Name: ______________________________

Teacher: ______________________________

Class: ______________________________

FORT STREET HIGH SCHOOL

2006

PRELIMINARY SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: FINAL PRELIMINARY EXAMINATION

Mathematics Extension I

TIME ALLOWED: 2 HOURS

Outcomes Assessed	Questions	Marks
Demonstrates appropriate mathematical techniques in basic algebra, equations and geometry.	1, 2	
Manipulates algebraic expressions to solve problems from topic areas such as functions, trigonometry and quadratics.	3, 5	
Demonstrates skills in the processes of calculus and applies them appropriately.	4, 6	
Synthesises mathematical solutions to harder problems and communicates them in appropriate form.	7	

Question	1	2	3	4	5	6	7	Total	%
Marks	/12	/12	/12	/12	/12	/12	/12	/84	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated on the right thus: [1]
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- **Each new question is to be started in a new booklet**

Question One: 12 marks (Start a new booklet)

a) Express $\dfrac{1+\sqrt{8}}{1-3\sqrt{2}}$ with a rational denominator. [2]

b) If $H=(1-k)^n$, find the value of k (to 4 significant figures) given $H=0.13$ and $n=7$ [2]

c) Express $\dfrac{1-x^{-1}}{x^{-1}-x^{-2}}$ in its simplest form [2]

d) Find the values of θ, with $0^\circ \le \theta \le 360^\circ$, for which $2\cos\theta=-\sqrt{2}$ [2]

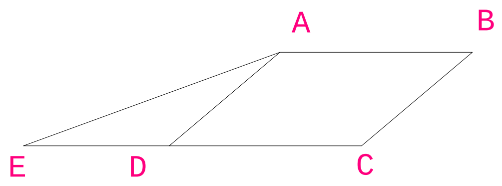
e) Find the value of $\dfrac{a^2b^3}{c^2}$ given $a=\left(\dfrac{3}{4}\right)^3$, $b=\left(\dfrac{2}{3}\right)^2$ and $c=\left(\dfrac{1}{2}\right)^4$ as a fraction. [2]

f) Sketch the graph of $y=|2x-1|$ [2]

<u>Question Two</u>: 12 marks (Start a new booklet)

a) Solve for x: $\dfrac{2}{x} > x - 1$. [3]

b) ABCD is a parallelogram with CD produced to E so that ED =AD.



Prove that $\angle ABC = 2\angle DEA$. [2]

c) Given $\log 2 = a$ and $\log 3 = b$, find $\log 2.25$ in terms of a and b. [2]

d) For the lines $y = 3x - 1$ and $x - 2y + 5 = 0$:

i) Find the acute angle between these lines. [2]

ii) Using the k-method or otherwise, find the equation of the line through the intersection of these lines that passes through $P(1,-1)$ [3]

<u>Question Three</u>: 12 marks (Start a new booklet)

a) The angle of depression from the top of a 120m cliff down to a boat in line with the bottom of the cliff is 35°. Calculate the distance of the boat from the base of the cliff (to the nearest m). [2]

b) The quadratic equation $3x^2 - 5x + 2 = 0$ has roots α and β. Find the value of

i) $(\alpha - 1)(\beta - 1)$ [1]

ii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ [2]

c) Find the coordinates of the point that divides the interval between $A(1, -6)$ and $B(-4, 3)$ externally in the ratio $3:2$. [2]

d) Solve $x^4 - 3x^2 - 4 = 0$ for x. [3]

e) Prove $\frac{\cos\theta(\sin\theta + \cos\theta)}{(1 + \sin\theta)(1 - \sin\theta)} = 1 + \tan\theta$ [2]

Question Four: 12 marks (Start a new booklet)

a) Differentiate with respect to x:

i) $x^3 - 5x^2 + 3$ [1]

ii) $(5 - 2x^2)^7$ [1]

iii) $\dfrac{2x}{1 + 4x}$ [2]

b) Find the gradient function for $f(x) = \dfrac{1}{x}$ from first principles. [3]

c) For the curve $y = 4 + 3x - x^2$, find the:

i) equation of the tangent at $x = 1$ (in General Form) [3]

