Name: ______________________

Teacher: ______________________

Class: ______________________

FORT STREET HIGH SCHOOL

2009

PRELIMINARY SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: FINAL PRELIMINARY EXAMINATION

Mathematics Extension I

TIME ALLOWED: 1½ HOURS

Outcomes Assessed	Questions	Marks
Demonstrates appropriate mathematical techniques algebra, equations and graphs.	1	
Manipulates algebraic expressions to solve problems from the topic areas of calculus and circle geometry.	2	
Applies appropriate techniques to solve problems from the topic areas of co-ordinate geometry, plane geometry and 3D trigonometry.	3	
Applies appropriate techniques to solve problems in trigonometry.	4	
Applies appropriate techniques to solve problems in quadratics, locus and polynomials.	5	

Question	1	2	3	4	5	Total	%
Marks	/17	/13	/15	/15	/15	/75	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started in a new booklet

2009 Final Preliminary Exam

Question 1 (17 marks)

marks

a. Factorise $3x^2 - 5x - 2$ 1

b. Solve $|2x-3| > 5$ 2

c. Find the values of a and b given that $(ax-3)^2 + b \equiv 4x^2 - 12x + 15$ 2

d. A weight oscillates up and down at the end of an elastic string. Its depth below the point of suspension, t seconds after it starts, is $(12 + 3\cos 2\pi t)$ cm. At what time will its depth be 9cm? 2

e. 3

i. What is the period of $y = 3 + \sin 2x$

ii. What is the amplitude of $y = 3 + \sin 2x$

iii. Draw a neat sketch of $y = 3 + \sin 2x$ where $0 \le x \le 2\pi$

f. Draw a sketch of the function $f(x) = \dfrac{1}{x^2 - 1}$, stating the domain and range 4

g. Solve $\dfrac{4x}{x+2} \le 1$ 3

Question 2 (13 marks)

marks

a. The interval AB has end points $A(-4,1)$ and $B(8,7)$. Find the coordinates of the point P that divides the interval AB internally in the ratio $1:3$ 2

b. Differentiate with respect to x 8

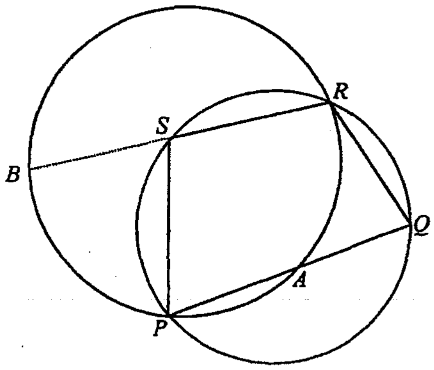
i. $y = \sqrt{x^2 + 5}$

ii. $y = \dfrac{1}{2x^3}$

iii. $y = 2x\sqrt{x}$

iv. $y = \dfrac{x-3}{\sqrt{x}}$

c. *3*



A circle through the points P, A and R cuts RS produced at B. Prove that $AB \parallel SQ$.

Question 3 (15 marks)

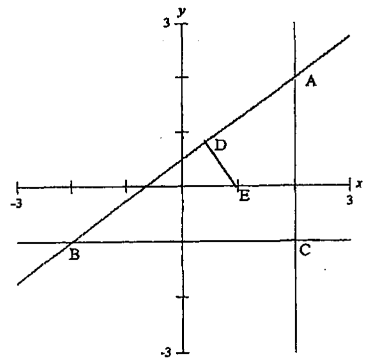
marks

a. $A(2,2), B(-2,-1), C(2,-1)$ are the vertices of a triangle and E is the points $(1,0)$

i. Show that the equation of AB is $3x-4y+2=0$ *2*

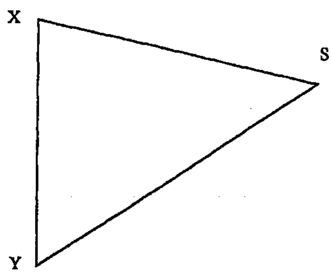
ii. D is the foot of the perpendicular from $E(1,0)$ to AB. Find the equation of ED *2*

iii. Find the coordinates of D. *2*



b. Use the k method to find the equation of the straight line through the point $(-4,-1)$ that passes through the intersection of the line $2x+y-1=0$ and $3x+5y+16=0$. *2*

c. Two lighthouses X and Y, 12 kilometres apart observe a ship S, at the same instant. X and Y are on a north-south line . From X, the ship is on a bearing of $105°$ and from Y the bearing of the ship is $058°$

