BAULKHAM HILLS HIGH SCHOOL

2015 YEAR 11
YEARLY EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- In Questions 11–14, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for careless or badly arranged work

Total marks – 70
Exam consists of 9 pages.

This paper consists of TWO sections.

<u>Section 1</u> – Page 2-4 (10 marks)

- Attempt Question 1-10
- Allow about **15** minutes for this section

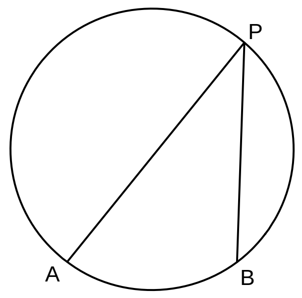
<u>Section II</u> – Pages 5-9 (60 marks)

- Attempt questions 11-14
- Allow about **1 hours and 45** minutes for this section

Section 1 –Multiple Choice (10 marks)
Attempt all questions.

Answer the following in the booklet provided.

1 AP is a diameter of the circle



If $\angle APB = 40°$, then $\angle PAB$ is

(A) $40°$

(B) $50°$

(C) $60°$

(D) not enough information

2 What is another expression for $\cos(x + y)$?

(A) $\cos x \cos y - \sin x \sin y$

(B) $\cos x \cos y + \sin x \sin y$

(C) $\sin x \cos y - \cos x \sin y$

(D) $\sin x \cos y + \cos x \sin y$

3 What is the acute angle to the nearest degree between the lines $y = 1 - 3x$ and $4x - 6y - 5 = 0$?

(A) $15°$

(B) $38°$

(C) $52°$

(D) $75°$

4 ${}^nC_r$ is equal to:

(A) $(n - r)!\,{}^nP_r$

(B) $\frac{{}^nP_r}{(n-r)!}$

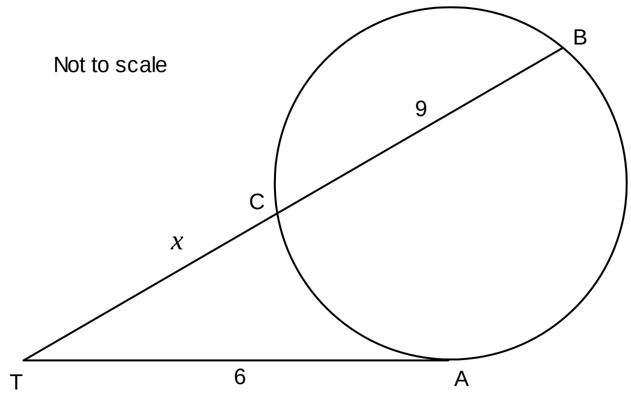
(C) $r!\,{}^nP_r$

(D) $\frac{{}^nP_r}{r!}$

5	Let $t = \tan\frac{\theta}{2}$ where $0° < \theta < 180°$ Which of the following gives the correct expression for $2\sin\theta + 2\cos\theta$? (A) $\frac{1+2t-t^2}{1+t^2}$ (B) $\frac{2+2t-2t^2}{1+t^2}$ (C) $\frac{2+4t-2t^2}{1+t^2}$ (D) $\frac{2-4t+2t^2}{1-t^2}$
6	The coordinates of a point that divides the interval $A(-2,7)$ to $B(12,0)$ externally in the ratio 4:3 is (A) $(-44,-28)$ (B) $(44,-28)$ (C) $(54,-21)$ (D) $(54,21)$
7	If $\cos\theta = -\frac{3}{5}$ and $0° < \theta < 180°$ then $\tan\frac{\theta}{2}$ is equal to: (A) $-\frac{1}{3}$ or 3 (B) $\frac{1}{3}$ or 3 (C) -2 (D) 2
8	The expression $\sin x - \sqrt{3}\cos x$ can be written in the form $2\sin(x+\beta)$. The value of β is (A) $30°$ (B) $-30°$ (C) $60°$ (D) $-60°$

9 The line TA is a tangent to the circle at A, and TB is a secant meeting the circle at B and C.

Given that $TA = 6$, $CB = 9$ and $TC = x$, what is the value of x?



