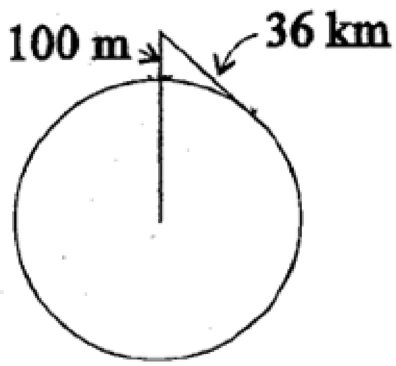
Question 1 (Start a new Booklet) (16 marks)

a) Solve the inequality $\frac{x-1}{x+1} \geq 2$ [3]

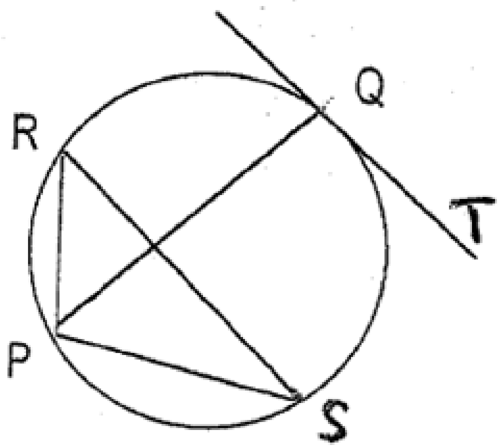
b) The point C $(-6,1)$ divides the interval AB externally in the ratio 3:1. If A has co-ordinates $(0,4)$ find the co-ordinates of B. [3]

c) From a cliff 100 metres high, the straight line distance to the horizon is 36 kilometres. Calculate the radius of the earth to the nearest kilometre. [3]



d) Find the obtuse angle between the lines $\frac{x}{7} + \frac{y}{5} = 1$ and $2x - 3y + 4 = 0$ to the nearest minute. [3]

e) The diagram shows a circle with a chord PQ and another chord RS which is parallel to the tangent at Q. Prove that the chord PQ bisects $\angle$RPS. [4]



Question 2 (Start a new Booklet) (15 marks)

a) Consider the polynomial $P(x) = x^3 - 5x + c$

i) Find the value of c if $x+2$ is a factor of $P(x)$. [1]

ii) For this value of c, find $Q(x)$ such that $P(x) = (x+2).Q(x)$. [2]

b) If $y = P(x)$ is an odd polynomial passing through $(8,-3)$ find the remainder when $y = P(x)$ is divided by ($x+8$). [2]

c) The quadratic equation $x^2 + 6x + c = 0$ has two real roots. These roots have opposite signs and differ by $2n$, where $n \neq 0$.

i) Show that $n^2 = 9 - c$. [3]

ii) Find the set of all possible values of n. [2]

d) If α, β, γ are the roots of the equation $x^3 + 2x^2 - x - 5 = 0$ find

i) $\alpha + \beta + \gamma$ [1]

ii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$ [2]

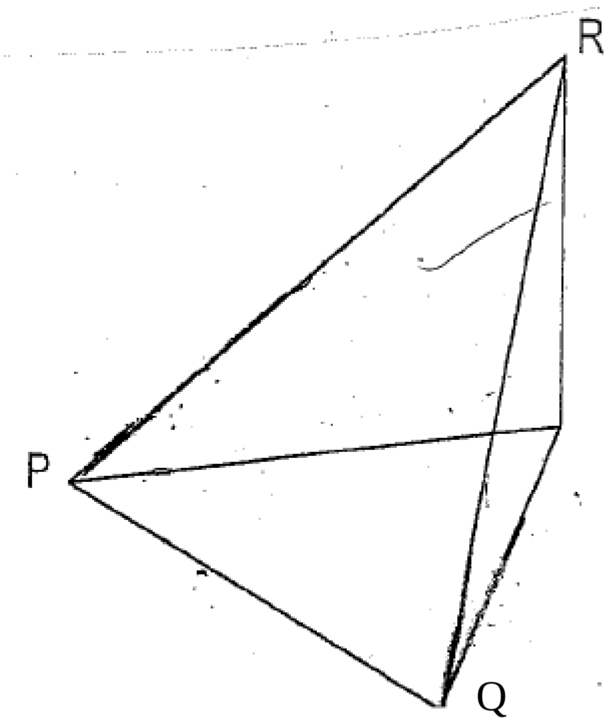
iii) $\alpha^2 + \beta^2 + \gamma^2$. [2]