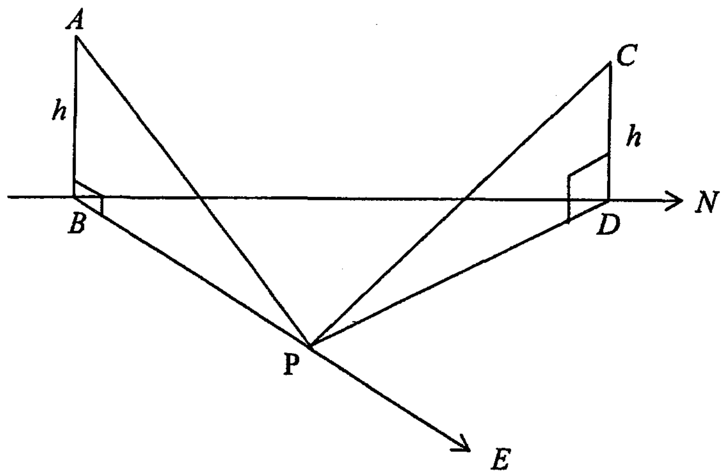


Copy the diagram onto your own paper.

i. Show that $\angle XSY = 47°$ *1*

ii. Find, correct to one decimal place the distance of the ship from the nearer lighthouse. *2*

d. AB and CD are towers of equal height (h). CD is due north of AB. From a point P on the same horizontal plane as the feet B and D of the towers, and bearing due east of the tower AB, the angles of elevation of A and C, the tops of the towers, are $47°$ and $31°$ respectively. If the distance between the towers is $88m$, find the height of the tower to the nearest metre. *4*



Question 4 (15 marks)

marks

a. Solve $\cos 2\alpha = \frac{1}{2}$, where $0 \le \alpha \le 2\pi$ *3*

b.

i. Prove that $\frac{\sin 2x}{1-\cos 2x} = \cot x$ *3*

ii. Hence, or otherwise, obtain a value for $\cot 67\frac{1}{2}°$ *3*

(Rationalise the denominator)

c. Find the general solution of the equation $\cos\left(x - \frac{\pi}{4}\right) = -\frac{1}{2}$. *2*

d. Express $7\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$ where $R > 0$ and $0 \le \alpha \le 360$. *4*
Hence solve the equation $7\cos\theta - \sin\theta = 5$ for $0 \le \theta \le 360$

Question 5 (15 marks)

marks

a. Find the coordinates of the vertex and the focus, and the equation of the directrix of the parabola with equation $x^2 - 10x - 16y - 7 = 0$ *5*

b. Factor and sketch the polynomial $P(x) = x^4 - x^3 - 19x^2 - 11x + 30$, indicating all intercepts with the axes. *5*

c. If α, β, γ are the roots of the equation $2x^3 + 5x - 3 = 0$,find the values of