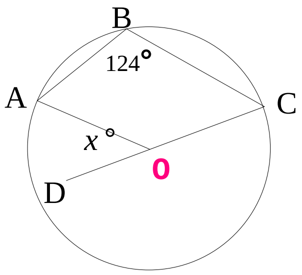
ii) find the exact distance of point $P(2, -1)$ from this tangent [2]

<u>Question Five</u>: 12 marks (Start a new booklet)

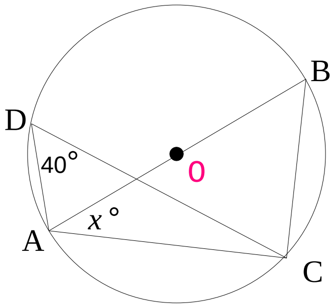
a) Find the values of a, b and c such that $2x^2 - 5x + 7$ can be expressed in the form $a(x+b)^2 + c$ [4]

b) Find the value of x, giving reasons.

i) [2]



ii) [2]



c) For what values of k does the quadratic $x^2 + (k+1)x - (k-1) = 0$ have distinct real roots? [4]

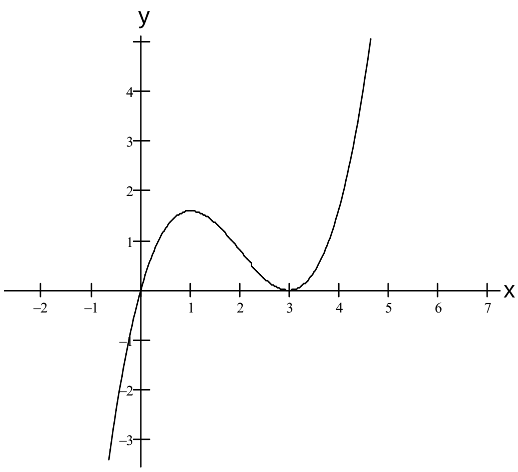
Question Six: 12 marks (Start a new booklet)

a) Points $P(2,4)$ and $Q(x_1, y_1)$ are on the parabola $y = x^2$:

i) Show the equation of the normal at point P is given by $x + 4y - 18 = 0$. [1]

ii) The tangent at Q is parallel to the normal at P. Find the coordinates of Q and the equation (in general form) of the tangent at Q. [3]

b) For the function $y = f(x)$, the graph of its derivative $y = f'(x)$ is shown below.



i) For what values of x is $f(x)$ rising? [1]

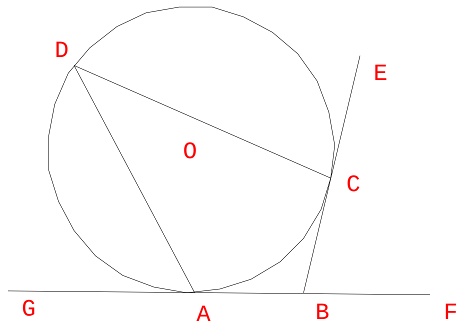
ii) What are the x-values of the stationary points of $f(x)$? [1]

iii) Given $x = -1$ is a root of $f(x)$, and the minimum value of $f(x)$ is -3, draw a neat sketch of a possible $y = f(x)$ for $-2 < x < 5$. [2]

c) For the curve given by $y = x^3 + 3x^2 - 9x - 7$, find any stationary points or points of inflection and determine their nature. [4]

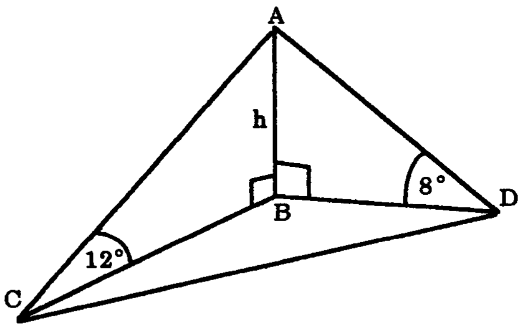
Question Seven: 12 marks (Start a new booklet)

a) A and C are two separate points on the circle, centre O, as shown in the diagram below. Tangents at A and C meet a B. D is a point on the circumference of the circle.



