2016 Preliminary
Assessment Task 3

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hour
- Write using black or blue pen
- Board-approved calculators may be used
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I Pages 2 – 4

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 5 – 12

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer page in the writing booklet for Questions 1 – 10.

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1 A parabola has parametric equations $x = -3t^2$ and $y = 6t$, what are the coordinates of its focus?

(A) $(0,3)$ (B) $(0,-3)$ (C) $(3,0)$ (D) $(-3,0)$

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2 How many arrangements of all of the letters of the word TRIGONOMETRY are possible?

(A) 59 875 200 (B) 119 750 400

(C) 239 500 800 (D) 479 001 600

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3 What is the exact value of $\tan(\theta - 180°)$, if $\cos\theta = -\frac{3}{4}$ and $\tan\theta > 0$?

(A) $-\frac{\sqrt{7}}{3}$ (B) $\frac{\sqrt{7}}{3}$ (C) $-\frac{3}{\sqrt{7}}$ (D) $\frac{3}{\sqrt{7}}$

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4 What is the size of the acute angle between the lines $2x - y = 0$ and $x + y = 0$, correct to the nearest degree?

(A) 18° (B) 19° (C) 71° (D) 72°

5 If $t = \tan\frac{\theta}{2}$, which of the following trigonometric ratios is equivalent to $\frac{1-t^4}{2t-2t^3}$?

(A) $\sin\theta$ (B) $\tan\theta$ (C) $\operatorname{cosec}\theta$ (D) $\cot\theta$

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6 If $x = 2at$ and $y = 3at^2$, which of the following is an expression for $\frac{dy}{dx}$?

(A) t (B) $2t$ (C) $3t$ (D) $6t$

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7 Given that $n! = n(n-1)(n-2)\times\cdots\times3\times2\times1$, which of the following expressions is equivalent to $\frac{1}{(n-1)!}+\frac{n^3+1}{(n+1)!}$?

(A) $\frac{n+1}{n!}$ (B) $\frac{n^2+1}{n!}$ (C) $\frac{n^2+2n+1}{n!}$ (D) $\frac{n^3+n^2+1}{n!}$

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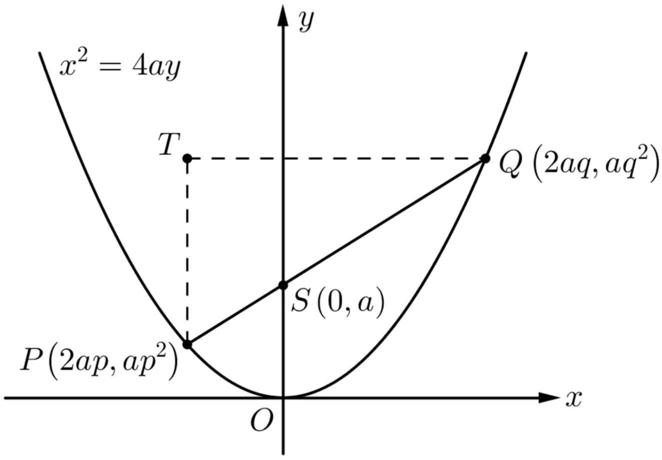
8 Given $x = \sqrt{2\cos2\theta}$ and $y = 3\sin^2\theta$, which of the following equations is correct?

(A) $y = \frac{3}{2}\left(1-x^2\right)$ (B) $y = \frac{3}{2}\left(2-x^2\right)$

(C) $y = \frac{3}{4}\left(1-x^2\right)$ (D) $y = \frac{3}{4}\left(2-x^2\right)$

9 $P\left(2ap, ap^2\right)$ and $Q\left(2aq, aq^2\right)$ are points on the parabola $x^2 = 4ay$. PQ is a focal chord of this parabola.

T is another point such that PT and QT are parallel to the y-axis and x-axis respectively, as shown below.



Which of the following equations best represents the locus of T?

(A) $xy = 4a^2$

(B) $xy = 4a^3$

(C) $x^2 y = 4a^2$

(D) $x^2 y = 4a^3$

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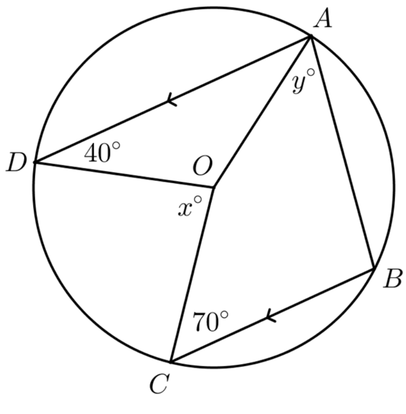
10 A, B, C and D are points on a circle with centre O. AD is parallel to BC as shown. It is given that $\angle ADO = 40^\circ$ and $\angle BCO = 70^\circ$.



What are the values of x and y?

(A) $x = 80$ and $y = 15$

(B) $x = 80$ and $y = 35$

(C) $x = 110$ and $y = 15$

(D) $x = 110$ and $y = 35$

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate section of the writing booklet. Extra writing paper is available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 section of the writing booklet.