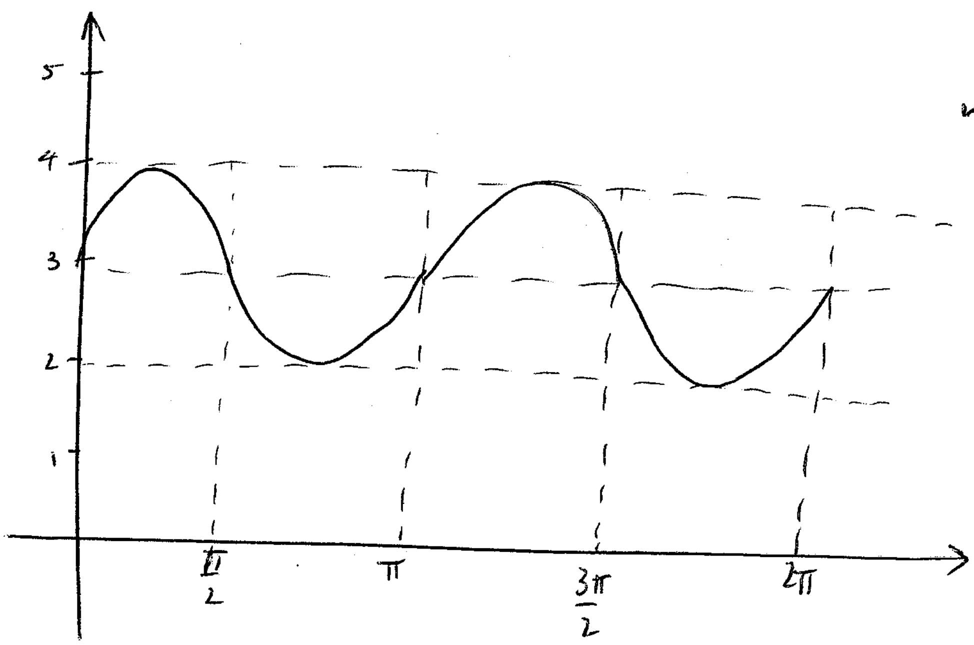
i. $\alpha + \beta + \gamma$ *1*

ii. $\alpha^2 + \beta^2 + \gamma^2$ *2*

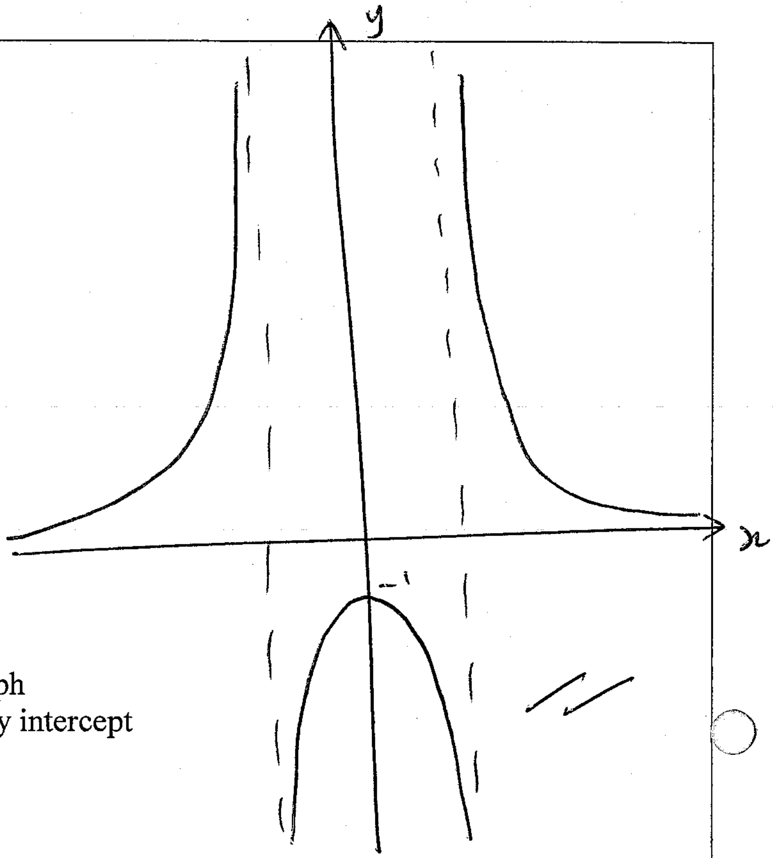
iii. $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$ *2*