Question 3 (Start a new Booklet) (13 marks)

a) Prove that $\dfrac{\sec^2 x}{\tan x} = \dfrac{1}{\sin x \cos x}$. [2]

b) Find the exact value of $\cos 15^0$. [2]

c) The diagram shows a mountain of height h metres. From a point P due south the angle of elevation of summit R is found to be 14^0. From another point Q, 7000 metres due east of P, the elevation of summit R is found to be 10^0.



i) Copy the diagram into your booklet and mark on all the information given. [1]

ii) Calculate the height of the mountain to the nearest metre. [4]

d)

i) If $t = \tan\dfrac{x}{2}$ write down expressions for $\sin x$ and $\cos x$ in terms of t. [1]

ii) Hence solve $4\cos x - 7\sin x = 1$ for $0^0 \le x \le 360^0$ correct to the nearest minute. [3]

Question 4 (Start a new Booklet) (12 marks)

a)

i) Show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. [2]

ii) Hence solve the equation $3\sin\theta - 4\sin^3\theta = -1$ for $0 \le \theta \le 2\pi$. [2]

b) Factorise $2\cos x\sin x - 2\sin x - \sqrt{3}\cos x + \sqrt{3}$. Hence find the solutions to $2\cos x\sin x - 2\sin x - \sqrt{3}\cos x + \sqrt{3} = 0$ in general form using radian measure. [3]

c) For the expression $6\cos x - 8\sin x$

i) Express this in the form $R\cos(x+\alpha)$ where α is an acute angle. [2]

Use this relationship to find

ii) the maximum value of $6\cos x - 8\sin x$. [1]

iii) the value of x to the nearest minute, for $0^\circ \le x \le 360^\circ$, when $6\cos x - 8\sin x = 5$. [2]

QUESTION 1.

Comments

(a)

i $\dfrac{x-1}{x+1} \geq 2 \qquad x \neq -1$

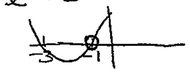
$(x+1)^2 \dfrac{x-1}{x+1} \geq 2(x+1)^2$

$x^2 - 1 \geq 2(x^2+2x+1)$

$x^2-1 \geq 2x^2+4x+2$

$0 \geq x^2+4x+3$ ✓

$(x+3)(x+1) \leq 0$



$-3 \leq x < -1$ ✓

✓ (both sol'ns)

Test: $x=0$

$\dfrac{-1}{1} \geq 2$ False

(b) $A(0,4) \quad B(x,y) \quad C(-6,1)$

$k:l$
$-3:1$

$x = \dfrac{kx_2+lx_1}{k+l} \qquad y = \dfrac{ky_2+ly_1}{k+l}$

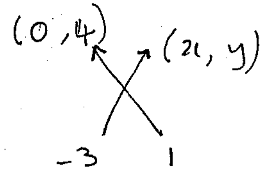
$-6 = \dfrac{-3x+0}{-2} \qquad 1 = \dfrac{-3y+4}{-2}$ ✓

$12 = -3x \qquad -2 = -3y+4$

$x = -4$ ✓ $\qquad 3y = 6$

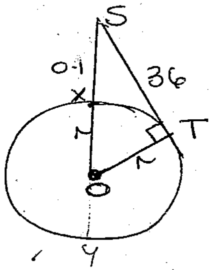
$y = 2$ ✓

$\therefore B(-4,2)$



$x=-6, \ y=1$

(c)



$\angle OTS = 90°$
(angle between radius + tangent) ✓

$r^2 + 36^2 = (r+0{\cdot}1)^2$
(using pythagoras) ✓ (with reason)