(A) 2

(B) 3

(C) 12

(D) 18

10 A point moves in the xy-plane such that $P(\tan\theta, \cot\theta)$ is its parametric presentation with parameter θ, where θ is any real number. The locus of P is then a

(A) Parabola

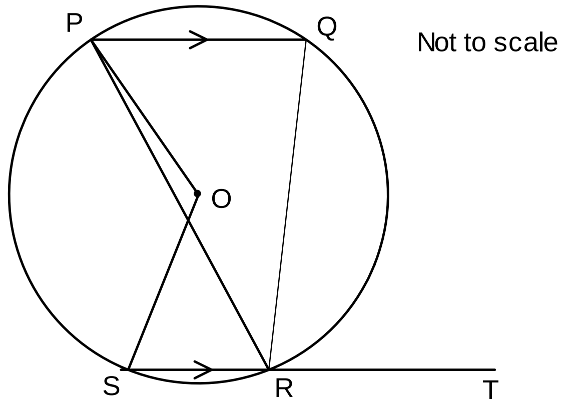
(B) Circle

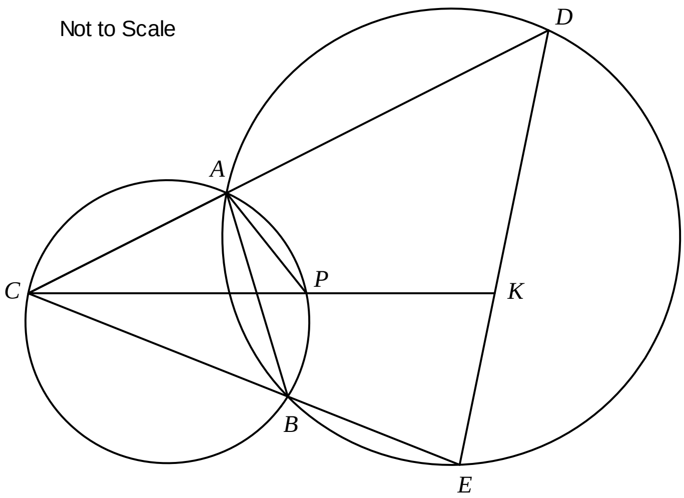
(C) Hyperbola

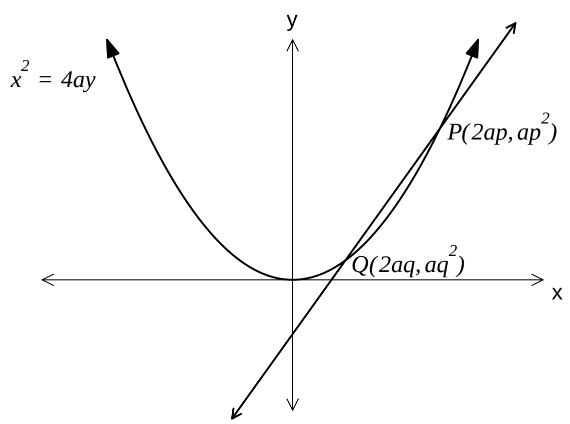
(D) Straight line

End of Section I

Section II – Extended Response
All necessary working should be shown in every question.

	Question 11 (15 marks) - Start on the appropriate page in your answer booklet	**Marks**
a)	Solve $$\frac{2x-3}{x-2} \geq 1$$	3
b)	The acute angle between the lines $y = (m+3)x$ and $y = mx$ is 45° (i) Show that $$\left|\frac{3}{m^2+3m+1}\right| = 1$$ (ii) Hence find all the values of m.	1 2
c)	In the diagram, PQ is parallel to ST and O is the centre of the circle. $\angle PQR = 80°$ and $\angle POS = 110°$  Copy the diagram into your booklet. Find the value of $\angle PRQ$ giving reasons.	3
d)	Solve $5\cos\theta + 12\sin\theta - 13 = 0$ over the domain $0^o \leq \theta \leq 360°$.	3
e)	How many distinct permutations of the letters of the word $D\ I\ V\ I\ D\ E$ are possible in a straight line when the word begins and ends with the letter D?	1
f)	If $\tan(A+B) = x$ and $\tan B = \frac{1}{3}$, express $\tan A$ in terms of x.	2
	End of Question 11	

