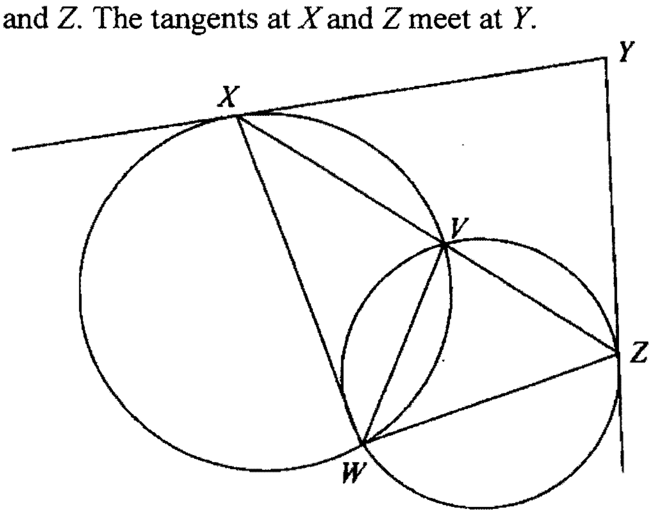
Question 12 ***(15 marks)*** Use a *separate* answer sheet

a) Find the number of ways in which 4 boys and 4 girls can be seated around a circular table so that no two people of the same sex is next to each other. 2

b) (i) Express $\sqrt{3}\cos\theta + \sin\theta$ in the form $A\sin(\theta + \alpha)$ 2

(ii) Hence solve $\sqrt{3}\cos\theta + \sin\theta = -\sqrt{3}$ for $0° \leq \theta \leq 360°$ 2

c) Two circles intersect at V and W as shown. A line through V cuts the two circles at X and Z. The tangents at X and Z meet at Y. 3



Copy or trace the diagram onto your answer sheet.

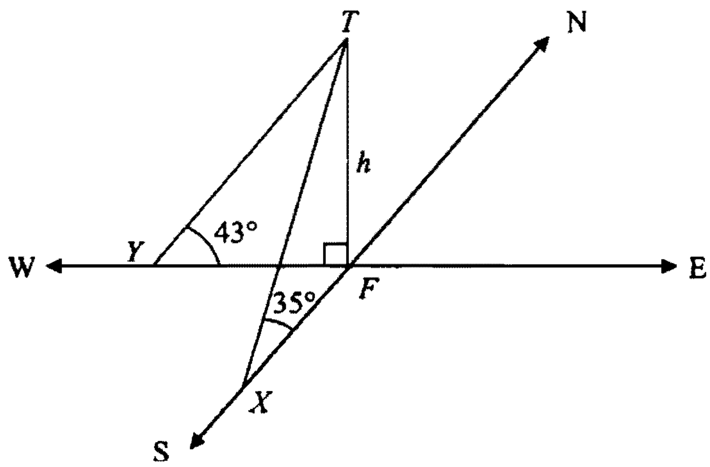
Prove $XYZW$ is a cyclic quadrilateral.

Question 12 continues on page 8

Marks

<u>Question 12</u>*...continued.*

d) Point X is due South and Point Y is due West of the foot F of the mountain FT, which has a height of h metres. From X and Y, the angles of elevation of T are 35° and 43° respectively. X and Y are 1200 metres apart.



(i) Prove that $XF = h \tan 55°$ *1*

(ii) Prove that $h = \dfrac{1200}{\sqrt{\tan^2 55° + \tan^2 47°}}$ *3*

e) If $2^a + 3^b = 17$ and $2^{a+2} - 3^{b+1} = 5$ find the values of a and b. *2*
Note: Full working must be shown in order to gain full marks.