Year 11 Extension 1 Assessment 3 Solutions

Q1

a.	$(3x+1)(x-2)$
b.	$\|2x-3\|>5$ $2x-3>5$ $2x>8$ $x>4$ $-(2x-3)>5$ $2x-3<-5$ $2x<-2$ $x<-1$
c.	$(ax-3)^2 \equiv 4x^2-12x+15$ $a^2x^2-6ax+9+b \equiv 4x^2-12x+15$ $a=2$ $9+b=15$ $b=6$
d.	$9=12+3\cos 2\pi t$ $-3=3\cos 2\pi t$ $-1=\cos 2\pi t$ $\pi=2\pi t$ $t=\frac{1}{2}s$
e.	i $T=\pi$ ii $a=1$
c.	

f.



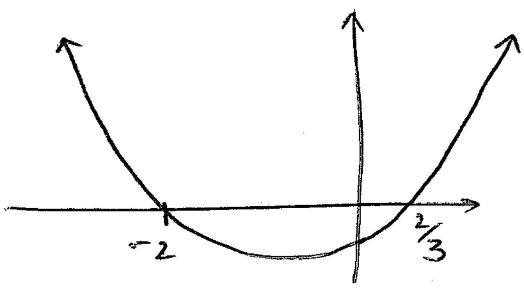
1 mark for the correct shape of the graph
1 mark for the correct asymptotes and y intercept

Domain: All real x, $x \neq \pm 1$

Range : $-1 \geq y > 0$

g.

$$4x(x+2) \leq (x+2)^2$$
$$0 \leq (x+2)^2 - 4x(x+2)$$
$$0 \leq (x+2)(-3x+2)$$
$$(x+2)(3x-2) \leq 0$$
$$-2 < x \leq \frac{2}{3}$$



Q2

a.

$$x = \frac{mx_2 + nx_1}{m+n} \qquad y = \frac{my_2 + ny_1}{m+n}$$
$$x = \frac{1(8) + 3(-4)}{4} \qquad y = \frac{1(7) + 3(1)}{4}$$
$$x = \frac{-4}{4} \qquad y = \frac{10}{4}$$

$(-1, 2\frac{1}{2})$

b.

i. $y' = \frac{1}{2}(x^2+5)^{-\frac{1}{2}}.2x = \frac{x}{\sqrt{x^2+5}}$

ii.

$$y = \frac{1}{2}x^{-3}$$
$$y' = -\frac{3}{2}x^{-4} = -\frac{3}{2x^4}$$

iii.

$$y = 2x\sqrt{x} = 2x^{\frac{3}{2}}$$

$$y' = 2.\frac{3}{2}x^{\frac{1}{2}} = 3\sqrt{x}$$

iv.

$$y' = \frac{\sqrt{x}.1 - (x-3)\frac{1}{2}x^{-\frac{1}{2}}}{x}$$

$$= \frac{\sqrt{x} - \frac{x-3}{2\sqrt{x}}}{x}$$

$$= \frac{x+3}{2x\sqrt{x}}$$

c.

$\angle BRP = \angle BAP$ (angles at the circ. subtended by the same arc)
$\angle SRP = \angle SQA$ (angles at the circ. subtended by the same arc)
$\angle BAP = \angle SQA$
$\angle BAP$ AND $\angle SQA$ are corresponding angles
$\therefore BA \parallel SQ$

Q3

a. P

i. $m = \frac{-1-2}{-2-2} = \frac{3}{4}$

$$y - 2 = \frac{3}{4}(x-2)$$

$$4y - 8 = 3x - 6$$

$$0 = 3x - 4y + 2 \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

ii.

$$m = -\frac{4}{3}$$

$$y - 0 = -\frac{4}{3}(x-1)$$

$$4x + 3y - 4 = 0 \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

iii.