Question 12 (15 marks) - Start on the appropriate page in your answer booklet		**Marks**
a)	Simplify $$\frac{4^n + 8(2^n)}{2^{n-1}}$$	**2**
b)	From a group of 7 girls and 6 boys, 3 girls and 2 boys are chosen.	
	(i) How many different groups of 5 are possible?	**2**
	(ii) If the group of 5 stand in a line, how many arrangements are possible if the boys stand together?	**2**
c)	In the diagram, two circles intersect at A and B. CAD, CBE, CPK and DKE are straight lines.	
		
	Draw the diagram into your answer booklet.	
	(i) Explain why $\angle APC = \angle ABC$.	**1**
	(ii) Hence, or otherwise, show that $ADKP$ is a cyclic quadrilateral.	**2**
d)	(i) Sketch the graph of $y = \|1 - 2x\|$.	**2**
	(ii) Hence or otherwise solve $\|1 - 2x\| \leq x$.	**2**
e)	Prove that $\cos 2\theta + \tan\theta \sin 2\theta = 1$.	**2**
	End of Question 12	

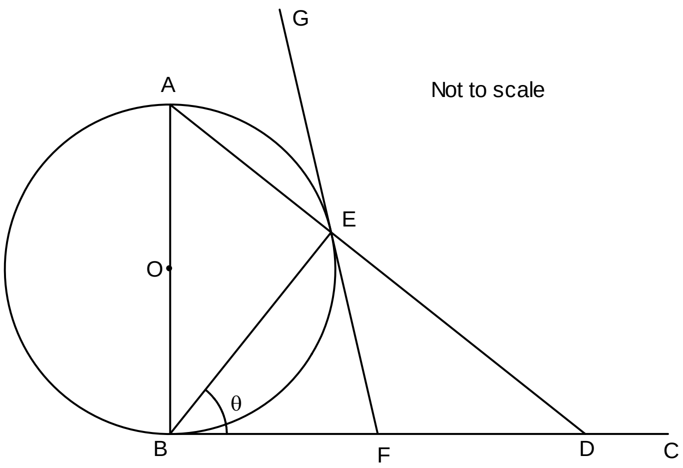
Question 13 (15 marks) - Start on the appropriate page in your answer booklet		Marks
a)	4 men, 2 women and a child sit at a round table. In how many ways can these 7 people be arranged (i) Without restrictions. (ii) If the child is seated between the two men.	 1 1
b)	Consider the function $f(x) = \frac{x}{4-x^2}$ (i) State the domain of the function. (ii) Show that the function is an odd function. (iii) Show that the function is increasing throughout its domain. (iv) Evaluate $\lim_{x \to \infty} f(x)$. (v) Using the above information sketch the function $y = f(x)$ showing any essential features.	 1 2 2 1 2
c)	The diagram below shows the parabola $x^2 = 4ay$ and the points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$  (i) Show that the equation of the chord PQ is $2y = (p+q)x - 2apq$. (ii) The line joining P and Q passes through the point $(0, -2a)$. Show that $pq = 2$. (iii) The normals to the parabola $x^2 = 4ay$ at points P and Q intersect at K. The coordinates of K are $\left(-apq(p+q), a(p^2+q^2+pq+2)\right)$ (DO NOT PROVE THIS) Find the equation of the locus of K.	 2 1 2
	End of Question 13	