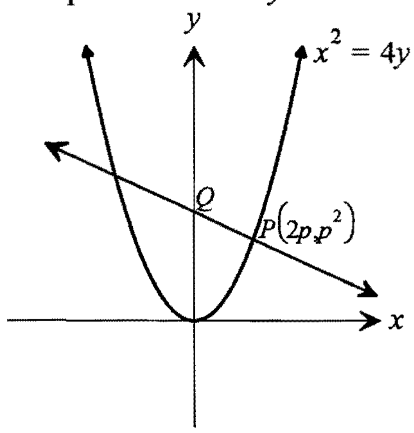
End of Question 12

Marks

Question 13 ***(15 marks)*** Use a *separate* answer sheet

a) Prove the identity $\dfrac{1-\tan^2 A}{1+\tan^2 A} \equiv \cos 2A$ *2*

b) $P(2p, p^2)$ is a point on the parabola $x^2 = 4y$



(i) Prove that the equation of the normal to the parabola at P has the equation *2*

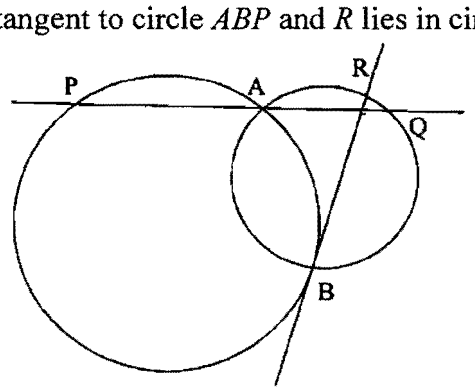
$$x + py = p^3 + 2p$$

(ii) The normal meets the axis of the parabola at Q. Find the coordinates of Q. *1*

(iii) Find the coordinates of R, the midpoint of PQ. *1*

(iv) Show that the locus of R is a parabola and find its vertex. *2*

c) Two circles cut at A and B. A line through A meets one circle at P and the other at Q. BR is a tangent to circle ABP and R lies in circle ABQ. *3*



Copy or trace the diagram onto your answer sheet.

Prove $PB || QR$.

d) How many ways can the letters of **SUTHAHARAN** be arranged, if;

(i) there are no restrictions? *1*

(ii) the vowels are grouped together? *1*

(iii) no two vowels are next to each other? *2*

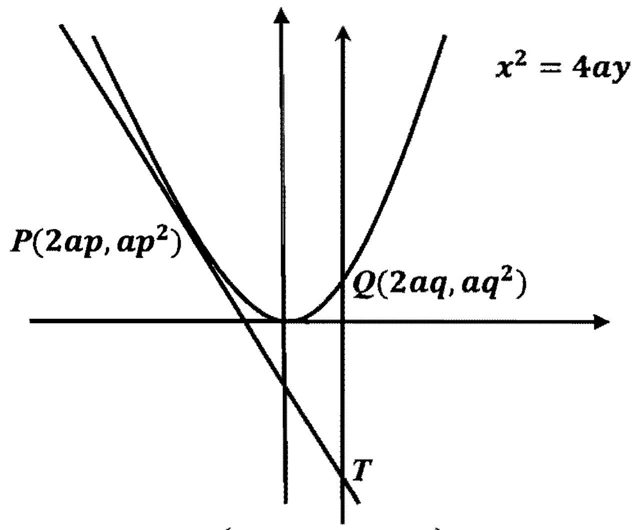
Question 14 ***(15 marks)*** Use a *separate* answer sheet

a) Find the horizontal asymptote of the function $y = \dfrac{3x^2 - 4x + 1}{2x^2 - 1}$ *1*

b) In Powerball 6 numbers are drawn from the first barrel containing 40 numbers and then a single number is drawn from a second barrel containing 20 numbers. How many different draws are possible in Powerball? *2*

c) Solve $|2x - 1| - |x| \leq 0$ *3*

d) Two points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$. The tangent at P has the equation $y = px - ap^2$ (Do NOT prove this).
The tangent at P and the line through Q parallel to the y axis intersect at T.