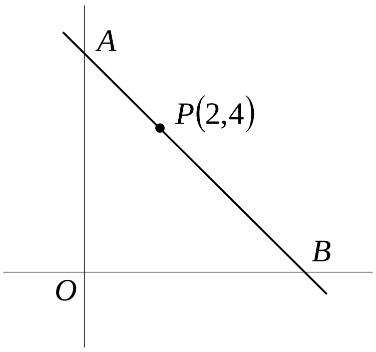
Prove that quadrilateral ABCD can never be cyclic. [4]

b) AB is a tower of height h metres. From points C and D in the same plane as the base of the tower, the angles of elevation to the top of the tower are 12° and 8° respectively, as shown in the diagram opposite. From the base of the tower, the bearing of points C and D are $229^\circ T$ and $187^\circ T$ respectively. Find the height of the tower (to the nearest metre) if C is 400m from D. [3]



(Question 7 continues over)

c) A straight line passing through the point P(2,4) cuts the positive axes at A and B.



i) Show the equation of the line is $y = mx - 2m + 4$, where m is the gradient of the line. [1]

ii) Show that points A and B, where the line meets the axes, have coordinates $(0, 4 - 2m)$ and $\left(\frac{2m-4}{m}, 0\right)$ respectively. [1]

iii) Find the value of m for which the area of triangle AOB is least. [3]

Solutions: Preliminary Final Exam Extension 1 2006.

Question One:

a) $= \dfrac{1-\sqrt{8}}{1-3\sqrt{2}} \times \dfrac{1+3\sqrt{2}}{1+3\sqrt{2}}$ **[1]**

$= \dfrac{(1-2\sqrt{2})(1+3\sqrt{2})}{(1-18)}$

$= \dfrac{1-2\sqrt{2}+3\sqrt{2}-12}{17}$

$= \dfrac{3\sqrt{2}-11}{17}$ **[1]**

b) $H = (1-k)^n$

i.e. $\sqrt[7]{0.13} = 1-k$ **[1]**

or $k = 1-\sqrt[7]{0.13}$

$\therefore \quad k = 0.252828189$

i.e. $k = 0.2528$ (to 4 sig. figs) **[1]**

c) $\dfrac{1-x^{-1}}{x^{-1}-x^{-2}}$

$= \dfrac{1-\frac{1}{x}}{\frac{1}{x}-\frac{1}{x^2}}$ **[1]**

$= \dfrac{\frac{x-1}{x}}{\frac{x-1}{x^2}}$

$= \dfrac{x-1}{x} \times \dfrac{x^2}{x-1}$

$= x$ **[1]**

d) $2\cos\theta = -\sqrt{2}$

$\cos\theta = \dfrac{-\sqrt{2}}{2}$

$\theta = \cos^{-1}\left(\dfrac{-1}{\sqrt{2}}\right)$

i.e. base ϕ is 45° **[1]**

Cos is negative in Q's 2 & 3

$\therefore \theta = 180-\phi, \ 180+\phi$

i.e. $\theta = 135^\circ, \ 225^\circ$ **[1]**

e) $a = \left(\dfrac{3}{4}\right)^3, \ b = \left(\dfrac{2}{3}\right)^2 \ c = \left(\dfrac{1}{2}\right)^4$

$\therefore \dfrac{a^2b^3}{c^2}$

$= \dfrac{\left(\left(\frac{3}{4}\right)^3\right)^2\left(\left(\frac{2}{3}\right)^2\right)^3}{\left(\left(\frac{1}{2}\right)^4\right)^2}$