Question 14 (15 marks) - Start on the appropriate page in your answer booklet

a) Simplify fully: **3**

$$\cos^2 A + \cos^2(120° + A) + \cos^2(120° - A).$$

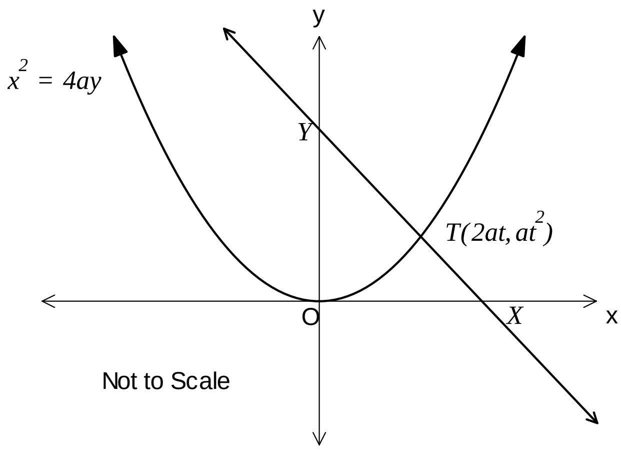
b) In the diagram, AB is the diameter of the circle centre O, and BC is a tangent at B. The line AD intersects the circle at E and BC at D. The tangent to the circle at E intersects BC at F. Let $\angle EBF = \theta$.



(i) Prove $\angle FED = 90° - \theta$. **2**

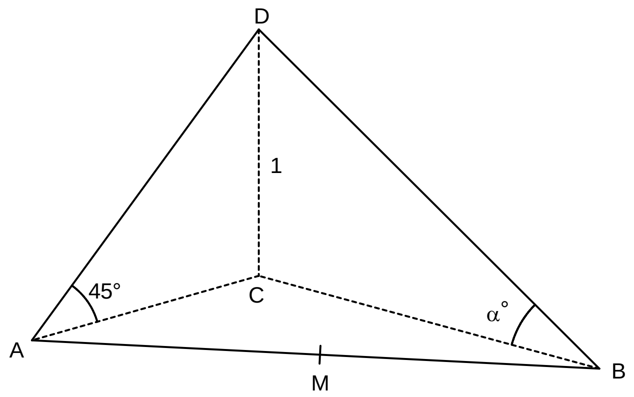
(ii) Prove $BF = FD$. **2**

c) In the diagram, $T(2at, at^2)$ is a point on the parabola $x^2 = 4ay$



(i) Show that the normal to the parabola at T has equation $x + ty = 2at + at^3$. **2**

(ii) This normal cuts the x and y axes at X and Y respectively. **2**
Find the ratio of $\frac{TX}{TY}$ in simplest terms.

d)	CD is a vertical pole of height 1 metre that stands with its base C on horizontal ground. A is a point due South of C such that the angle of elevation of D from A is $45°$. B is a point due East of C such that the angle of elevation of D from B is α . M is the midpoint of AB.  	
	(i) Show that $AB = \text{cosec}\alpha$.	**2**
	(ii) Find an expression for CM in terms of $\text{cosec}\alpha$.	**2**
	End of Examination	

13 a	For which x- values on the curve $y = \frac{x}{x+1}$ is the curve increasing?	**2**
13 b	(i) If $a = \frac{15bx}{3b+5x}$ express x in terms of a and b Hence express $\sqrt{\frac{3b-a}{5x-a}}$ in terms of a and b	**2** **2**
	(ii)	

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Answers

Multiple choice

1. B (∠ in a semicircle)

2. A

3. D. $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{-3-\frac{2}{3}}{1+-3\times\frac{2}{3}}\right|$

$\theta = 75^\circ$ (nearest degree)

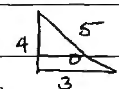
4. D.

5. C $\quad 2\times\frac{2t}{1+t^2} + \frac{2\times(1-t^2)}{1+t^2} = \frac{4t+2-2t^2}{1+t^2}$

6. C $\quad \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right) \quad m:n = 4:3$

$\left(\frac{4\times 12 - 3\times -2}{4-3}, \frac{4\times 0 - 3\times 7}{4-3}\right) = (54, -21)$

~~7.~~ D. (Answer missing)



2nd quad. $\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$

$-\frac{4}{3} = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$

$(2\tan\frac{\theta}{2}+1)(\tan\frac{\theta}{2}-2) = 0$

$\tan\frac{\theta}{2} = 2$

8. D. $\tan\beta = \frac{\sqrt{3}}{1}$

$\beta = 60$ but $(x+\beta)$ should be $(x-\beta)$

$\therefore \beta = -60$

9. B. $AT^2 = TB\times TC$

$6^2 = (x+9)x$

10. C. $x = \tan\theta \qquad y = \cot\theta$

$= \frac{1}{\tan\theta}$

$= \frac{1}{x}$

$\therefore xy = 1$

10 marks

Question 11.