(i) Show that T has coordinates $(2aq, 2apq - ap^2)$ *1*

(ii) Show that the coordinates of M, the midpoint of PT are $\{a(p + q), apq\}$ *1*

(iii) Prove that if PQ is a focal chord, then $pq = -1$ *2*

(iv) Determine the locus of M, if PQ is a focal chord. *1*

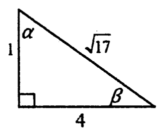
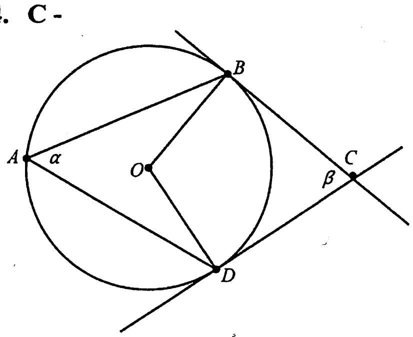
e) If $\tan\alpha$ and $\tan\beta$ are the two values of $\tan\theta$ which satisfy the quadratic equation $a\tan^2\theta + b\tan\theta + c = 0$;

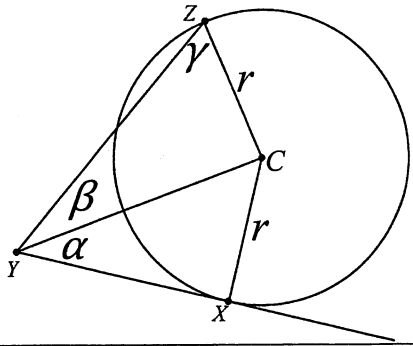
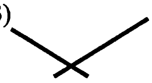
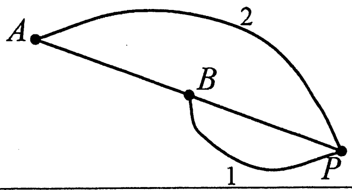
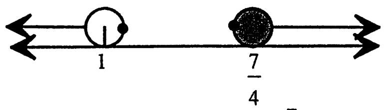
(i) Find $\tan(\alpha + \beta)$ *2*

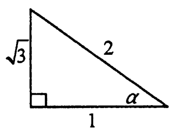
(ii) Show that $\tan^2(\alpha - \beta) = \dfrac{b^2 - 4ac}{(a + c)^2}$ *2*

End of paper

BAULKHAM HILLS HIGH SCHOOL
YEAR 11 EXTENSION YEARLY 2014 SOLUTIONS

Solution	Marks	Comments
SECTION I		
1. A - $\cos 2x = 2\cos^2 x - 1$ $= 2\left(\frac{1}{\sqrt{5}}\right)^2 - 1$ $= -\frac{3}{5}$	1	
2. D - $x = t^2 \Rightarrow x \geq 0$ $y = t^4$ $= (t^2)^2$ $= x^2$ $\therefore y = x^2, x \geq 0$	1	
3. C - Let $\angle POX = \alpha$ $\tan\alpha = 4$ Let $\angle QOX = \beta$ $\tan\beta = \frac{1}{4}$ $\cos\angle POQ = \cos(\alpha - \beta)$ $= \cos\alpha\cos\beta + \sin\alpha\sin\beta$ $= \left(\frac{1}{\sqrt{17}}\right)\left(\frac{4}{\sqrt{17}}\right) + \left(\frac{4}{\sqrt{17}}\right)\left(\frac{1}{\sqrt{17}}\right)$ $= \frac{8}{17}$  	1	
4. C -  $\angle OBC = \angle ODC = 90°$ (radius ⊥ tangent) $\angle BOD = 2\alpha$ ($\angle$ at centre , twice $\angle$ at circumference on same arc) $\angle OBC + \angle BCD + \angle CDO + \angle DOB = 360°$ ($\angle$ "sum" of a quadrilateral) $\therefore 90° + \beta + 90° + 2\alpha = 360°$ $2\alpha + \beta = 180°$	1	
5. C - ${}^7C_3 \times {}^4C_2 \times 5!$ $= 35 \times 6 \times 120$ $= 25200$	1	
6. B - $x^2 = 4a(mx + b)$ $x^2 - 4amx - 4ab = 0$ $\Delta = (-4am)^2 - 4(1)(-4ab)$ $= 16a^2m^2 + 16ab$ Line will be a tangent when $\Delta = 0$ $16a^2m^2 + 16ab = 0$ $am^2 + b = 0$	1	
7. B - $x^2 + y^2 + 4x\cos\theta + 8y\sin\theta + 10 = 0$ $(x + 2\cos\theta)^2 + (y + 4\sin\theta)^2 = 4\cos^2\theta + 16\sin^2\theta - 10$ $= 12\sin^2\theta - 6$ $\therefore 12\sin^2\theta - 6 > 0$ $\sin^2\theta > \frac{1}{2}$ $\sin\theta > \frac{1}{\sqrt{2}}$ (quad 1 quad 2 $\therefore \sin\theta > 0$) $45° < \theta < 135°$	1	