(a) $A(-3,6)$ and $B(1,2)$ are two points. Find the coordinates of the point $P(x,y)$ which divides the interval *AB* externally in the ratio $3:1$. **2**

(b)

(i) Express $\sqrt{3}\sin\theta+\cos\theta$ in the form $A\sin(\theta+\alpha)$, where $A>0$ and $0^\circ<\alpha<90^\circ$. **2**

(ii) Hence, or otherwise, solve $\sqrt{3}\sin\theta+\cos\theta=1$ for $0^\circ\le\theta\le 360^\circ$. **2**

(c) The lines $y=mx$ and $y=2mx$, where $m>0$, are inclined to each other at an angle θ such that $\tan\theta=\dfrac{1}{3}$.

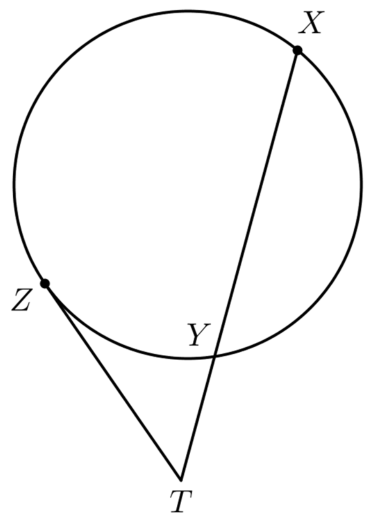
(i) Show that $2m^2-3m+1=0$. **2**

(ii) Hence find the possible values of *m*. **1**

Question 11 continues over the page

(d) Solve $\dfrac{2x-1}{x} \geq x$. **3**

(e) In the following diagram, X, Y and Z are concyclic points. The tangent at Z meets XY produced at T.



(i) Given that $TZ = 24$ cm, $XY = 14$ cm and $TY = x$ cm, find the value of x. **2**

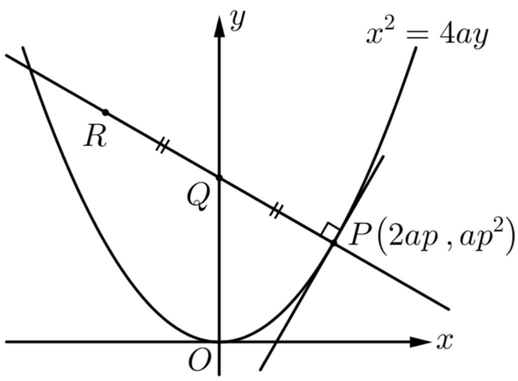
(ii) Calculate the length of YZ, given that YZ is the diameter of the circle passing through T, Y and Z. **1**

End of Question 11

Question 12 (15 marks) Use the Question 12 section of the writing booklet.

(a) The diagram below shows a variable point $P\left(2ap, ap^2\right)$ on parabola $x^2 = 4ay$.

The normal to the parabola at *P* intersects the *y*-axis at *Q*. *R* is a point on the normal such that *Q* is the midpoint of *PR*.



(i) Show that the equation of the normal at *P* has equation $x + py - 2ap - ap^3 = 0$. **2**

(ii) Find the coordinates of *Q*. **1**

(iii) Show that the locus of the point *R* is another parabola. Find the equation of this parabola in Cartesian form. **2**

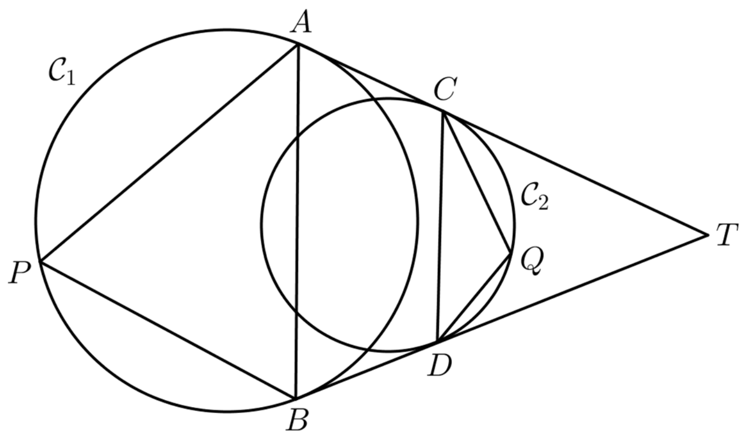
(iv) Hence state the coordinates of the vertex of the parabola defined by the locus of the point *R*. **1**

(b) Sketch the region on the number plane where the following inequalities hold simultaneously, showing any points of intersection: **3**

$$y \le \frac{3}{|x|} \text{ and } y \ge \frac{|x|}{3}$$

Question 12 continues over the page

(c) In the diagram below, AC and BD are tangents to both circles $\mathcal{C}_1$ and $\mathcal{C}_2$. AC and BD produced, meet at T. P and Q are points on the circumference of the circles $\mathcal{C}_1$ and $\mathcal{C}_2$ respectively.