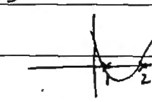
a) $\frac{2x-3}{x-2} \geq 1$

$\frac{2x-3-x+2}{x-2} \geq 0$ ① identifies $x = 2, 1$

$(x-2)^2 \times \frac{(x-1)}{\cancel{x-2}} \geq 0 \times (x-2)^2$ ① correct answer

① $x \neq 2$

$(x-2)(x-1) \geq 0$



$\therefore x \leq 1$ or $x \geq 2$ but $x \neq 2 \therefore x \leq 1 \quad x > 2$

b) $l_1: m_1 = m+3 \qquad l_2: m_2 = m \qquad \theta = 45$

i) $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\tan 45 = \left|\frac{m+3-m}{1+(m+3)m}\right|$ ① should have either — 1st 2 lines

$1 = \left|\frac{3}{m^2+3m+1}\right|$

ii) two cases ① for solving absolute value equations

$m^2+3m+1 = 3 \qquad m^2+3m+1 = -3$

$m^2+3m-2 = 0 \qquad m^2+3m+4 = 0$

$m = \frac{-3\pm\sqrt{9+8}}{2} \qquad m = \frac{-3\pm\sqrt{9-16}}{2}$

$= \frac{-3\pm\sqrt{17}}{2}$ ① the soln $\qquad \therefore$ no soln

3 marks

c) $\angle PRS = \frac{1}{2}\angle POS$ (∠ at centre twice ∠ at circumference on same arc)

$= 55^\circ$ ①

$\angle QRT = \angle PQR$ (alternate ∠s, PQ ∥ ST)

$= 80^\circ$ ①

$\angle PRQ + \angle PRS + \angle QRT = 180^\circ$ (∠ sum straight line)

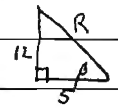
$\therefore \angle PRQ = 180 - (55+80)$ ①

$= 45^\circ$.

d) $5\cos\theta + 12\sin\theta = 13 \qquad 0 \le \theta \le 360^\circ$ (3 marks)

$5\cos\theta + 12\sin\theta = R\cos(\theta - \beta)$

where R :



$R = \sqrt{144+25}$

$= 13$

$\tan\beta = \frac{12}{5}$

$\beta = 67^\circ 23'$ (1)

state if corrected off?

(1)

$\therefore \quad 13\cos(\theta - 67^\circ 23') = 13 \qquad 0 \le \theta - 67^\circ 23' \le 292^\circ 37'$

$\theta - 67^\circ 23' = 0,\ 360$ ← not in range

$\theta = 67^\circ 23'$. (1)

OR.

t method:

$5\frac{(1-t^2)}{1+t^2} + \frac{12 \times 2t}{1+t^2} = 13 \qquad$ where $t = \tan\frac{\theta}{2}$.

$5 - 5t^2 + 24t = 13 + 13t^2$ (1)

$18t^2 - 24t + 8 = 0$

$9t^2 - 12t + 4 = 0$

$(3t-2)^2 = 0$

$t = \frac{2}{3} \qquad \tan\frac{\theta}{2} = 2/3$

$\frac{\theta}{2} = 33.69\ldots$ (1)

$\theta = 67^\circ 23'$ (1)

test 180°

LHS $= 5\cos 180 + 12\sin 180^\circ$

$= -5$

$\ne 13 \qquad \therefore$ no soln $\qquad \therefore \theta = 67^\circ 23'$.