Solution	Marks	Comments
8. A –  $\sin\alpha = \frac{r}{YC}$ $YC = \frac{r}{\sin\alpha}$ $\frac{\sin\beta}{r} = \frac{\sin\gamma}{YC}$ $= \frac{\sin\alpha\sin\gamma}{r}$ $\therefore \sin\beta = \sin\alpha\sin\gamma$	1	
9. A – Using the second equation to eliminate x we see that $2(k-y)+ky=4$ $(k-2)y=4-2k$ $y=\frac{2(2-k)}{k-2}$, if $k \neq 2$ $\therefore \quad y=-2$ if $k \neq 2$ If $k=2$; $2x+2y=4$ $x+y=2$ So $x=1$, $y=1$ is a solution	1	
10. D - N will be a square number when the index is even i.e. divisible by 2 $N=2^k \times 4^m \times 8^n$ $=2^{k+2m+3n}$ $=2^{2\left(m+n+\frac{k+n}{2}\right)}$ So index will be even whenever $k+n$ is even	1	
SECTION II		
QUESTION 11		
11a) (i) $A(2,-3)$  $B(-1,7)$ $-2:1$ $P=\left(\frac{-2\times -1+2\times 1}{-2+1}, \frac{-2\times 7+1\times -3}{-2+1}\right)$ $=\left(\frac{4}{-1}, -\frac{17}{-1}\right)$ $=(-4,17)$	2	**2 marks** • Correct answer **1 mark** • Uses a correct method
11a) (ii)  $\therefore B$ is midpoint of $AB \Rightarrow$ the ratio is 1:1	1	**1 mark** • Correct answer
11b) $\frac{3}{x-1} \leq 4$ $x-1 \neq 0$ $x \neq 1$ $\frac{3}{x-1}=4$ $3=4x-4$ $4x=7$ $x=\frac{7}{4}$  $x<1$ or $x \geq \frac{7}{4}$	3	**3 marks** • Correct graphical solution on number line or algebraic solution, with correct working **2 marks** • Bald answer • Identifies the two correct critical points via a correct method • Correct conclusion to their critical points obtained using a correct method **1 mark** • Uses a correct method • Acknowledges a problem with the denominator. **0 marks** • Solves like a normal equation , with no consideration of the denominator.