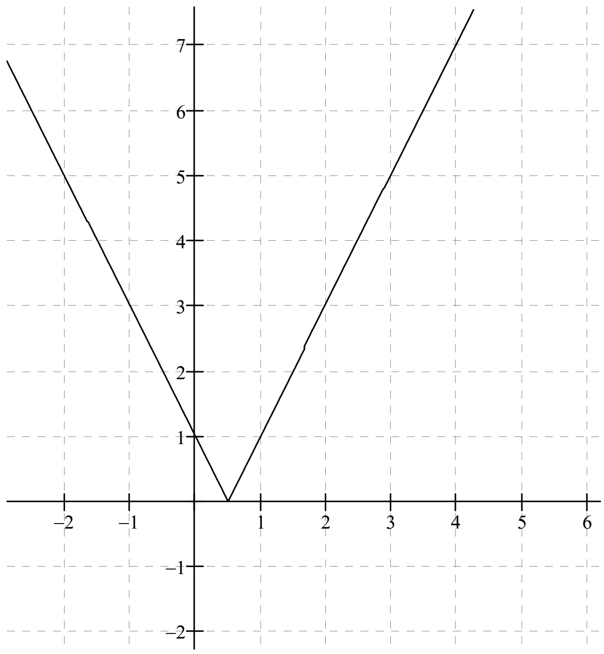
$= \dfrac{3^6}{4^6} \times \dfrac{2^6}{3^6} \div \dfrac{1}{2^8}$ **[1]**

$= \dfrac{3^6.2^6.2^8}{2^{12}.3^6}$

$= 2^2$

$= 4$ **[1]**

f) Sketch $y = |2x-1|$



"Vertex" correct **[1]**

Shape with correct y intercept **[1]**[2]

Question Two:

a) $\times x^2$: $2x > x^3 - x^2$

i.e. $x^3 - x^2 - 2x < 0$

$\therefore \quad x(x^2 - x - 2) < 0$

Boundary points when

$x(x^2 - x - 2) = 0$

$x(x-2)(x+1) = 0$

i.e. $x = -1, \ 0, \ 2$ **[1]**

so for $\frac{2}{x} > x - 1$

testing points between boundaries: **[1]**

$x = -2; \quad x = \frac{-1}{2}; \quad x = 1; \quad x = 3$

LHS:	LHS:	LHS:	LHS:
$= \frac{2}{-2}$	$= \frac{2}{\frac{-1}{2}}$	$= \frac{2}{1}$	$= \frac{2}{3}$
= -1	= -4	= 2	
RHS:	RHS:	RHS:	RHS:
$= -2 - 1$	$= \frac{-1}{2} - 1$	$= 1 - 1$	$= 3 - 1$
$= -3$	$= -1\frac{1}{2}$	= 0	=2
<LHS	>LHS	<LHS	>LHS
False	True	False	True

$\therefore \quad -1 < x < 0, \quad x > 2$ is soln. **[1]**

(any equivalent method ok)

b) Let $\angle DEA = x$

$\therefore \angle EAD = x$ (base $\angle$'s Isos Δ)

$\therefore \angle CDA = x + x$ (ext. $\angle$ of Δ EDA **[1]**

$= 2x$

$\therefore \angle ABC = 2x$ (op. $\angle$'s ||gram =) **[1]**

$\therefore \angle ABC = 2\angle DEA$

c) $\log 2.25$

$= \log \frac{9}{4}$

$= \log \frac{3^2}{2^2}$

$= \log 3^2 - \log 2^2$ **[1]**

$= 2\log 3 - 2\log 2$

$= 2a + 2b$ **[1]**

d) $y = 3x - 1$ and $x - 2y + 5 = 0$

i) $m_1 = 3$ and $m_2 = \frac{1}{2}$, so

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{3 - \frac{1}{2}}{1 + 3 \times \frac{1}{2}}\right|$ **[1]**

