Name: ______________________________

Teacher: ______________________________

Class: ______________________________

FORT STREET HIGH SCHOOL

2007

PRELIMINARY SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: FINAL PRELIMINARY EXAMINATION

Mathematics Extension I

TIME ALLOWED: 1½ HOURS

Outcomes Assessed	Questions	Marks
Demonstrates appropriate mathematical techniques basic algebra, equations and geometry.	1	
Manipulates algebraic expressions to solve problems from the topic areas of functions and co-ordinate geometry.	2	
Manipulates algebraic expressions to solve problems from the topic areas of trigonometry, quadratics, locus and the parabola.	3, 5	
Demonstrates skills in the processes of calculus and applies them appropriately.	4	
Synthesises mathematical solutions to harder problems and communicates them in appropriate form.	6	

Question	1	2	3	4	5	6	Total	%
Marks	/10	/10	/10	/10	/10	/10	/60	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started on a new page

Question 1 (10 marks)

(a) Calculate $\dfrac{171.8}{14.3\times 2.3}$ correct to 2 decimal places. **1**

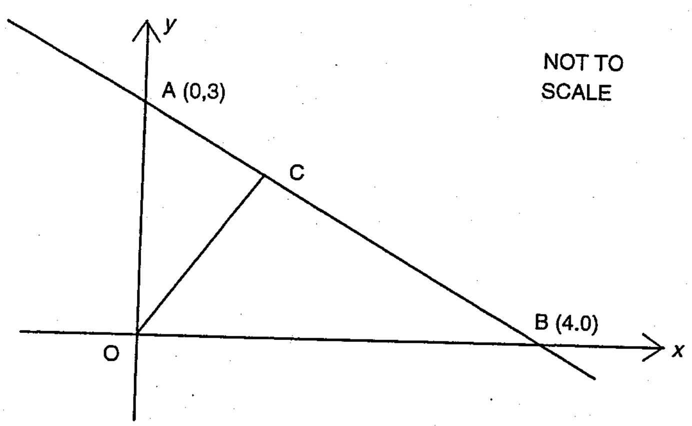
(b) Solve: $\dfrac{4x}{x+1}\le 1$ **3**

(c) Differentiate $(2x^2-5)^7$ **1**

(d) Find x and y if $\dfrac{4^x}{16}=8^{x+y}$ and $2^{2x+y}=128$ **4**

(e) Write down the domain of the function $f(x)=\dfrac{x^2}{x^2-1}$ **1**

Question 2 (10 marks)



In the diagram above AOB is a triangle. The coordinates of O, A and B are (0,0), (0,3) and (4,0) respectively. Point C lies on AB. Copy the diagram.

(a) Show that the equation of AB is $3x+4y-12=0$. **1**

(b) The equation of OC is $4x-3y=0$. Explain why OC is perpendicular to AB. **2**

(c) Prove that $\triangle AOC$ is similar to $\triangle OBC$. **2**

(d) Show that $\dfrac{OC}{AC}=\dfrac{BC}{OC}=\dfrac{4}{3}$ **1**

(e) Hence or otherwise, find the ratio of the areas of triangles AOC and OBC. **2**

(f) Find the coordinates of point which divides AB externally in the ratio 1:3. **2**

Question 3 (10 marks)

(a) The acute angle between the lines $y=(m+2)x$ and $y=mx$ is 45^0.

(i) Show that $\left|\frac{2}{m^2+2m+1}\right|=1$ **2**

(ii) Hence find any values of m. **3**

(b) Solve $\sin\theta=\frac{1}{\sqrt{2}}$ for $0\le\theta\le 2\pi$ **1**

(c) (i) Show that $\cot\theta-\cot 2\theta=\operatorname{cosec}2\theta$. **3**

(ii) Hence find the exact value of $\cot 15^0$ **1**

Question 4 (10 marks)

(a) A function is defined by $f(x)=x^2(2x-3)$.

(i) Find the turning points for the curve $y=f(x)$ and determine their nature. **3**

(ii) Find any points of inflexion. **2**

(iii) Sketch the graph of $y=f(x)$ showing essential features including x intercepts. **2**

(b) Find the equation of the tangent to the curve $y=\frac{x+1}{x-1}$ at the point P (2,3). **3**

Question 5 (10 marks)

(a) For the parabola $8y = x^2 + 2x + 25$:

(i) Find the coordinates of the vertex. **2**

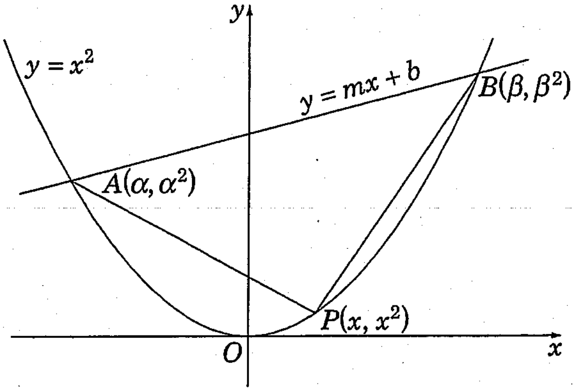
(i) Find the coordinates of the focus. **1**

(ii) Determine the equation of the directrix. **1**

(b) Consider the equation $x^2 + (k+2)x + 4 = 0$. For what values of k does the equation have real roots? **3**

(c) Let α and β be the roots of the equation $x^2 - 5x + 2 = 0$. Find the value of $(\alpha + 1)(\beta + 1)$. **3**

Question 6 (10 marks)



The parabola $y = x^2$ and the line $y = mx + b$ intersect at the points $A(\alpha, \alpha^2)$ and $B(\beta, \beta^2)$ as shown in the diagram.

(a) Explain why $\alpha + \beta = m$ and $\alpha\beta = -b$. **2**

(b) Given that $(\alpha - \beta)^2 + (\alpha^2 - \beta^2)^2 = (\alpha - \beta)^2\left[1 + (\alpha + \beta)^2\right]$, show that the distance

$$AB = \sqrt{(m^2 + 4b)(1 + m^2)}.$$ **3**

(c) The point $P(x, x^2)$ lies in the parabola between A and B. Show that the area of the triangle ABP is given by

$$\frac{1}{2}(mx - x^2 + b)\sqrt{m^2 + 4b}$$ **2**

(Hint: find the perpendicular distance from the point P to the line $y = mx + b$)

(d) The point P in part (c) is chosen so that the area of the triangle ABP is a maximum. Find the coordinates of P in terms of m. **3**

End of examination