You may ask for extra writing paper if you need more space to answer question 11

e. DIVIDE

D - - - - D $\quad \therefore$ no. of ways $= \frac{4!}{2!}$

$= 12$. (1)

f. $\tan(A+B) = x \qquad \tan B = \frac{1}{3}$

$\therefore \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$x = \frac{\tan A + \frac{1}{3}}{1 - \frac{1}{3}\tan A}$. (1)

$\frac{3x - x\tan A}{3} = \frac{3\tan A + 1}{3}$

$3x - 1 = 3\tan A + x\tan A$

$\tan A = \frac{3x-1}{3+x}$ (1)

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Question 12.

a) $\dfrac{4^n + 8(2^n)}{2^{n-1}} = \dfrac{2^{2n} + 2^3 . 2^n}{2^n . 2^{-1}}$ (1)

$= \dfrac{2^n(2^n + 2^3)}{2^n . 2^{-1}}$

$= 2^{n+1} + 2^4$ or $2^{n+1} + 16.$ (1)

b) i) no. of ways $= {}^7C_3 \times {}^6C_2$ — (1) recognising ${}^nC_r$

$= 525$ — (1) answer.

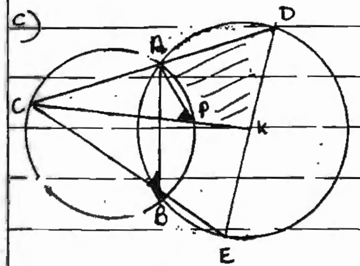
ii) boys together $= 2!$

no. of ways in a line $= 4! \times 2!$ — (1) putting boys together

$= 48$

(1) answer.

c)



i) $\angle APC = \angle ABC$ (1)

($\angle$s at circumference equal on same arc/chord)

ii) In cyclic quad ADEB

$\angle ADE = \angle ABC$ (external $\angle$ equal interior opposite $\angle$) (1)

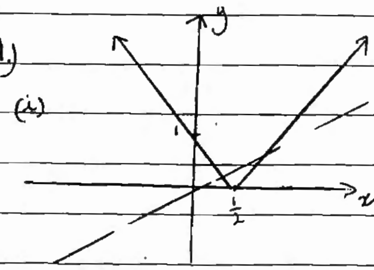
but $\angle ABC = \angle APC$ (part i)

and $\angle APC = \angle ADK = \angle ABC$

$\therefore$ external $\angle$ equals interior opposite $\angle$ (1)

$\therefore$ ADKP is a cylic quadrilateral

d) (i)



← not required in solution

(1) correct shape

(1) axes labelled. intercepts shown.

ii) $1 - 2x \le x$ $\qquad$ $-1 + 2x \le x$

$1 \le 3x$ $\qquad$ $-1 \le -x$

$x \ge \frac{1}{3}$ $\qquad$ $x \le 1$ — (1) correct values

$\therefore \frac{1}{3} \le x \le 1$ — (1) put together in one inequality

e) Prove: $\cos 2\theta + \tan\theta \sin 2\theta = 1$

LHS $= 2\cos^2\theta - 1 + \dfrac{\sin\theta}{\cos\theta} \times 2\sin\theta\cos\theta$ (1)

$= 2\cos^2\theta + 2\sin^2\theta - 1$

$= 2(\cos^2\theta + \sin^2\theta) - 1$ (1)

$= 1$

$=$ RHS.

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You may ask for extra writing paper if you need more space to answer question 12

Question 13 **BOS#:** ____________

a) 4 men 2 women 1 child = 7

i) no. of ways = $6!$

$= 720$ (1)

ii) no. of ways = ${}^4C_2 \times 2! \times 4!$

$= 288$ (1)

b) $f(x) = \frac{x}{4-x^2}$

i) D: all real x, $x \neq \pm 2$ (1)

ii) $f(x) = \frac{x}{4-x^2}$

$f(-x) = \frac{-x}{4-(-x)^2}$ (1) should show substitute

$= -\frac{x}{4-x^2} = -f(x)$ ∴ odd function. (1) concluding

iii) $f(x) = \frac{x}{4-x^2}$

$f'(x) = \frac{(4-x^2)\times 1 - x(-2x)}{(4-x^2)^2}$ (1) $f'(x)$

(1) concluding

$= \frac{4+x^2}{(4-x^2)^2}$ as numerator and denominator are >0 for all values of x