$= \left|\frac{2\frac{1}{2}}{2\frac{1}{2}}\right|$

$= 1$

$\therefore \theta = \tan^{-1}(1)$

$= 45^\circ$ **[1]**

ii) Required equation is given by

$3x - y - 1 + k(x - 2y + 5) = 0$ **[1]**

Substituting for $P(1, -1)$

$3 \times 1 - (-1) - 1 + k(1 - 2 \times (-1) + 5) = 0$

i.e. $3 + k(8) = 0$

So $k = \frac{-3}{8}$ **[1]**

$\therefore$ equation is

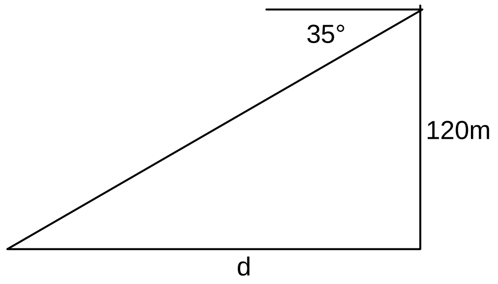
$3x - y - 1 + \left(\frac{-3}{8}\right)(x - 2y + 5) = 0$

$24x - 8y - 8 + (-3x + 6y - 15) = 0$

$\therefore 21x - 2y - 23 = 0$ **[1]**

Question Three:

a)



$\tan 35 = \dfrac{120}{d}$ **[1]**

$d = \dfrac{120}{\tan 35}$

$d = 171.3777608$

$d = 171m$ **[1]**

b) $\alpha + \beta = \dfrac{5}{3}; \quad \alpha\beta = \dfrac{2}{3}$

i) $(\alpha - 1)(\beta - 1)$

$= \alpha\beta - (\alpha + \beta) + 1$

$= \dfrac{5}{3} - \dfrac{2}{3} + 1$

$= 2$ **[1]**

ii) $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$

$= \dfrac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$

$= \dfrac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$ **[1]**

$= \dfrac{\left(5/3\right)^2 - 2 \times 2/3}{\left(2/3\right)^2}$

$= \dfrac{(25 - 12)/9}{4/9}$

$= \dfrac{13}{4}$

$= 3\dfrac{1}{4}$ **[1]**

c) $A(1, -6)$ and $B(-4, 3)$

Ratio $-3:2$ (-ve for external)

$\left(\dfrac{1 \times 2 + (-4) \times (-3)}{-3 + 2}, \dfrac{(-6) \times 2 + 3 \times (-3)}{-3 + 2}\right)$

[1]

$= \left(\dfrac{14}{-1}, \dfrac{-21}{-1}\right)$

$= (-14, 21)$ **[1]**

d) Solve $x^4 - 3x^2 - 4 = 0$

Let $u = x^2$

$\therefore u^2 - 3u - 4 = 0$ **[1]**

$(u + 1)(u - 4) = 0$

i.e. $u = -1, \ 4$

$\therefore x^2 = -1$ (no real solution) **[1]**

or $x^2 = 4$

$\therefore x = \pm 2$ **[1]**

e) Prove

$\dfrac{\cos\theta(\sin\theta + \cos\theta)}{(1 + \sin\theta)(1 - \sin\theta)} = 1 + \tan\theta$

LHS $= \dfrac{\cos\theta(\sin\theta + \cos\theta)}{(1 + \sin\theta)(1 - \sin\theta)}$

$= \dfrac{\cos\theta(\sin\theta + \cos\theta)}{(1 - \sin^2\theta)}$

$= \dfrac{\cos\theta(\sin\theta + \cos\theta)}{\cos^2\theta}$ **[1]**

$= \dfrac{(\sin\theta + \cos\theta)}{\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\cos\theta}$ **[1]**

$= \tan\theta + 1$

= RHS

Question Four:

a) Differentiations:

i) $3x^2 - 5$ **[1]**

ii) $7(5-2x^2)^6(-4x)$

$= -28x(5-2x^2)^6$ **[1]**

iii) $\dfrac{2x}{1+4x}$: $u = 2x, v = 1+4x$; $du = 2; dv = 4$

$\therefore y' = \dfrac{(1+4x)\times 2 - 2x \times 4}{(1+4x)^2}$ **[1]**

$= \dfrac{2+8x-8x}{(1+4x)^2}$

$= \dfrac{2}{(1+4x)^2}$ **[1]**

b) $f(x) = \dfrac{1}{x}$ and $f(x+h) = \dfrac{1}{x+h}$

$\therefore \dfrac{dy}{dx} = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \dfrac{\frac{1}{x+h} - \frac{1}{x}}{h}$ **[1]**