Copy or trace the diagram into your answer booklet.

(i) Show that $AC = BD$. **1**

(ii) Hence show that $AB \parallel CD$. **2**

(iii) Prove that $\angle APB + \angle CQD = 180°$. **2**

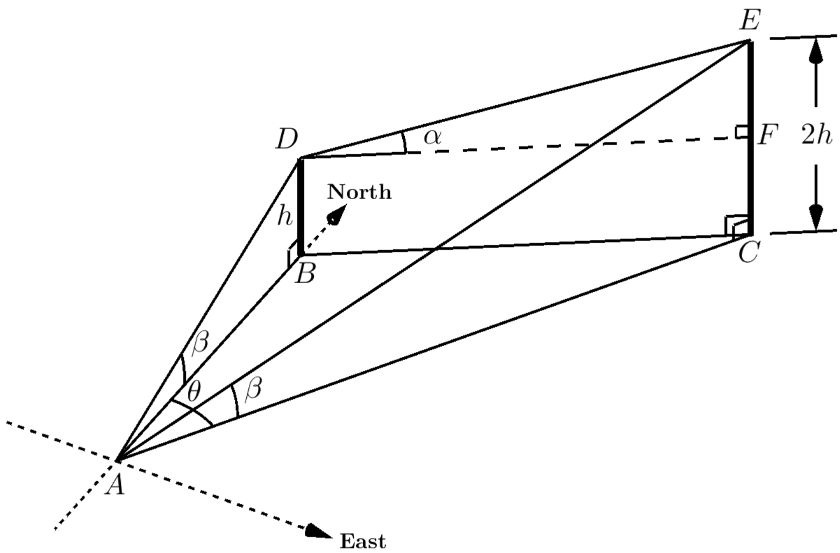
(iv) Explain why $ACDB$ is a cyclic quadrilateral. **1**

End of Question 12

Question 13 (15 marks) Use the Question 13 section of the writing booklet.

(a) A man standing at point A can see two vertical towers, BD and CE. A, B and C are on level ground, with B due north of A, and C on a bearing of θ from A. The height of tower BD is h metres, while tower CE is twice as tall.

The angle of elevation from A to the top of each tower is β. The angle of elevation to the top of tower CE from the top of tower BD is α as shown.



(i) Show that $AC = 2h\cot\beta$. **1**

(ii) Find similar expressions for AB and BC. **1**

(iii) Hence, or otherwise, show that $\cos\theta = \dfrac{5\cot^2\beta - \cot^2\alpha}{4\cot^2\beta}$. **2**

Question 13 continues over the page

(b) Timothy, Kate and six other people go through a doorway one at a time.

(i) In how many ways can the eight people go through the doorway if there are no restrictions? **1**

(ii) In how many ways can the eight people go through the doorway if Timothy goes through the doorway after Kate with no one in between? **1**

(iii) Find the number of ways in which the eight people can go through the doorway if Timothy goes through the doorway after Kate. **1**

(c) Ten people want to dine at a local restaurant.

(i) In how many ways can they all sit on around a circular table? **1**

(ii) When they arrived at the restaurant however, the only seating available for them is at two circular tables, one that seats six people, and another that seats four. How many different seating arrangements are now possible? **2**

(iii) Given this two-table seating arrangement, Jack and Jill insists on sitting on the same table, in how many different ways can this be done? **2**

(d) A cubic function whose equation is $f\left(x\right) = -2x^3 + px^2 - qx + 5$, where p and q are constants, has at most 2 stationary points.

(i) Show that if $f\left(x\right)$ is to have any stationary points, then $p^2 - 6q \geq 0$. **2**

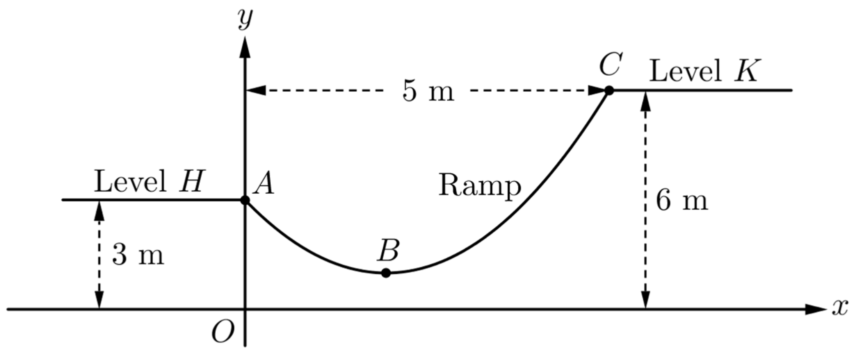
(ii) Describe what happens to the stationary points when $p^2 - 6q = 0$. **1**

End of Question 13

Question 14 (15 marks) Use the Question 14 section of the writing booklet.