Solution	Marks	Comments
11c) $y = 2x - 1 \Rightarrow m_1 = 2$ $3x + 2y - 4 = 0 \Rightarrow m_2 = -\frac{3}{2}$ $\tan\alpha = \left\| \dfrac{2 - \left(-\frac{3}{2}\right)}{1 + 2\left(-\frac{3}{2}\right)} \right\|$ $= \left\| \dfrac{4+3}{2-6} \right\|$ $= \dfrac{7}{4}$ $\alpha = 60°$	2	**2 marks** • Correct answer **1 mark** • Finds **both** slopes correctly • Substitutes into a correct formula
11d) (i) $\angle RSQ = 72°$ (alternate segment theorem)	1	**1 mark** • Correct answerer with correct reason
11d) (ii) $\angle RSP = \angle RSQ + \angle QSP$ (common $\angle$) $= 72° + 61°$ $= 133°$ $\angle PQR + \angle RSP = 180°$ (opposite $\angle$'s in a cyclic quadrilateral are supplementary) $\angle PQR = 180° - 133°$ $= 47°$	1	**1 mark** • Correct answerer with correct reason
11d) (iii) $\angle ROQ = 2\angle RSQ$ ($\angle$ at centre = twice $\angle$ at circumference standing on same arc) $= 2 \times 72°$ $= 144°$	1	**1 mark** • Correct answerer with correct reason
11e) (i) $\cos 105° = \cos(60° + 45°)$ $= \cos 60°\cos 45° - \sin 60°\sin 45°$ $= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= \dfrac{1-\sqrt{3}}{2\sqrt{2}}$ $= \dfrac{\sqrt{2}-\sqrt{6}}{4}$	2	**2 marks** • Correct solution **1 mark** • Uses $\cos(\alpha + \beta)$ formula correctly
11e) (ii) $\cos 105° = \cos\left(2 \times 52\frac{1}{2}°\right)$ $\dfrac{\sqrt{2}-\sqrt{6}}{4} = 2\cos^2 52\frac{1}{2}° - 1$ $2\cos^2 52\frac{1}{2}° = \dfrac{4+\sqrt{2}-\sqrt{6}}{4}$ $\cos^2 52\frac{1}{2}° = \dfrac{4+\sqrt{2}-\sqrt{6}}{8}$ $\cos 52\frac{1}{2}° = \dfrac{\sqrt{4+\sqrt{2}-\sqrt{6}}}{2\sqrt{2}}$ as $\cos 52\frac{1}{2}° > 0$	2	**2 marks** • Correct solution **1 mark** • Uses double angle formula in a logical manner in an attempt to find a solution, or equivalent.
QUESTION 12		
12a) In order to separate the boys and girls they must sit alternately in the circle Arrangements =3!4! = 144	2	**2 marks** • Correct answer **1 mark** • Evidence of taking into account that there are (n – 1)! Arrangements in a circle • Arranging boys and girls seperately
12b) (i)  $\alpha = \tan^{-1}\sqrt{3}$ $= 60°$ $\sqrt{3}\cos\theta + \sin\theta = 2\sin(\theta + 60°)$	2	**2 marks** • Correct solution **1 mark** • Correctly finds A or α
12b) (ii) $\sqrt{3}\cos\theta + \sin\theta = -\sqrt{3}$ $2\sin(\theta + 60°) = -\sqrt{3}$ $\sin(\theta + 60°) = -\dfrac{\sqrt{3}}{2}$ quadrants 3 & 4 $\sin\alpha = \dfrac{\sqrt{3}}{2}$ $\alpha = 60°$ $\theta + 60° = 240°, 300°$ $\theta = 180°, 240°$	2	**2 marks** • Correct solution **1 mark** • Obtains $(\theta + 60) = 240°, 300°$ • Subtracts 60 to obtain answers for θ