$= \lim_{h \to 0} \dfrac{\frac{x-(x+h)}{x(x+h)}}{h}$

$= \lim_{h \to 0} \dfrac{-h}{hx(x+h))}$ **[1]**

$= \lim_{h \to 0} \dfrac{-1}{x^2+hx}$

$= \dfrac{-1}{x^2}$ **[1]**

c) $y = 4 + 3x - x^2$

i) $y' = 3 - 2x$ **[1]**

so for tangent at $x = 1$: $y' = 1$

and $y = 6$ **[1]**

$\therefore y - 6 = 1(x-1)$

or $0 = x - y + 5$ **[1]**

ii) For distance of $P(2,-1)$ from tangent:

$d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \dfrac{|2-(-1)+5|}{\sqrt{1^2+(-1)^2}}$ **[1]**

$= \dfrac{|8|}{\sqrt{2}}$

$= \dfrac{8\sqrt{2}}{2}$

$= 4\sqrt{2}$ **[1]**

Question Five:

a) $a(x+b)^2+c$

$=ax^2+2abx+b^2+c$

i.e.

$2x^2-5x+7\equiv ax^2+2abx+b^2+c$ **[1]**

$\therefore a=2$ **[1]**

also $2ab=-5$

$\therefore b=-\frac{5}{4}$ **[1]**

and $b^2+c=7$

$\therefore c=7-\left(\frac{-5}{4}\right)^2$

so $c=\frac{87}{16}$

or $c=5\frac{7}{16}$ **[1]**

b)

i) Reflex $\angle AOC=2\times124^\circ$
(angle at centre double angle at circumference) **[1]**

$\therefore \angle AOC=248^\circ$

but $\therefore \angle AOC=x^\circ+\angle DOC$

where $\angle DOC$ is a st.line.

$\therefore 248=x+180$

i.e. $x=68$ **[1]**

ii) $\angle ADC=\angle ABC$ (in same seg. Standing on same arc)

$\therefore \angle ABC=40^\circ$

$\angle ACB=90^\circ$ (angle in semicircle) **[1]**

$\therefore 40+90+x=180$ (Δ sum)

so $x=50$ **[1]**

c) For $x^2+(k+1)x-(k-1)=0$

Distinct real roots when $\Delta>0$.

i.e. $(k+1)^2-4\times1\times-(k-1)>0$

$k^2-2k+1+4k-8>0$

$\therefore k^2+2k-7>0$ **[1]**

Boundaries when $\Delta=0$

i.e. when $k^2+2k-7=0$

$\therefore k=\frac{-(2)\pm\sqrt{2^2-4\times1\times-7}}{2\times1}$

$k=\frac{-2\pm\sqrt{32}}{2}$

$k=\frac{-2\pm\sqrt{32}}{2}$

$k=\frac{-2\pm4\sqrt{2}}{2}$

$k=-1\pm2\sqrt{2}$ **[1]**

Testing k=0 in $k^2+2k-7>0$

LHS= -7

<0

<RHS

i.e. false so 0 is not a solution. **[1]**

$\therefore k<-1-2\sqrt{2}$

or $k>-1+2\sqrt{2}$ **[1]**

Question Six:

a) $P(2,4)$ and $Q(x_1,y_1)$ on $y=x^2$

$\frac{dy}{dx}=2x$

i) At $P(2,4)$:

$\frac{dy}{dx}=4$

So gradient of normal is $\frac{-1}{4}$:

$\therefore (y-4)=\frac{-1}{4}(x-2)$ **[1]**

$4y-16=-x+2$

i.e. $x+4y-18=-x+2$

ii) At $Q(x_1,y_1)$, $\frac{dy}{dx}=2x_1$ but also

$\frac{dy}{dx}=\frac{-1}{4}$

$\therefore 2x_1=\frac{-1}{4}$ **[1]**

i.e. $x_1=\frac{-1}{8}$

$\therefore y_1 = \frac{1}{64}$ so Q is $\left(\frac{-1}{8}, \frac{1}{64}\right)$ **[1]**

Equation of the tangent is:

$\left(y - \frac{1}{64}\right) = \frac{-1}{4}\left(x - \frac{-1}{8}\right)$

$4y - \frac{1}{16} = -x - \frac{1}{8}$

$\therefore 64y - 1 = -16x - 2$

or $16x + 64y + 1 = 0$ **[1]**

b)

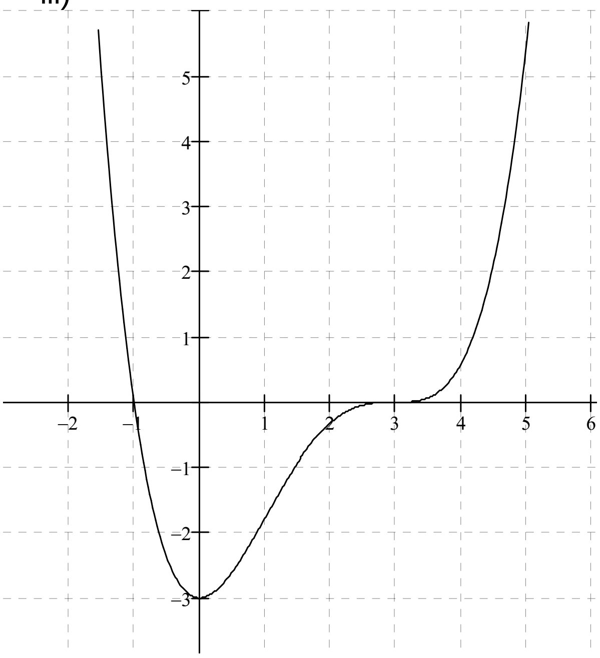
i) $f(x)$ is rising when $f'(x) > 0$

i.e. for $x > 0$ **[1]**

ii) Stat points when $f'(x) = 0$

i.e. when $x = 0$ or $x = 3$ **[1]**

iii)



Correct roots **[1]**

Correct shape **[1]**

c) $y = x^3 + 3x^2 - 9x - 7$

$\therefore y' = 3x^2 + 6x - 9$

and $y'' = 6x + 6$

Stat Points when $y' = 0$:

$\therefore 0 = 3x^2 + 6x - 9$

or $0 = x^2 + 2x - 3$

i.e. $0 = (x+3)(x-1)$

$\therefore x = -3$ or $x = 1$

y-values are:

$x = -3$

$y = (-3)^3 + 3(-3)^2 - 9(-3) - 7$

$y = 20$, so point is $(-3, 20)$

$x = 1$

$y = (1)^3 + 3(1)^2 - 9(1) - 7$

$y = -12$, so point is $(1, -12)$

Nature:

$x = -3$: $y'' = 6(-3) + 6$

i.e. $y'' = -12$ so ccd $\Rightarrow$ max t.p.

$x = 1$: $y'' = 6(1) + 6$

i.e. $y'' = 12$ so ccu $\Rightarrow$ min t.p.

Possible inflection point(s) when $y'' = 0$:

i.e. $0 = 6x + 6$

so $x = -1$

y-value is:

$y = (-1)^3 + 3(-1)^2 - 9(-1) - 7$

$y = 4$, so point is $(-1, 4)$

Testing:

For $x = -1 - \varepsilon$

$y'' = 6(-1 - \varepsilon) + 6$

$y'' = -6\varepsilon$

$y'' < 0$

For $x = -1 + \varepsilon$

$y'' = 6(-1 + \varepsilon) + 6$

$y'' = 6\varepsilon$

$y'' > 0$

$\therefore$ as concavity changes, $(-1, 4)$ is a point of inflection.