(a) Prove that $(\cos 3\phi - \cos\phi)\cos\phi + (\sin 3\phi + \sin\phi)\sin\phi = 0$. **2**

(b) The city council of Gausstown has decided to build a skateboard ramp for its teenagers. The structure consists of two levels, *H* and *K*, and the ramp itself as shown in the diagram. Gausstown engineers believe that if the ramp has a gradient greater than 3 at any point, the ramp will be too dangerous to use. Below is a cross-section of the proposed ramp.



Taking the *x*-axis as ground level, the ramp *ABC* can be modelled by the following equation:

$$y = \frac{8}{15}x^2 - \frac{31}{15}x + 3,\ 0 \le x \le 5 \text{ (Do NOT prove this.)}$$

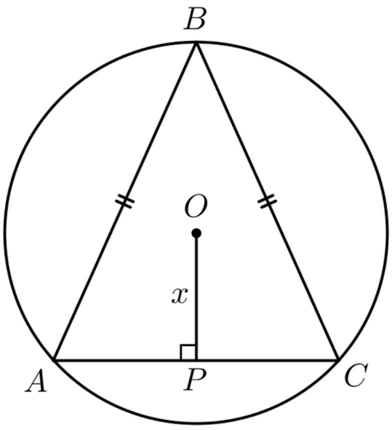
(i) If point *B* is the lowest point along the ramp, how far off the ground is point *B*? **1**

(ii) Justify with calculations as to why this proposed design is not safe for use. **3**

(iii) The engineers plan to move point *C* closer to point *B* along the ramp until it becomes safe to use. What is the minimum amount that Level *K* must be lowered for this to happen? **2**

Question 14 continues over the page

(c) An isosceles triangle ABC, where $AB = BC$, is inscribed in a circle of radius 10 cm and centre O. It is given that $OP = x$ cm and $AC \perp OP$.



(i) Show that the area A, of ΔABC, is given by $A = (10 + x)\sqrt{100 - x^2}$. **2**

(ii) Show that $\dfrac{dA}{dx} = \dfrac{100 - 10x - 2x^2}{\sqrt{100 - x^2}}$. **2**

(iii) Hence prove that the triangle with maximum area is equilateral. **3**

End of Paper

YEAR 11 ASSESSMENT TASK 3 2016
MATHEMATICS EXTENSION 1
MARKING GUIDELINES

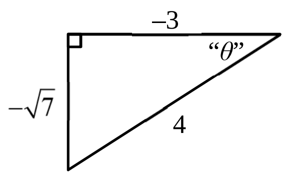
Section I

Multiple-choice Answer Key

Question	Answer
1	D
2	A
3	B
4	D
5	C

Question	Answer
6	C
7	B
8	D
9	D
10	D

Questions 1 – 10

Sample solution	
1.	$\left.\begin{array}{r} x=-3t^2 \\ y=6t \end{array}\right\} y^2=-12x$ $\therefore \text{Vertex}=(-3,0)$
2.	$\text{Number of arrangements}=\frac{12!}{2!2!2!}$ $=59\ 875\ 200$
3.	$\cos\theta=-\frac{3}{4}$ and $\tan\theta>0$ $\therefore \theta$ is in the third quadrant.  $\tan(\theta-180°)=\tan(\theta+180°)$ (adding a revolution) $=\tan\theta$ $=\frac{\sqrt{7}}{3}$
4.	$2x-y=0 \qquad x+y=0$ $y=2x \qquad y=-x$ $\tan\theta=\left\lvert\frac{2-(-1)}{1+2\times(-1)}\right\rvert$ $\tan\theta=3$ $\theta=\tan^{-1}3$ $\theta=72°$ (nearest degree)
5.	$\frac{1-t^4}{2t-2t^3}=\frac{\cancel{(1-t^2)}(1+t^2)}{2t\cancel{(1-t^2)}}$ $=\frac{1}{\sin\theta}\quad\left(\text{if } t=\tan\frac{\theta}{2}\right)$ $=\operatorname{cosec}\theta$

6.

$$x = 2at \qquad y = 3at^2 \qquad \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dx}{dt} = 2a \qquad \frac{dy}{dt} = 6at \qquad = 6at \times \frac{1}{2a}$$

$$= 3t$$

7.

$$\frac{1}{(n-1)!} + \frac{n^3+1}{(n+1)!} = \frac{n}{n!} + \frac{(n+1)(n^2-n+1)}{(n+1)!}$$

$$= \frac{n}{n!} + \frac{n^2-n+1}{n!}$$

$$= \frac{n^2+1}{n!}$$

8.

$$x = \sqrt{2\cos 2\theta} \qquad \cos 2\theta = 1 - 2\sin^2\theta$$

$$x^2 = 2\cos 2\theta \qquad \frac{x^2}{2} = 1 - \frac{2y}{3}$$

$$\frac{x^2}{2} = \cos 2\theta \qquad \frac{2y}{3} = 1 - \frac{x^2}{2}$$

$$y = 3\sin^2\theta \qquad y = \frac{3}{2}\left(\frac{2-x^2}{2}\right)$$

$$\frac{y}{3} = \sin^2\theta \qquad = \frac{3}{4}\left(2-x^2\right)$$

9.