	Solution	Marks	Comments
12c)	$\angle YXZ = \angle XWV$ (alternate segment theorem) $\angle YZX = \angle ZWV$ (") $\angle XWZ = \angle XWV + \angle ZWV$ (common $\angle$) $\therefore \angle XWZ = \angle YXZ + \angle YZX$ $\angle XYZ + \angle YXZ + \angle YZX = 180°$ ($\angle$ sum ΔXYZ) $\therefore \angle XYZ + \angle XWZ = 180°$ Thus $XYZW$ is a cyclic quadrilateral (opposite $\angle$'s are supplementary)	3	**3 marks** • Correct solution **2 marks** • Correct solution with poor reasoning • Significant progress towards solution with good reasoning. **1 mark** • Significant progress towards solution with poor reasoning. • Progress towards solution with good reasoning.
12d) (i)	$\frac{h}{XF} = \tan 35°$ $XF = \frac{h}{\tan 35°}$ $XF = h\tan 55°$	1	**1 mark** • Correct working in order to establish result
12d) (ii)	Similarly $YF = h\tan 47°$ ΔFXY is right angled $XY^2 = XF^2 + YF^2$ $1200^2 = h^2\tan^2 55° + h^2\tan^2 47°$ $h^2(\tan^2 55° + \tan^2 47°) = 1200^2$ $h^2 = \frac{1200^2}{\tan^2 55° + \tan^2 47°}$ $h = \frac{1200}{\sqrt{\tan^2 55° + \tan^2 47°}}$	3	**3 marks** • Correctly establishes result **2 marks** • Uses Pythagoras Theorem in an attempt to make h^2 the subject **1 mark** • States a correct expression for YF • Establishes ΔFXY is right angled
12e)	$2^a + 3^b = 17 \Rightarrow 3 \times 2^a + 3^{b+1} = 51$ (+) $2^{a+2} - 3^{b+1} = 5$ $2^{a+2} - 3^{b+1} = 5$ $(3 + 2^2)2^a = 56$ $7 \times 2^a = 56$ $2^a = 8$ $a = 3$ $\therefore 8 + 3^b = 17$ $3^b = 9$ $b = 2$ $\therefore a = 3$, $b = 2$	2	**2 marks** • Correct solution **1 mark** • Finds either a or b • Bald answer
	QUESTION 13		
13a)	$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1 - \frac{\sin^2 A}{\cos^2 A}}{1 + \frac{\sin^2 A}{\cos^2 A}}$ $= \frac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A}$ $= \frac{\cos 2A}{1}$ $= \cos 2A$	2	**2 marks** • Correct solution **1 mark** • Uses a trig identity or relationship in a logical attempt to solve the problem
13b) (i)	$4y = x^2$ $y = \frac{x^2}{4}$ $\frac{dy}{dx} = \frac{x}{2}$ when $x = 2p$ $\frac{dy}{dx} = p$ $\therefore$ slope of normal is $-\frac{1}{p}$ $y - p^2 = -\frac{1}{p}(x - 2p)$ $py - p^3 = -x + 2p$ $x + py = p^3 + 2p$	2	**2 marks** • Correct solution **1 mark** • Correctly finds the slope of the normal

Solution	Marks	Comments
13b) (ii) y intercept occurs when $x = 0$ $py = p^3 + 2p$ $y = p^2 + 2$ $\therefore Q$ is $(0, p^2 + 2)$	1	**1 mark** • Correct answer
13b) (iii) $R\left(\frac{2p+0}{2}, \frac{p^2+2p}{2}\right)$ $= \left(p, \frac{p^2}{2} + p\right)$	1	**1 mark** • Correct answer
13b) (iii) $x = p$ $y = \frac{p^2}{2} + p$ $y = \frac{1}{2}x^2 + x$ $\therefore$ locus is the parabola $y = x^2 + 1$ which has vertex $(0,1)$	2	**2 marks** • Correct answer **1 mark** • Shows that the locus is a quadratic
13c) $\angle APB = \angle RBA$ (alternate segment theorem) $\angle RBA = \angle RQA$ ($\angle$'s in the same segment are =) $\therefore \angle APB = \angle RQA$ Thus $PB \parallel QR$ (alternate $\angle$'s are =)	3	**3 marks** • Correct solution **2 marks** • Correct solution with poor reasoning • Significant progress towards solution with good reasoning. **1 mark** • Significant progress towards solution with poor reasoning. • Progress towards solution with good reasoning.
13d) (i) Ways $= \frac{10!}{3!2!}$ $= 302400$	1	**1 mark** • Answer may be left in factorial notation.
13d) (ii) Ways $= \frac{4!}{3!} \times \frac{7!}{2!}$ $= 10080$	1	**1 mark** • Answer may be left in factorial notation.
13d) (iii) Ways $= \frac{6!}{2!} \times {}^7C_4 \times \frac{4!}{3!}$ $= 50400$	2	**2 marks** • Correct answer, may be left unsimplified **1 mark** • Progress towards correct answer • Part (i) minus part (ii)
QUESTION 14		
14a) $\lim_{x \to \infty} \frac{3x^2 - 4x + 1}{2x^2 - 1} = \frac{3}{2}$ $\therefore$ horizontal asymptote is $y = \frac{3}{2}$	1	**1 mark** • Correct solution
14b) Draws $= {}^{40}C_6 \times {}^{20}C_1$ $= 3838380 \times 20$ $= 76767600$	2	**2 marks** • Correct answer **1 mark** • Calculates correct number of combinations for one barrel • Uses permutation instead of combination