$(1)\times 4-(2)\times -3$

$12x-16y+8-12x-9y+12=0$

$-25y+20=0$

$y=\frac{20}{25}=\frac{4}{5}$ ✓

$4x+3\left(\frac{4}{5}\right)-4=0$

$4x=4-\frac{12}{5}$

$4x=\frac{8}{5}$

$x=\frac{8}{20}=\frac{2}{5}$ ✓

b. $(2x+y-1)+k(3x+5y+16)=0$

at $(-4,-1)$

$(2(-4)-1-1)+k(3(-4)+5(-1)+16)=0$

$-10-k=0$

$k=-10$ ✓

$(2x+y-1)-10(3x+5y+16=0$

$2x+y-1-30x-50y-160=0$

$-28x-49y-161=0$

$4x+7y+23=0$ ✓

c.

i. $\angle YXS=75°$ (supplementary angles)
$\angle XSY=180-75-58=47°$ (angle sum of a triangle) ✓

ii.

$\frac{12}{\sin 47}=\frac{SY}{\sin 75}$

$SY=\frac{12}{\sin 47}\times\sin 75$

$SY=15.85$ ✓

OR 1 mark for correct answer and a mark for a reason

$\frac{12}{\sin 47}=\frac{SX}{\sin 58}$

$SX=\frac{12}{\sin 47}\times\sin 58$ ✓

$SX=13.91$

d.

$$\tan 47 = \frac{h}{PB} \qquad \tan 31 = \frac{h}{PD}$$

$$PB = \frac{h}{\tan 47} \qquad PD = \frac{h}{\tan 31}$$

$$88^2 + PB^2 = PD^2$$

$$88^2 = \left(\frac{h}{\tan 31}\right)^2 - \left(\frac{h}{\tan 47}\right)^2$$

$$88^2 = h^2\left(\frac{1}{\tan^2 31} - \frac{1}{\tan^2 47}\right)$$

$$88^2 = h^2\left(\cot^2 31 - \cot^2 47\right)$$

$$h^2 = \frac{88^2}{\tan^2 59 - \tan^2 43}$$

$$h = 64$$

Q4

a.

$$\cos 2\alpha = \frac{1}{2} \qquad 0 \le \alpha \le 2\pi$$

$$0 \le 2\alpha \le 4\pi$$

$$2\alpha = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$$

$$\alpha = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

b.

i.

$$LHS = \frac{\sin 2x}{1 - \cos 2x}$$

$$= \frac{2\sin x \cos x}{1 - 2\cos^2 x + 1}$$

$$= \frac{2\sin x \cos x}{2 - 2\cos^2 x}$$

$$= \frac{2\sin x \cos x}{2\sin^2 x}$$

$$= \frac{\cos x}{\sin x}$$

$$= \cot x = RHS$$

ii.

$$\cot 67\frac{1}{2}° = \frac{\sin 2\left(67\frac{1}{2}\right)}{1-\cos\left(67\frac{1}{2}\right)}$$

$$= \frac{\sin 135}{1-\cos 135}$$

$$= \frac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}+1}$$

$$= \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$= \sqrt{2}-1$$

c.

$$\cos\left(x-\frac{\pi}{4}\right) = -\frac{1}{2}$$

$$\left(x-\frac{\pi}{4}\right) = 2n\pi \pm \frac{2\pi}{3}$$

$$x = 2n\pi \pm \frac{2\pi}{3} + \frac{\pi}{4}$$

$$x = 2n\pi + \frac{11\pi}{12}, 2n\pi - \frac{5\pi}{12}$$

d.

$$R = \sqrt{7^2+1^2} = \sqrt{50} = 5\sqrt{2}$$

$$\tan\alpha = \frac{1}{7} = 8.1°$$

$$5\sqrt{2}\cos(x+8.1) = 5$$

$$\cos(x+8.1) = \frac{1}{\sqrt{2}}$$

$$x+8.1 = 45$$

$$x = 36.9°$$

Q5

a.

$$x^2 - 10x - 16y - 7 = 0$$

$$x^2 - 10x + 25 = 16y + 7 + 25$$

$$(x-5)^2 = 16y + 32$$

$$(x-5)^2 = 16(y+2)$$

vertex $(5,-2)$
$a = 4$
focus $(5,2)$
directrix $y = -6$

b.