$r^2 + 36^2 = r^2 + 0{\cdot}2r + 0{\cdot}01$

$r = \dfrac{36^2 - 0{\cdot}01}{0{\cdot}2}$

$r = 6479{\cdot}95$ ✓

$= 6480$

$\therefore$ the radius of the earth 6480 km. ✓

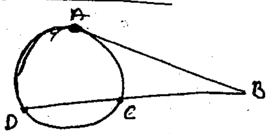
PYTHAGORAS

Note Can also use property

$ST^2 = SX \cdot SY$

$36^2 = 0{\cdot}1(0{\cdot}1 + 2r) \Rightarrow r = 6480.$

Property



$AB^2 = BC \cdot BD$

(d) $\dfrac{x}{7} + \dfrac{y}{5} = 1 \qquad 2x-3y+4=0.$

$5x + 7y - 35 = 0 \qquad y = \dfrac{2x}{3} + \dfrac{4}{3}$

$y = \dfrac{-5x}{7} + \dfrac{35}{7} \qquad m_2 = \dfrac{2}{3}$

$m_1 = -\dfrac{5}{7}$

$\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

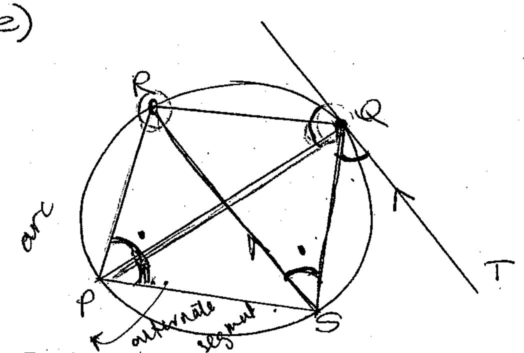
$= \left|\dfrac{-\frac{5}{7} - \frac{2}{3}}{1 - \frac{5}{7}\times\frac{2}{3}}\right|$ ✓ Correct substitution

$= \left|\dfrac{-\frac{29}{21}}{1 - \frac{10}{21}}\right|$

$\theta = 69°14'$ ✓ acute angle

$\therefore$ Obtuse angle $= 110°46'$. ✓ Obtuse

(e)



By construction: RQ and SQ

Join RQ & SQ.

$\angle TQS = \angle QPS$ ($\angle$ between tangent + chord equals $\angle$ in alternate segment) ✓

$\angle TQS = \angle QSR$ (alt. $\angle$'s, $QT \parallel RS$ given) ✓

$\angle QSR = \angle RPQ$ ($\angle$'s standing on the same arc PS) ✓

$\therefore \angle TQS = \angle RPQ$ (both equal to $\angle QSR$) ✓

$\therefore$ PQ bisects $\angle RPS$.

Question 2.

a) i) $P(x) = x^3 - 5x + C$

$P(-2) = -8 + 10 + C = 0$ as $(x+2)$ is a factor.

$\therefore C = -2$

ii) $\therefore P(x) = x^3 - 5x - 2$ OR

$$
\begin{array}{r}
x^2 - 2x - 1 \\
x+2 \overline{)\, x^3 - 5x - 2} \\
x^3 + 2x^2 \quad - \\
\hline
-2x^2 - 5x - 2 \\
-2x^2 - 4x \quad - \\
\hline
-x - 2 \\
-x - 2 \\
\hline
0
\end{array}
$$

✓ division for process

$\therefore P(x) = (x+2)(x^2 - 2x - 1)$

$\therefore \underline{Q(x) = x^2 - 2x - 1.}$

(b) $y = P(x)$

thro' $(8, -3) \Rightarrow P(8) = -3$

for an odd fn $P(x) = -P(-x)$

$\therefore -P(-8) = -3$

$\therefore P(-8) = 3$

relationship

The remainder when $P(x)$ is divided by $(x+8)$ is given by $P(-8)$.

$\therefore$ remainder $= 3$.

(c) $x^2 + 6x + c = 0$.

$\therefore a = 1,\ b = 6,\ c = c$

let the roots be $\alpha,\ \alpha + 2n$

now, $\alpha + \alpha + 2n = \frac{-b}{a}$

$2\alpha + 2n = -6$

$\alpha + n = -3$

$\underline{\alpha = -3 - n}$

$p(x) = x^3 - 5x - 2$

$p(x) = (x+2)(x^2 + bx - 1)$

$2b - 1 = -5$

$2b = -4$

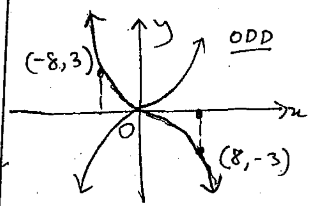
$b = -2$

$\therefore p(x) = (x+2)(x^2 - 2x \ldots$

OR use Remainder theorem:

ODD: reflected on the ~~origin~~

Hence: $y = 3$



Q2 (c) (cont'd) $x^2 + 6x + c = 0$

Also $\alpha(\alpha + 2n) = \frac{c}{a}$

$(-3-n)(-3-n+2n) = c$

$(-3-n)(-3+n) = c$

$9 - n^2 = c$

$\therefore \underline{n^2 = 9 - c}$ as req'd.