Marking:

Finds correct Stat points **[1]**

Nature tests correct **[1]**

Finds Inflection point **[1]**

Tests nature correctly **[1]**

Question Seven:

a) Join AC

Let $\angle CAB = x^\circ$

Now $AB = BC$ (tangents from external point equal)

$\therefore \Delta ABC$ is isosceles

$\therefore \angle ACB = x^\circ$ (base $\angle$'s Isos Δ)

$\therefore \angle ABC = (180-2x)^\circ$ (Δ sum) **[1]**

Also, $\angle ADC = x^\circ$ (angle in alt. seg.)

For $ABCD$ to be cyclic, opposite angles must be supplementary;

i.e. $\angle ADC + \angle ABC = 180^\circ$ **[1]**

i.e. $x + (180-2x) = 180$

which gives $x = 0$ **[1]**

but A and C are separate points, so $x > 0$ **[1]**

$\therefore ABCD$ cannot be cyclic.

b) $\angle CBD = 229^\circ - 187^\circ$

$\therefore \angle CBD = 42^\circ$

Also, $\dfrac{h}{BD} = \tan 8^\circ$

$\therefore BD = h\cot 8^\circ$

and $\dfrac{h}{BC} = \tan 12^\circ$

$\therefore BC = h\cot 12^\circ$

(correct expressions for BC, BD) **[1]**

In Δ CBD:

$CD^2 = BD^2 + BC^2 - 2.BC.BD.\cos\angle CBD$

$400^2 = (h\cot 8^\circ)^2 + (h\cot 12^\circ)^2$

$- 2\times h\cot 8^\circ \times h\cot 12^\circ \times \cos 42^\circ$

(correct substitutions) **[1]**

$$h^2 = \frac{400^2}{\cot^2 8 + \cot^2 12 - 2\cot 8\cot 12\cos 42}$$

$$\therefore h^2 = \frac{160000}{23.00821324}$$

$h^2 = 6954.03847$

$\therefore h = 83.39087762$

$\therefore h = 83$ (nearest m) **[1]**

c) $P(2,4)$

i) $(y - y_1) = m(x - x_1)$ with $P(2,4)$ gives:

$(y-4) = m(x-2)$

$y - 4 = mx - 2m$ **[1]**

or $y = mx - 2m + 4$

ii) For A, $x = 0$, subst in (i) gives

$y = -2m + 4$

$\therefore A(0, 4-2m)$

For B, $y = 0$, subst in (i) gives

$0 = mx - 2m + 4$

or $mx = 2m - 4$

so $x = \dfrac{2m-4}{m}$

$\therefore B\left(\dfrac{2m-4}{m}, 0\right)$

(Correctly shown) **[1]**

iii) Area of ΔAOB is given by

$A = \dfrac{1}{2}bh$ where

$b = \dfrac{2m-4}{m}$ and $h = 4 - 2m$

$\therefore A = \dfrac{1}{2}\times\dfrac{(2m-4)}{m}\times(4-2m)$

i.e. $A = \dfrac{-16+16m-4m^2}{2m}$

or $A = 8 - \dfrac{8}{m} - 2m$ **[1]**

$\therefore \dfrac{dA}{dm} = \dfrac{8}{m^2} - 2$

and $\dfrac{d^2A}{dm^2} = \dfrac{-16}{m^3}$

Min/Max when $\dfrac{dA}{dm} = 0$

i.e. $0 = \dfrac{8}{m^2} - 2$

or $0 = 8 - 2m^2$

$\therefore 2m^2 = 8$

$m^2 = 4$

so $m = \pm 2$ **[1]**

at $m = 2$

$\dfrac{d^2A}{dm^2} = \dfrac{-16}{2^3}$

<0 ccd $\Rightarrow$ max t.p.

at $m = -2$

$\dfrac{d^2A}{dm^2} = \dfrac{-16}{(-2)^3}$

>0 ccu $\Rightarrow$ min t.p. **[1]**

$\therefore m = -2$ is the gradient that gives the least area for ΔAOB