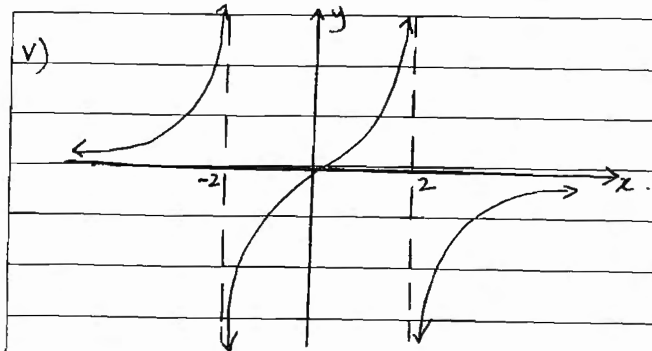
∴ $f(x) = y$ is always increasing for its domains.

iv) $\lim_{x\to\infty} \frac{x}{4-x^2} = \lim_{x\to\infty} \frac{\frac{x}{x^2}}{\frac{4}{x^2} - \frac{x^2}{x^2}}$

$= \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{4}{x^2} - 1}$ (1)

$= 0$ ∴ curve approaches 0 as $x \to \infty$

v)



(1) asymptotes and centre

(1) curve/graph.

c) i) PQ: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{ap^2 - aq^2}{2ap - 2aq}$ (1)

$= \frac{p+q}{2}$

$y - y_1 = m(x - x_1)$ using $P(2ap, ap^2)$

$y - ap^2 = \left(\frac{p+q}{2}\right)(x - 2ap)$

$y - ap^2 = \left(\frac{p+q}{2}\right)x - \frac{2ap^2}{2} - \frac{2apq}{2}$ (1)

$y = \left(\frac{p+q}{2}\right)x - apq$

∴ $2y = (p+q)x - 2apq$.

ii) sub in $(0, -2a)$

∴ $-4a = 0 - 2apq$. (1)

$pq = 2$.

iii) $x = -apq(p+q)$

$= -2a(p+q)$

$p+q = \frac{x}{-2a}$ (1)

$y = a(p^2 + q^2 + pq + 2)$

$= a((p+q)^2 - pq + 2)$

$= a\left(\left(\frac{-x}{2a}\right)^2 - 2 + 2\right)$

$= \frac{x^2}{4a}$

∴ $x^2 = 4ay$. (1)

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Question 14 **BOS#:**________________

a) $\cos^2 A + \cos^2(120° + A) + \cos^2(120° - A)$

$= \cos^2 A + \frac{1}{2}(\cos 2(120° + A) + 1) + \frac{1}{2}(\cos(2(120 - A)) + 1)$

$= \cos^2 A + \frac{1}{2}\cos(240 + 2A) + \frac{1}{2} + \frac{1}{2}\cos(240 - 2A) + \frac{1}{2}$

$= 1 + \cos^2 A + \frac{1}{2}(\cos 240 \cos 2A - \sin 240 \sin 2A)$
$+ \frac{1}{2}(\cos 240 \cos 2A + \sin 240 \sin 2A)$

$= 1 + \cos^2 A + \cos 240 \cos 2A$ $(\cos 240 = -\frac{1}{2})$

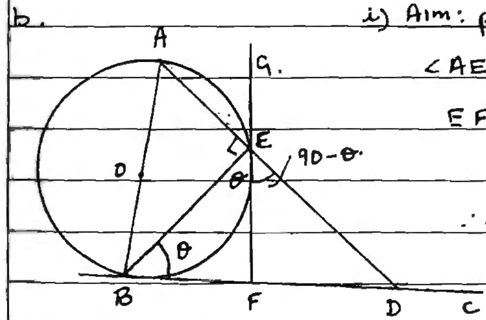
$= 1 + \cos^2 A - \frac{1}{2}(2\cos^2 A - 1)$

$= 1 + \cos^2 A - \cos^2 A + \frac{1}{2}$ (2) working towards...