$c = 9 - n^2$

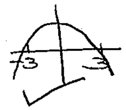
ii) Since roots are opposite in sign $\Rightarrow$ product is $-$ve.

ie $c < 0$ reasoning.

$\therefore 9 - n^2 < 0$ (from above)

$(3-n)(3+n) < 0$

$\{n : n < -3,\ n > 3.\}$



(d) $x^3 + 2x^2 - x - 5 = 0$.

$a = 1 \quad b = 2 \quad c = -1 \quad d = -5$.

i) $\alpha + \beta + \gamma = \frac{-b}{a}$

$= \underline{-2}$

ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

$= \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

$= \frac{-1}{5}$

$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = -1$

$\alpha\beta\gamma = \frac{-d}{a} = 5$

iii) $\alpha^2 + \beta^2 + \gamma^2$

$= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$

$= 4 - 2 \times -1$

$= \underline{6.}$

$n = \pm\sqrt{9 - c}$

QUESTION 3

a) $\frac{\sec^2 x}{\tan x} = \frac{1}{\sin x \cos x}$

$$\text{L.H.S} = \frac{1}{\cos^2 x} \times \frac{1}{\tan x}$$ ✓ substitution

$$= \frac{1}{\cos^2 x} \times \frac{\cos x}{\sin x}$$ ✓ simplifying.

$$= \frac{1}{\cos x \sin x}$$

$$= \text{RHS.}$$

b) $\cos 15° = \cos(45° - 30°)$

$$= \cos 45° \cos 30° + \sin 45° \sin 30°$$ ✓ expansion.

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} \left(\frac{\times \sqrt{2}}{\sqrt{2}}\right)$$

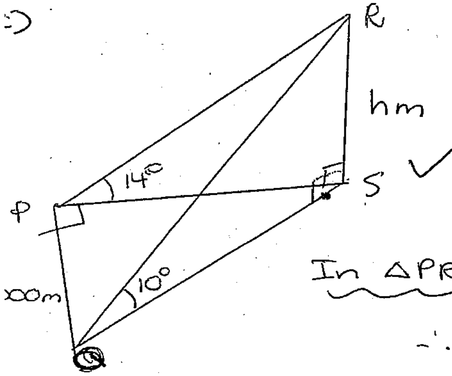
$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$ ✓ either.

$$\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{(\sqrt{3}+1)}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

c)



✓ all info on diagram.

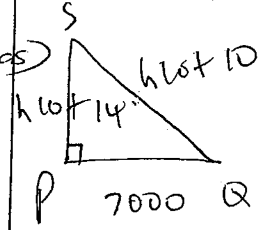
In $\triangle PRS$: $\tan 14 = \frac{h}{PS}$

$$\therefore PS = \frac{h}{\tan 14}$$

In $\triangle RQS$: $\tan 10 = \frac{h}{QS}$

$$QS = \frac{h}{\tan 10}$$ ✓ expression for PS, QS.

In $\triangle PQS$



$$QS^2 = 7000^2 + PS^2$$ (using pythagoras)

$$\frac{h^2}{\tan^2 10} = 7000^2 + \frac{h^2}{\tan^2 14}$$ ✓

$$\frac{h^2}{\tan^2 10} - \frac{h^2}{\tan^2 14} = 7000^2$$

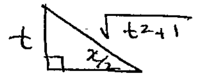
$$h^2\left(\frac{1}{\tan^2 10} - \frac{1}{\tan^2 14}\right) = 7000^2$$

$$h^2 = \frac{7000^2}{\frac{1}{\tan^2 10} - \frac{1}{\tan^2 14}}$$ ✓ for expression in h^2

$$h = 1745.8\ldots$$

$\therefore$ the height of mountain is 1746 m (nearest m). ✓ answer

(d) $t = \tan\frac{x}{2}$



$$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$$

* $\sin x = \frac{2t}{1+t^2}$
* $\cos x = \frac{1-t^2}{1+t^2}$

← both ✓

$$4\cos x - 7\sin x = 1 \qquad 0° \le x \le 360°$$

$$4\left(\frac{1-t^2}{1+t^2}\right) - 7\left(\frac{2t}{1+t^2}\right) = 1$$

$$4 - 4t^2 - 14t = 1 + t^2$$

$$5t^2 + 14t - 3 = 0$$ ✓ quadratic eqn in t.

$$t = \frac{-14 \pm \sqrt{14^2 - 4\times5\times-3}}{10}$$

$$= \frac{-14 \pm 16}{10}$$

$$t = -3, \frac{1}{5}$$ ✓ both solns for t.