$$T = \left(2ap, aq^2\right) \qquad y = aq^2$$

$$= a \times \left(\frac{-1}{p}\right)^2$$

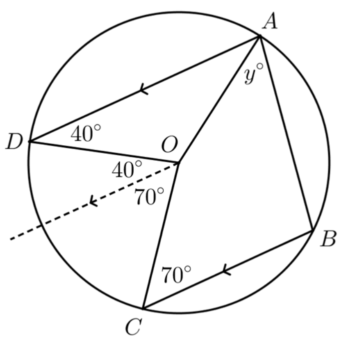
$pq = -1$ (PQ is a focal chord) $\qquad = \dfrac{a}{p^2}$

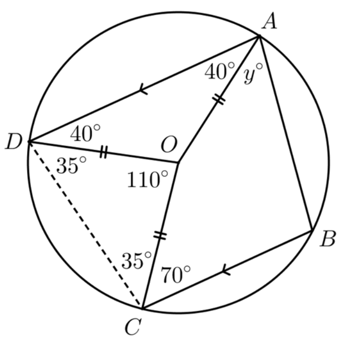
$$q = \frac{-1}{p} \qquad = \frac{a}{\left(\frac{x}{2a}\right)^2}$$

$$x = 2ap \qquad = \frac{4a^3}{x^2}$$

$$\frac{x}{2a} = p \qquad x^2y = 4a^3$$

10.





Construct a ray from *O*, parallel to *AD* and *BC*, as shown in the left hand diagram.
Using alternate angles and a summation, $x = 110$

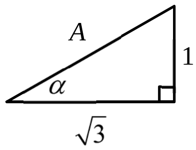
Construct DC as shown in the right hand diagram. Using isosceles triangles, angles are as shown, then:

$35° + 70° + 40° + y° = 180°$ (opposite angles of a cyclic quadrilateral are supplementary)

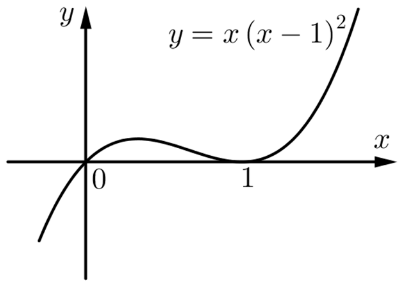
$$y = 35$$

Section II

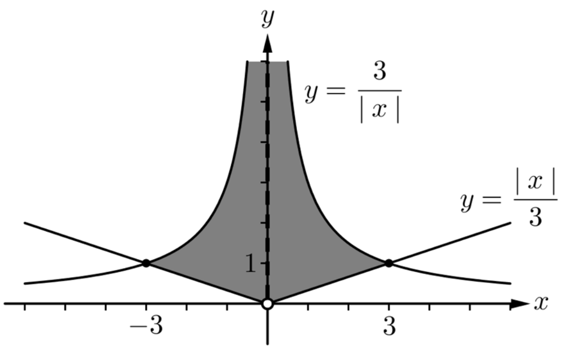
Question 11

	Sample solution	Suggested marking criteria
(a)	$A(-3,6) \qquad B(1,2)$ $3 : -1$ $P=\left(\dfrac{3\times1+(-1)\times(-3)}{3+(-1)},\dfrac{3\times2+(-1)\times6}{3+(-1)}\right)$ $=(3,0)$	• 2 – correct solution • 1 – uses the division of an interval formula, with substitution
(b) (i)	$\sqrt{3}\sin\theta+\cos\theta\equiv A\sin(\theta+\alpha)$ $=A\sin\theta\cos\alpha+A\cos\theta\sin\alpha$ $A\cos\alpha=\sqrt{3} \qquad A\sin\alpha=1$ $\cos\alpha=\dfrac{\sqrt{3}}{A} \qquad \sin\alpha=\dfrac{1}{A}$  $A^2=1^2+\left(\sqrt{3}\right)^2$ $=4$ $A=2 \quad (A>0)$ $\tan\alpha=\dfrac{1}{\sqrt{3}}$ $\alpha=30° \quad (0°<\alpha<90°)$ $\therefore \sqrt{3}\sin\theta+\cos\theta=2\sin(\theta+30°)$	• 2 – correct solution • 1 – correct value of A or α – attempts to use the auxiliary angle method, or equivalent merit
(b) (ii)	$\sqrt{3}\sin\theta+\cos\theta=1, \quad 0°\le\theta\le360°$ $2\sin(\theta+30°)=1, \quad 30°\le\theta+30°\le390°$ $\sin(\theta+30°)=\dfrac{1}{2}$ $\theta+30°=30°, 150°, 390°$ $\theta=0°, 120°, 360°$	• 2 – correct solution • 1 – attempts to evaluate θ in the correct domain – obtains a subset of the correct answers
(c) (i)	$\tan\theta=\left\|\dfrac{m_1-m_2}{1+m_1m_2}\right\|$ $\dfrac{1}{3}=\left\|\dfrac{2m-m}{1+2m\times m}\right\|$ $\dfrac{1}{3}=\left\|\dfrac{m}{1+2m^2}\right\|$ $\dfrac{1}{3}=\dfrac{m}{1+2m^2} \quad \left(\text{Since } m>0 \text{ and } 1+2m^2>0\right)$ $1+2m^2=3m$ $2m^2-3m+1=0$	• 2 – correct solution, justifying the removal of the absolute value signs • 1 – establishes $\dfrac{1}{3}=\left\|\dfrac{m}{1+2m^2}\right\|$
(c) (ii)	$2m^2-3m+1=0$ $2m^2-2m-m+1=0$ $2m(m-1)-(m-1)=0$ $(m-1)(2m-1)=0$ $\therefore m=1 \quad \text{or} \quad m=\dfrac{1}{2}$	• 1 – correct solution