$= \frac{3}{2}$ many methods

$\left[\frac{3}{2}\cos^2 \alpha \text{ (1) mark only}\right]$ (1) final answer

b.



i) Aim: prove $\angle FED = 90° - \theta$. (2)

$\angle AEB = 90°$ ($\angle$ in a semicircle)

$EF = FB$ (tangents from an external point are equal)

$\therefore \angle FBE = \angle FEB$ ($\angle$s opposite equal sides are equal)

$\therefore A\hat{E}B + B\hat{E}F + F\hat{E}D = 180°$ ($\angle$ sum straight line)

$\angle FED = 180 - 90 - \theta$

$= 90 - \theta$

ii) Prove: $BF = FD$.

$\angle EFD = 2\theta$ (external $\angle$ of $\triangle BEF$ equal to interior opposite $\angle$s) (2)

then $\angle FDE = 180 - E\hat{F}D - F\hat{E}D$ ($\angle$ sum $\triangle$)

$= 180 - (90 - \theta) - 2\theta$

$= 90 - \theta$

$\therefore F\hat{D}E = F\hat{E}D$

and $FE = FD$ (sides opposite equal $\angle$s are equal)

but $BF = FE$ (proved above)

$\therefore \underline{BF = FD}$.

c. i) $x^2 = 4ay$ at $T(2at, at^2)$

$y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{x}{2a}$ (1) establishing gradient

at $x = 2at$ $m_1 = t$ $m_2 = -\frac{1}{m_1}$

$= -\frac{1}{t}$.

normal: $y - y_1 = m(x - x_1)$

$y - at^2 = -\frac{1}{t}(x - 2at)$ (1) this line

$ty - at^3 = -x + 2at$

$x + ty = 2at + at^3$.

ii) at X $y = 0$ $\therefore$ $x = 2at + at^3$ $(2at + at^3, 0)$

at Y $x = 0$ $\therefore$ $y = 2a + at^2$ $(0, 2a + at^2)$

length $TY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2at)^2 + (at^2 - 2a - at^2)^2}$

$= \sqrt{4a^2t^2 + 4a^2}$

$= 2a\sqrt{t^2 + 1}$

(similarly) length $TX = \sqrt{(2at + at^3 - 2at)^2 + (0 - at^2)^2}$

$= \sqrt{a^2t^6 + a^2t^4}$

$= at^2\sqrt{t^2 + 1}$ (1) method (ratio / distance ?)

$\therefore \frac{TX}{TY} = \frac{at^2\sqrt{t^2+1}}{2a\sqrt{t^2+1}}$

$= \frac{t^2}{2}$. (1) $\frac{t^2}{2}$.

d) in $\triangle ADC$ in $\triangle BDC$	① establish $\triangle$'s.
$\tan 45 = \frac{DC}{AC}$ $\tan\alpha = \frac{DC}{CB}$	① $AB^2 = 1+\cot^2\alpha$

$AC = 1$ $CB = \frac{1}{\tan\alpha} = \cot\alpha.$

i) now $A\hat{C}B = 90°$

in $\triangle ACB$

$AC^2 + CB^2 = AB^2$ (pythagoras' theorem)

$\therefore AB^2 = 1 + \cot^2\alpha$

$= \operatorname{cosec}^2\alpha$

$\therefore AB = \operatorname{cosec}\alpha.$

ii) since $A\hat{C}B = 90°$ ① for valid method.

and m is the midpoint of AB.

then $A\hat{C}B$ is the ∟ in a semicircle through A, C, B with diameter AB.

$\therefore CM = \frac{1}{2} AB$

$= \frac{1}{2}\operatorname{cosec}\alpha.$ ① $\frac{1}{2}\operatorname{cosec}\alpha$.

/15.

[①/2 for assuming $\alpha = 45$ and achieving correct answer].