$5t$ -1

t 3

$$5t^2 + 14t - 3 = 0$$

$$(5t-1)(t+3) = 0$$

$0 \le x < 360°$

$0° \le \frac{x}{2} \le 180°$

$\tan\frac{x}{2} = -3$ | S A / T C

acute $\frac{x}{2} = 71°34'$

$\frac{x}{2} = 180 - 71°34'$

$\frac{x}{2} = 108°26'$

$x = 216°52'$

$\tan\frac{x}{2} = \frac{1}{5}$ | S A / T C

$\frac{x}{2} = 11°19'$

$x = 22°37'$

$\therefore x = 22°37', 216°52'$

QUESTION 4

a) (i) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$L.H.S = \sin(2\theta + \theta)$

$= \sin 2\theta\cos\theta + \sin\theta\cos 2\theta$

$= 2\sin\theta\cos^2\theta + \sin\theta(\cos^2\theta - \sin^2\theta)$

$= 2\sin\theta\cos^2\theta + \sin\theta(1-\sin^2\theta) - \sin^3\theta$

$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - \sin^3\theta - \sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$

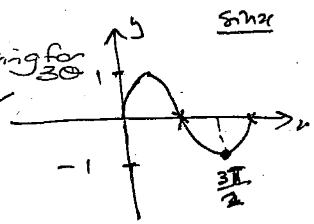
$= RHS$

ii) $3\sin\theta - 4\sin^3\theta = -1$ $\qquad 0 \le 3\theta \le 6\pi$

$\sin 3\theta = -1$

$3\theta = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}$ solving for 3θ

$\theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$.



b) $2\cos x\sin x - 2\sin x - \sqrt{3}\cos x + \sqrt{3}$

$= 2\sin x(\cos x - 1) - \sqrt{3}(\cos x - 1)$

$= (2\sin x - \sqrt{3})(\cos x - 1)$ correct factors

$\therefore (2\sin x - \sqrt{3})(\cos x - 1) = 0$.

$2\sin x - \sqrt{3} = 0$

$\sin x = \frac{\sqrt{3}}{2}$

$\sin x = \sin\frac{\pi}{3}$

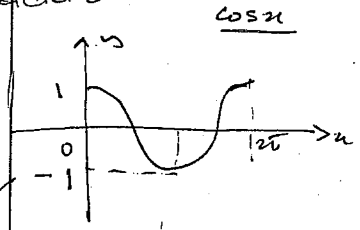
$x = n\pi + (-1)^n \frac{\pi}{3}$

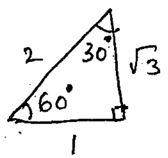
$\cos x = 1$

$\cos x = 0, 2\pi$

$x = 2n\pi$

$x = 2\pi n$



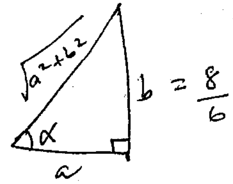


Question 4 (cont'd.)

(c) (i) $6\cos x - 8\sin x \equiv R\cos(x+\alpha)$

$R = \sqrt{6^2+8^2}$

$= 10$.



$\tan\alpha = 53°8'$

$\therefore 6\cos x - 8\sin x \equiv 10\cos(x+\alpha)$

$\frac{3}{5}\cos x - \frac{4}{5}\sin x \equiv \cos(x+\alpha)$

$\equiv \cos x\cos\alpha - \sin\alpha\cos x$

$\therefore \left.\begin{array}{l}\cos\alpha = \frac{3}{5} \\ \sin\alpha = \frac{4}{5}\end{array}\right\} \Rightarrow \tan\alpha = \frac{4}{3}$

$\alpha = 53°8'$ resolving α.

$\therefore 6\cos x - 8\sin x \equiv 10\cos(x + 53°8')$

ii) Max value $= 10$

iii) $6\cos x - 8\sin x = 5$

$\therefore 10\cos(x+53°8') = 5$

$\cos(x+53°8') = \frac{1}{2}$.

acute $(x+53°8') = 60°$

$x + 53°8' = 60°, 300°$

$x = 6°52', 246°52'$