Question 11 (continued)

Sample solution			Suggested marking criteria
(d)		$\frac{2x-1}{x} \geq x$ $x(2x-1) \geq x^3$ $2x^2 - x \geq x^3$ $0 \geq x^3 - 2x^2 + x$ $x(x^2 - 2x + 1) \leq 0$ $x(x-1)^2 \leq 0$ $x < 0$ or $x = 1$ 	• 3 – correct solution • 2 – obtains "$x \leq 0$ or $x = 1$" as the solution, or equivalent merit – recognises that $x = 0$ is not part of the solution (eg. stating "$x < 0$" only as the solution) • 1 – obtains $x \leq 0$ – attempts to solve the inequality using a valid method
(e)	(i)	$TY \times TX = TZ^2$ (the square of the tangent is equal to the product of the intercepts of secants from a point) $x(x+14) = 24^2$ $x^2 + 14x - 576 = 0$ $(x+32)(x-18) = 0$ $x = 18 \quad (x > 0)$	• 2 – correct solution • 1 – forms an appropriate quadratic equation
	(ii)	If YZ is the diameter of the circle passing through T, Y and Z, then $\angle YTZ = 90°$ (angle in a semicircle) By Pythagoras' Theorem: $YZ^2 = TY^2 + TZ^2$ $YZ^2 = 18^2 + 24^2$ $YZ = 30 \quad (YZ > 0)$	• 1 – correct solution

Question 12

Sample solution		Suggested marking criteria
(a)	(i) $x^2 = 4ay$ $\frac{x^2}{4a} = y$ $\frac{x}{2a} = \frac{dy}{dx}$ At P, $\frac{dy}{dx} = \frac{2ap}{2a}$ $= p$ $\therefore m_N = -\frac{1}{p}$ $y - ap^2 = -\frac{1}{p}(x - 2ap)$ $py - ap^3 = -x + 2ap$ $x + py - 2ap - ap^3 = 0$	• 2 – correct solution • 1 – finds the gradient of the normal at P
	(ii) Let $x = 0$: $x + py - 2ap - ap^3 = 0$ $py - 2ap - ap^3 = 0$ $py = 2ap + ap^3$ $y = 2a + ap^2$ $\therefore Q = (0, 2a + ap^2)$	• 1 – correct solution
	(iii) Let $R = (x, y)$ $Q(0, 2a + ap^2) = \left(\frac{x + 2ap}{2}, \frac{y + ap^2}{2}\right)$ $\frac{x + 2ap}{2} = 0$ $x + 2ap = 0$ $x = -2ap$ $-\frac{x}{2a} = p$ $\frac{y + ap^2}{2} = 2a + ap^2$ $y + ap^2 = 4a + 2ap^2$ $y = 4a + ap^2$ $y = 4a + a\left(-\frac{x}{2a}\right)^2$ $= 4a + \frac{x^2}{4a}$ $y - 4a = \frac{x^2}{4a}$ $x^2 = 4a(y - 4a)$	• 2 – correct solution • 1 – establishes the equation – eliminating the parameter p
	(iv) $x^2 = 4a(y - 4a)$ Vertex $= (0, 4a)$	• 1 – correct solution
(b)	Points of intersection: $\frac{3}{\lvert x \rvert} = \frac{\lvert x \rvert}{3}$ $9 = (\lvert x \rvert)^2$ $9 = x^2$ $\pm 3 = x$ $y = \frac{3}{\lvert \pm 3 \rvert}$ $= 1$  	• 3 – correct region, must exclude the origin and the y-axis • 2 – recognises that the region is the intersection of a region of the hyperbolic function and a region of the absolute value function, or equivalent merit, showing correct scale. • 1 – correctly sketches the region of $y \le \frac{3}{\lvert x \rvert}$ – correctly sketches the region of $y \ge \frac{\lvert x \rvert}{3}$ – sketches the lines $y = \frac{\lvert x \rvert}{3}$ and $y = \frac{3}{\lvert x \rvert}$

Question 12 (continued)