Solution	Marks	Comments
14c) $\lvert 2x-1\rvert - \lvert x\rvert \le 0$ $x<0$: $1-2x-(-x)\le 0$; $1-x\le 0$; $x\ge 1$; $\therefore$ no solutions $0\le x\le \frac{1}{2}$: $1-2x-x\le 0$; $-3x\le -1$; $x\ge \frac{1}{3}$; $\frac{1}{3}\le x\le \frac{1}{2}$ $x>\frac{1}{2}$: $2x-1-x\le 0$; $x-1\le 0$; $x\le 1$; $\frac{1}{2}<x\le 1$ $\therefore \frac{1}{3}\le x\le 1$	3	**3 marks** • Correct solution **2 marks** • Significant progress towards solution • Correct graph **1 mark** • Investigates logical cases • Attempts to draw the graph $y=\lvert 2x-1\rvert-\lvert x\rvert$
14d) (i) when $x=2aq$; $y=p(2aq)-ap^2$ $=2apq-ap^2$ $\therefore T$ is $\left(2aq\,,2apq-ap^2\right)$	1	**1 mark** • Correct solution
14d) (ii) $\left(\frac{2ap+2aq}{2},\frac{ap^2+2apq-ap^2}{2}\right)$ $=\left(\frac{2a(p+q)}{2},\frac{2apq}{2}\right)$ $=(a(p+q),apq)$	1	**1 mark** • Correct solution
14d) (iii) $m_{PQ}=m_{SP}$ $\frac{ap^2-aq^2}{2ap-2aq}=\frac{ap^2-a}{2ap-0}$ $\frac{a(p+q)(p-q)}{2a(p-q)}=\frac{a\left(p^2-1\right)}{2ap}$ $\frac{p+q}{2}=\frac{p^2-1}{2p}$ $p^2+pq=p^2-1$ $pq=-1$	2	**2 marks** • Correct solution **1 mark** • Finds the slope of the chord PQ
14d) (iv) $y=apq$ $pq=-1\Rightarrow y=-a$ Locus is the line $y=-a$	1	**1 mark** • Correct solution
14e) (i) sum of the roots $=-\frac{b}{a}$ product of the roots $=\frac{c}{a}$ $\tan\alpha+\tan\beta=-\frac{b}{a}$ $\tan\alpha\tan\beta=\frac{c}{a}$ $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$ $=\frac{-\frac{b}{a}}{1-\frac{c}{a}}$ $=-\frac{b}{a-c}$ $=\frac{b}{c-a}$	2	**2 marks** • Correct solution **1 mark** • Finds $\tan\alpha+\tan\beta$ and/or $\tan\alpha\tan\beta$ • Correctly uses $\tan(\alpha+\beta)$ expansion
14e) (ii) $\tan^2(\alpha-\beta)=\left(\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}\right)^2$ $=\frac{(\tan\alpha+\tan\beta)^2-4\tan\alpha\tan\beta}{(1+\tan\alpha\tan\beta)^2}$ $=\frac{\left(\frac{b}{a}\right)^2-\frac{4c}{a}}{\left(1+\frac{c}{a}\right)^2}$ $=\frac{\frac{b^2}{a^2}-\frac{4ac}{a^2}}{\frac{(a+c)^2}{a^2}}$ $=\frac{b^2-4ac}{(a+c)^2}$	2	**2 marks** • Correct solution **1 mark** • Rewrites $\tan^2(\alpha-\beta)$ in terms of known results