$P(x) = x^4 - x^3 - 19x^2 - 11x + 30$

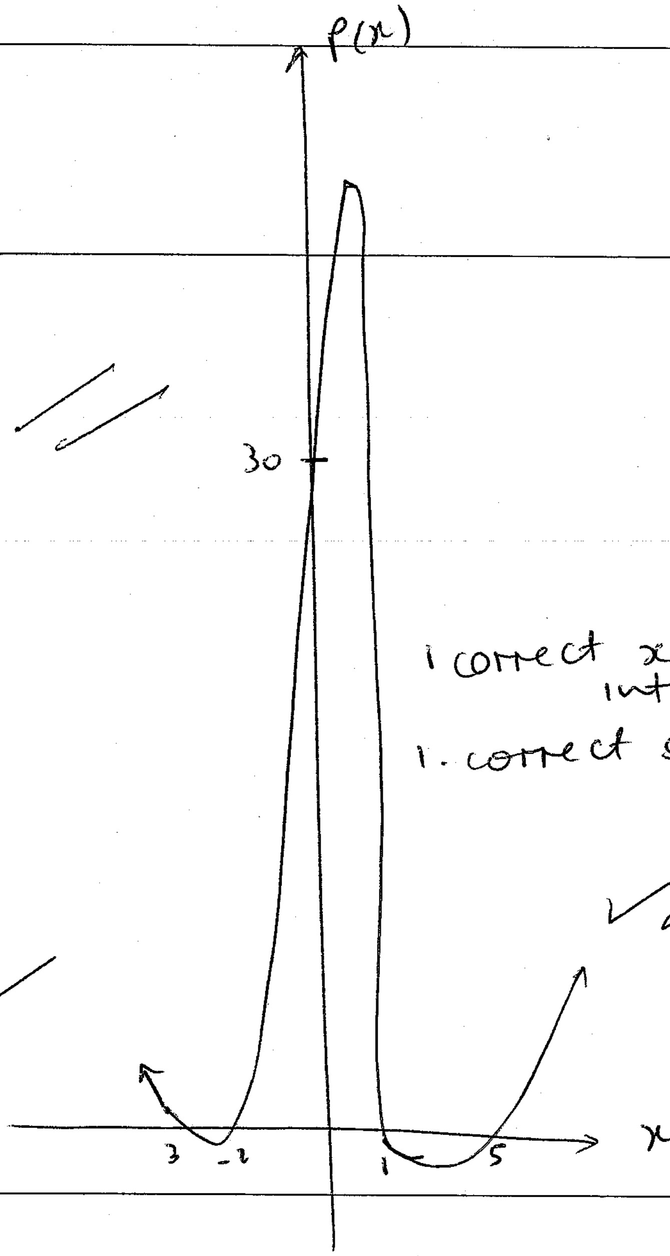
$P(1) = 1 - 1 - 19 - 11 + 30 = 0$

$P(-2) = 16 + 8 - 76 + 22 + 30 = 0$

$\therefore (x-1)$ *and* $(x+2)$ are factors

$$
\begin{array}{r}
x^2 - 2x - 15 \\
x^2 + x - 2 \overline{)\, x^4 - x^3 - 19x^2 - 11x + 30} \\
\underline{-(x^4 + x^3 - 2x^2)} \\
-2x^3 - 17x^2 - 11x + 30 \\
\underline{-(-2x^3 - 2x \quad + 4x)} \\
-15x^2 - 15x + 30 \\
\underline{-(-15x^2 - 15x + 30)} \\
0
\end{array}
$$

$P(x) = (x-1)(x+2)(x^2 - 2x - 15)$
$P(x) = (x-1)(x+2)(x-5)(x+3)$



1 correct x & y intercept
1. correct shape

c. $2x^3 + 0x^2 + 5x - 3 = 0$

i. $\alpha + \beta + \gamma = 0$

ii.

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$

$\alpha^2 + \beta^2 + \gamma^2 = 0 - 2\left(\frac{5}{2}\right)$

$= -5$

iii.

$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

$= \frac{5}{3}$