Sample solution		Suggested marking criteria
(c)	(i) $TA = TB$ (tangents from an external point to $\mathcal{C}_1$ are equal) $TC = TD$ (tangents from an external point to $\mathcal{C}_2$ are equal) $AC = TA - TC$ $= TB - TD$ $= BD$	• 1 – correct solution
	(ii) $AC = BD$ (from (i)) $TC = TD$ (tangents from an external point to $\mathcal{C}_2$ are equal) $\dfrac{AC}{BD} = \dfrac{TC}{TD}$ $(=1)$ $\therefore AB \parallel CD$ (ratio of intercepts are equal)	• 2 – correct solution • 1 – significant progress towards showing $AB \parallel CD$
	(iii) $\angle APB = \angle CAB$ (angle between tangent and chord at the point of contact is equal to the angle at the circumference in the alternate segment) $\angle CQD = \angle ACD$ (angle between tangent and chord at the point of contact is equal to the angle at the circumference in the alternate segment) $\angle CAB + \angle ACD = 180°$ (co-interior angles are supplementary, $AB \parallel CD$) $\angle APB + \angle CQD = 180°$	• 2 – correct solution • 1 – identifying a pair of equal angles using the alternate segment theorem
	(iv) $\angle CAB = \angle TCD$ (corresponding angles are equal, $AB \parallel CD$) $\angle CAB = \angle DBA$ (base angles of isosceles ΔTAB are equal, $TA = TB$) $\angle TCD = \angle DBA$ *ACDB* is a cyclic quadrilateral (exterior angle is equal to the interior opposite angle)	• 1 – correct solution

Question 13

Sample solution			Suggested marking criteria
(a)	(i)	In ΔAEC: $\tan\beta = \frac{EC}{AC}$ $\tan\beta = \frac{2h}{AC}$ $AC = \frac{2h}{\tan\beta}$ $= 2h\cot\beta$	• 1 – correct solution
	(ii)	In ΔADB: $\tan\beta = \frac{DB}{AB}$ $\tan\beta = \frac{h}{AB}$ $AB = \frac{h}{\tan\beta}$ $= h\cot\beta$ In ΔDFE, $\tan\alpha = \frac{EF}{DF}$ $\tan\alpha = \frac{h}{DF}$ $DF = \frac{h}{\tan\alpha}$ $= h\cot\alpha$ $BC = DF = h\cot\alpha$	• 1 – correct expressions for *AB* and *BC*
	(iii)	In ΔABC, $\cos\theta = \frac{AB^2 + AC^2 - BC^2}{2\times AB\times AC}$ $= \frac{(h\cot\beta)^2 + (2h\cot\beta)^2 - (h\cot\alpha)^2}{2\times h\cot\beta\times 2h\cot\beta}$ $= \frac{\cancel{h^2}\cot^2\beta + 4\cancel{h^2}\cot^2\beta - \cancel{h^2}\cot^2\alpha}{4\cancel{h^2}\cot^2\beta}$ $= \frac{5\cot^2\beta - \cot^2\alpha}{4\cot^2\beta}$	• 2 – correct solution • 1 – uses the cosine rule, showing appropriate substitutions
(b)	(i)	Number of ways $= 8!$ $= 40320$	• 1 – correct solution
	(ii)	Treating Kate and Timothy as one entity (in that order): Number of ways $= 7!$ $= 5040$	• 1 – correct solution
	(iii)	Either Timothy will go through the door after Kate, or the other way around, therefore: Number of ways $= \frac{1}{2}\times 40320$ $= 20160$	• 1 – correct solution

Question 13 (continued)

Sample solution			Suggested marking criteria
(c)	(i)	Number of ways $= 9!$ $= 362880$	• 1 – correct solution
	(ii)	Number of ways $= {}^{10}C_6 \times 5! \times 3!$ $= 151200$	• 2 – correct solution • 1 – uses a combination to select people to sit around a table
	(iii)	Number of ways $= \left({}^{8}C_4 \times 5! \times 3!\right) + \left({}^{8}C_2 \times 3! \times 5!\right)$ $= 70560$	• 2 – correct solution • 1 – correct evaluates the number of ways one of the two tables can be filled if Jack and Jill sits on that table
(d)	(i)	$f(x) = -2x^3 + px^2 - qx + 5$ $f'(x) = -6x^2 + 2px - q$ Stationary points exist if $f'(x) = 0$ has solutions: $f'(x) = 0$ $-6x^2 + 2px - q = 0$ $\Delta \geq 0$ $(2p)^2 - 4 \times (-6) \times (-q) \geq 0$ $4p^2 - 24q \geq 0$ $p^2 - 6q \geq 0$	• 2 – correct solution • 1 – obtains the equation $-6x^2 + 2px - q = 0$ – significant progress towards a correct solution by using the discriminant
	(ii)	Stationary points exist if $p^2 - 6q \geq 0$. There will be exactly two stationary points if $p^2 - 6q > 0$. When $p^2 - 6q = 0$, the two stationary points will coincide and become a horizontal point of inflexion.	• 1 – correct description

Question 14

Sample solution			Suggested marking criteria
(a)	(i)	$$\begin{aligned}\text{LHS} &= (\cos 3\phi - \cos\phi)\cos\phi + (\sin 3\phi + \sin\phi)\sin\phi \\ &= \cos 3\phi\cos\phi - \cos^2\phi + \sin 3\phi\sin\phi + \sin^2\phi \\ &= \cos(3\phi - \phi) - (\cos^2\phi - \sin^2\phi) \\ &= \cos 2\phi - \cos 2\phi \\ &= 0 \\ &= \text{RHS}\end{aligned}$$	• 2 – correct solution • 1 – recognises the cosine of a difference – recognises the cosine of a double angle
(b)	(i)	At the vertex: $$x = -\frac{b}{2a} = -\frac{\left(-\frac{31}{15}\right)}{2\times\frac{8}{15}} = \frac{31}{16}$$ $$y = \frac{8}{15}\times\left(\frac{31}{16}\right)^2 - \frac{31}{15}\times\frac{31}{16} + 3 = \frac{479}{480}$$ Point B is $\frac{479}{480}$ metres off the ground.	• 1 – correct solution
	(ii)	$$y = \frac{8}{15}x^2 - \frac{31}{15}x + 3$$ $$\frac{dy}{dx} = \frac{16}{15}x - \frac{31}{15}$$ By the geometry of the parabola, the gradient is largest at point C: $$\frac{dy}{dx} = \frac{16}{15}\times 5 - \frac{31}{15} = \frac{49}{15}$$ The gradient here is greater than 3, therefore the proposed ramp design is not safe for use.	• 3 – correct solution • 2 – uses $\frac{dy}{dx}$ in an attempt to justify why the ramp is not safe for use • 1 – correct expression for $\frac{dy}{dx}$
	(iii)	For the ramp to be safe for use, the gradient at C can at most equal to 3: $$\begin{aligned}\frac{dy}{dx} &= 3 \\ \frac{16}{15}x - \frac{31}{15} &= 3 \\ 16x - 31 &= 45 \\ 16x &= 76 \\ x &= \frac{19}{4}\end{aligned}$$ $$y = \frac{8}{15}\times\left(\frac{19}{4}\right)^2 - \frac{31}{15}\times\frac{19}{4} + 3 = 5\frac{13}{60}$$ Therefore, Level K must be lowered by a minimum of $\frac{47}{60}$ metres.	• 2 – correct solution • 1 – correctly solves $\frac{dy}{dx} = 3$

Question 14 (continued)

Sample solution			Suggested marking criteria
(c)	(i)	By Pythagoras' Theorem in ΔAOP: $AO^2 = OP^2 + AP^2$ $10^2 = x^2 + AP^2$ $100 - x^2 = AP^2$ $\sqrt{100 - x^2} = AP \quad (AP > 0)$ $A = \frac{1}{2} \times AC \times BP$ $= \frac{1}{2} \times 2\sqrt{100 - x^2} \times (10 + x)$ $= (10 + x)\sqrt{100 - x^2}$	• 2 – correct solution • 1 – correct expression for *AP*
	(ii)	$\frac{dA}{dx} = \sqrt{100 - x^2} + \frac{1}{2}(100 - x^2)^{-\frac{1}{2}} \times (-2x) \times (10 + x)$ $= \sqrt{100 - x^2} - \frac{x(10 + x)}{\sqrt{100 - x^2}}$ $= \frac{100 - x^2 - 10x - x^2}{\sqrt{100 - x^2}}$ $= \frac{100 - 10x - 2x^2}{\sqrt{100 - x^2}}$	• 2 – correct solution • 1 – attempts to use the product rule to find an expression for $\frac{dA}{dx}$
	(iii)	Area is a maximum when: $\frac{dA}{dx} = 0$ $\frac{100 - 10x - 2x^2}{\sqrt{100 - x^2}} = 0$ $100 - 10x - 2x^2 = 0$ $50 - 5x - x^2 = 0$ $(5 - x)(x + 10) = 0$ $\therefore x = 5 \quad (x > 0)$ In ΔAOP, when $x = 5$: $AP^2 + 5^2 = 10^2$ $AP^2 = 100 - 25$ $AP = \sqrt{75} \quad (AP > 0)$ $AP = 5\sqrt{3}$ $AC = 2 \times AP$ $= 10\sqrt{3}$ In ΔABP, when $x = 5$: $AB^2 = (5\sqrt{3})^2 + 15^2$ $AB^2 = 75 + 225$ $AB = \sqrt{300} \quad (AP > 0)$ $AB = 10\sqrt{3}$ $AB = BC = AC = 10\sqrt{3}$ Therefore, when the area is a maximum, ΔABC is equilateral.	• 3 – correct solution • 2 – significant progress towards showing the equilateral triangle – showing the area is a maximum at $x = 5$ • 1 – correctly evaluates *x* at which the area is a minimum